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MORE POWER AND LESS LOADS IN WIND FARMS: "HEAT AND FLUX'

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Abstract

We consider a farm as a single energy extracting body instead of a superposition of individual energy extractors i.e. wind turbines. As a result we found two new hypotheses called Heat and Flux. Both hypotheses reveal that the classical operation of turbines in a wind farm at the Lanchester-Betz optimum does not lead to maximum farm output. However, when the turbines at the windward side of the farm are operated below their optimum, then the power of the turbines under the lee increases in such a way that the net farm production increases slightly. Next to this production advantage of Heat and Flux operation there is also a loading advantage. The average axial loading of the upwind turbines of a farm is reduced in a 'Heat and Flux'-farm. As a result those turbines generate less turbines so that the fatigue loads of the downwind turbines reduce too. The results were confirmed by in a boundary layer tunnel by means of differential measurements between a 'Heat and Flux'-farm and a classical farm.

Keywords: wind turbine farm, axial induction, production, wake, interference

1. Introduction

Heat and Flux refers to a new concept to increase the production and to reduce the loads of wind turbines in a farm. This is becoming of increasing importance since wind power increases by about 30% annually and most new turbines are installed in farms. Recent work [1] emphasises the importance even more: when 6 GW of wind power is installed offshore in an area of 200 times 50 km, a resource reduction in the order of a metre per second wind speed at 100m height may turn up due to the roughness change of the sea.
Compared to our earlier paper [2] we derive the values for the axial induction for a cascade of turbines under the same assumptions as made by Lanchester and Betz. Furthermore we disclose a set of differential wind farm measurements in the boundary layer tunnel. By using the method of installing two wind turbine farms in the tunnel and carrying out differential experiments between the two farms we obtained much more reliable results. This is described in chapter 3. In chapter 4 we finally present our conclusion and the implications we can foresee for future wind farms.

2. Physics

We will show that the classical thought that wind turbines operated at the Lanchester-Betz limit [3,4] yield maximum production is not valid for wind turbines in a wind farm. Instead, we will show that the power of a farm reaches its maximum when turbines at the windward side of a farm are operated below the Lanchester-Betz limit. This will be explained by two different physical mechanisms: Heat and Flux. In literature [5] it can be found that the output of a wind farm can be enhanced by reducing the tip speed ratios at the upwind side of a farm. Simulation results have been the argument for this conclusion from 1988. This paper is valuable since it was the first publication of the idea to optimise a group of wind turbines differently than solitary turbines. However we have three physical arguments that clarify that the essential parameter is the axial induction factor and not the tip speed ratio.

2.1 Heat

By the end of the year 2000, at ECN, it was concluded that actuator discs extract 50% more kinetic energy from the wind than what they extract as useful power [6]. This was derived for the theoretical situation that losses due to 'technical imperfections' are excluded. Examples of such technical imperfections are the aerodynamic drag of the airfoils, losses in the bearings, losses in the generator or in the gear box. The classic thought is that the useful power equals about the extracted power, in other words, the textbooks on wind turbines describe the process of converting kinetic energy from the wind into mechanical energy in the rotor as a process without losses. The new insight is that an actuator disc extracts about 50% more kinetic energy from the wind for fundamental reasons. The additional 50% are dissipated as heat in the wake behind the disc.

The heat loss is not always 50% of the useful power, this percentage only holds for the Lanchester-Betz limit. For the more general situation it was derived in [6, 7] that the efficiency of an actuator disc

$$\eta = \frac{useful\ power}{used\ power}, \qquad (1)$$

wherein the useful power is the power that is made available by the actuator disc. The 'useful power' corresponds to the electrical power produced by a wind turbines, which does not have the 'technical imperfections' mentioned above. The used power is the total kinetic power decrease in the wind that results from the conversion process. The used power equals the useful power + the dissipated heat. In [6] it is shown that the efficiency depends on the axial induction factor a by

$$\eta = 1 - a. \qquad (2)$$

In this context it is relevant to mention that most literature introduces $C_p = 4a(1-a)^2$ as the efficiency. We think that this is not correct, C_p is the fraction of useful power extracted compared to the kinetic power that would flow through a disc without induction. So, it is not the efficiency, but a conversion factor for a process that converts without losses kinetic energy from the wind into mechanic energy of the rotor.

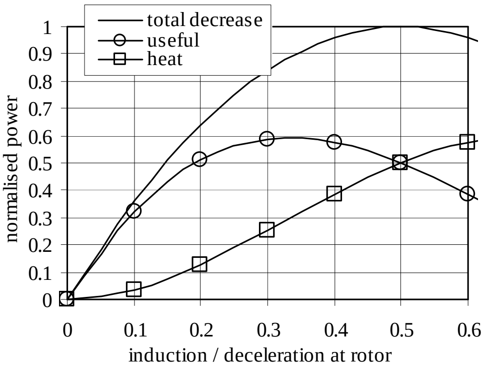

Figure 1: the classical Lanchester-Betz curve, our relation for the heat loss and the total decrease of kinetic energy in the wind.

Figure 1 shows three curves as a function of the induction factor a. The curve with the circles shows the useful power $4a(1-a)^2$ according to the classical Lanchester-Betz relation, which reaches its maximum of 16/27 (≈0.59) for a = 1/3. The lower curve shows the newly discovered effect of heat dissipation. It can be seen that the dissipated heat increases rapidly with increasing a around the Lanchester-Betz optimum. The upper curve shows the total amount of extracted kinetic energy from the wind by the disc. It can be seen that the useful power hardly decreases, when a becomes less, but not much less, than 1/3. However at the same time, the wasted heat reduces much, so that the disc would also extract less kinetic energy from the wind.

For solitary turbines the effect is of no practical relevance, for wind farms however, the situation is different. The waste of kinetic energy by converting it partly into heat is, of course, disadvantageous for the turbines under the lee.

In a farm it is therefore important that turbines at the windward side do not only extract much energy. The turbines should also dissipate little energy kinetic energy in the wind into heat, since this is a loss for the downwind turbines. In other words, the turbines should capture much energy but also at a high efficiency. Since the efficiency is given by 1-a, the farm output can be improved by simply reducing a at the upwind side.

As a result the upwind turbines will produce slightly less while the turbines under the lee will produce more. The average axial loading of the upwind turbines decreases. Those upwind turbines experienced the highest average loading in the farm, so the reduction makes the loading of the turbines in the farm more uniform. Furthermore the upwind turbines at a lower axial induction factor will generate a less pronounced wake and thus also less turbulence. This is favourable for the downwind turbines of which the fatigue loading is mainly caused by generated turbulence.

2.2 Flux

In 2002, further analysis of the Heat-theory confirmed the heat loss, but also showed that there were two other relevant effects, which both are in short denoted by the term 'flux'. Flux was discovered by running computer simulation with a model of a cascade of turbines like in figure 2, wherein the 'heat loss' could be switched off. In that case the two turbine together still produced more when the induction factor of the upwind turbines was lowered below the Lanchester-Betz optimum. Altogether we think that three effects given reason to decrease the axial induction factor at the upwind side of wind farms.

1. The above explained 'heat',
2. We add the index 1 to the upwind turbine in figure 2 and index 2 to the downwind turbines. When the induction factor of upwind turbine in figure 2 is reduced slightly (by $-\Delta a_1$) from its optimum, the power will hardly decrease since we are at the top of the curve 'useful' in figure 1, where $dP_1/da_1 = 0$. The wind speed in the wake of those wind turbines will rise by $d[V(1\text{-}2a_1)]/da_1 \cdot \text{-}\Delta a_1 = 2V \cdot \Delta a_1$, and turbines under lee will take advantage of this. Assuming that the power is proportional to the cube of the wind speed, the power of the second turbine will rise by $P_2 = P_2(V_2) - P_2(V_2+\Delta V_2) \approx 6 \cdot \Delta a_1 \cdot P_2(V_2)$.
3. With larger values of a, the expansion of the stream tube enclosing the rotor increases and therefore the cross section of the wake is enlarged. The larger the wakes of the upwind turbines, the larger the probability that wind turbines under the lee are located in those wakes (which causes production losses).

The 'flux'-effect 2 is illustrated by figure 2 for the situation of two turbines on a line parallel to the wind. The upper situation corresponds to the classical operation of both turbines at the Lanchester-Betz limit. The lower picture is the situation at 'Heat and Flux'-operation, where the axial induction of the upwind turbine is reduced. It can be seen that the resources of the second turbines are better in the lower half of figure 2 and that the wake of upwind turbine is smaller in this case. Figure 3 shows the outputs of the actuator discs as a function of the axial induction factor of the upwind turbine. The curve of the upwind disc (P_1) follows the classical Lanchester-Betz relation. The curve for the second turbine (P_2) is also a function of the axial induction of the upwind disc, since this influences its resources. The second disc always operates at its optimum at $a_2 = 1/3$. The total production of the turbines is denoted by the curve P_{tot}. This total equals P_1+P_2 and reaches its optimum for a_1=1/5 for the upwind turbine. Then it exceeds the useful power in the classical situation by 4.1%. The numerical result is valid for the situation that momentum flow into the wake from the undisturbed flow outside is neglected and that the wake of the first turbine is fully developed before it enters the pressure field of the second turbine. In appendix A we show that in the theoretical situation analogous to Froude's actuator disc model, the axial induction for the most upwind turbine in a row of 2 turbines is 1/5, for a row of 3 turbines it is 1/7 and for a row of n turbines it is $1/(2n+1)$.

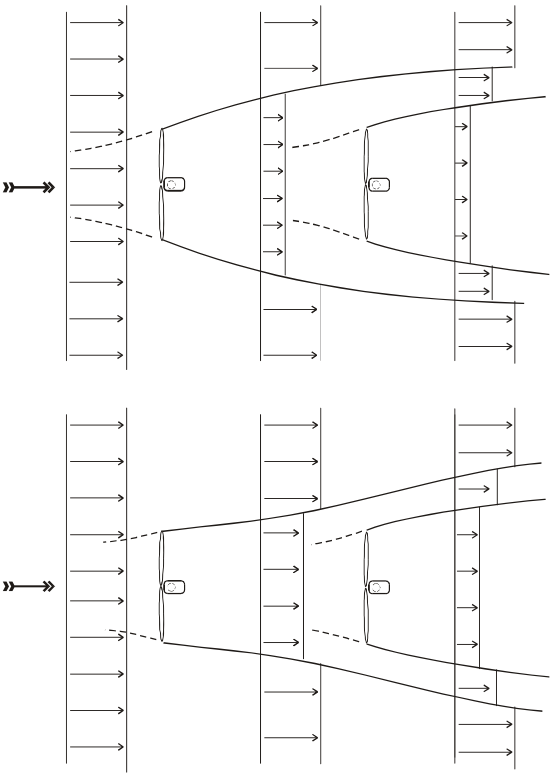

Figure 2: schematic drawing of the classical situation above and the 'heat and flux'-situation below [3]. In the latter case the induction of the windward turbine is below the Lanchester-Betz optimum. The result is a higher net output at less loads.

2.3 Heat and Flux

The new insights are denoted shortly by 'Heat and Flux'. The three effects summarised in chapter 2 all demand the same measure: reduction of the axial induction at the windward side of wind farms. This measure can be implemented easily in the control algorithms of both the turbines and the farm. The potential of Heat and Flux is such that ECN initiated several projects to further develop the physical understanding, to implement the effects in wind farm design codes and to test the hypotheses experimentally. Those projects are supported by Essent, Novem and Shell. Heat and Flux has effects on turbine control, on turbines rotors, on the layout of farms and on the control of farms. ECN has applied for a patent on those characteristics. The next chapter presents experimental results.

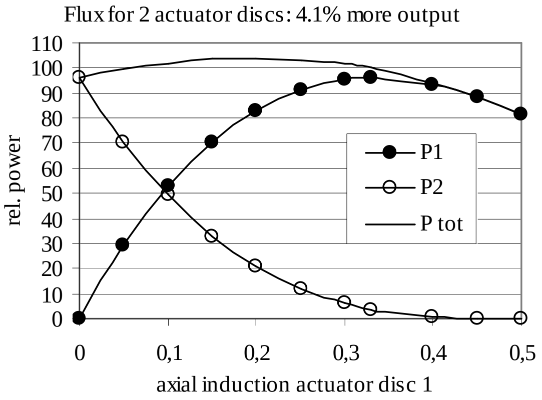

Figure 3: Two actuator discs are positioned on a line parallel to the wind. The downwind disc is fully in the wake of the upwind disc. The combination reaches maximum output when the axial induction factor of the upwind disc is 1/5 and that of the downwind one is 1/3. Then the output is 4% higher compared to the situation that both discs are operated at the Lanchester-Betz optimum (a=1/3).

3. Wind Tunnel Experiment

3.1 The Wind Tunnel

In order to study the effect of Heat and Flux in practice, we set-up an experiment with a model farm in the wind tunnel, see figure 4. The measurements took place in the boundary layer tunnel of TNO at Apeldoorn, the Netherlands. The boundary layer tunnel has a test section of over 10 metres length and a cross section of 3 metres width by 2 metres height. The wind speed can be adjusted between 1 and 13 m/s. The tunnel represented offshore conditions scaled by a factor of 400. The unscaled roughness length is about 0.1-0.2mm, therefore it had to be 0.00025-0.0005mm in the tunnel. With a traversing hot wire system, velocity profiles in the tunnel were measured and used to estimate z_0. The best fit occurred for z_0 = 0.00024mm, so the boundary layer profile was scaled correctly. If we estimate the turbulence intensity I by using $I = 1/\ln(z/z_0)$, we find 7.2%, which is rather well corresponding with the value of 8.5% ± 1% which was the experimental result of the hotwire which was sampled at 200Hz. For on scale offshore conditions an often found empirical value for the turbulence is 6%. Both values were determined for the turbine hub height of 265mm. The flow in the wind tunnel differs from the scaled flow in the planetary boundary layer in some aspects. One is that there is a negative pressure gradient in the tunnel in flow direction. This pressure gradient is created by the fan of the tunnel and is required to drive the flow. This implies that the important meteorological characteristic that the shear stress is not a function of height is not valid in the tunnel. In the case of an empty wind tunnel, the shear stress in the tunnel will decrease approximately proportional with height to zero at half tunnel height. This will be different when a wind farm is installed. The non constant shear and the acceleration of the air due to the pressure gradient will causes that the velocity profile in the tunnel will be less steep.

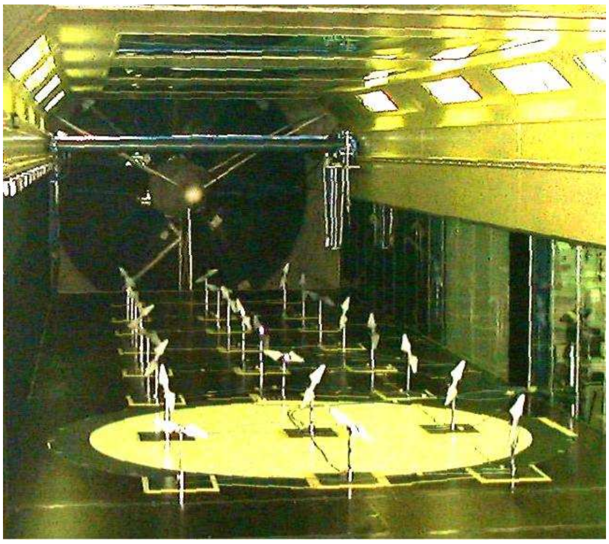

Figure 4: In the boundary layer tunnel of TNO, first measurements with a 1:400 scaled model of a wind farm confirmed Heat and Flux.

3.2 The Model Turbines

ECN designed 30 turbines of 25 cm diameter which should represent 1:400 scaled models of commercial wind turbines of 100m diameter and 100m hub height, see figure 5. The small turbines will have low Reynolds number and especially for Reynolds numbers below 40.000 the lift and drag characteristics of airfoils become very dependent on it. To keep the chord-based Reynolds numbers above this critical value the minimum diameter required was 25 cm for the case of a 2-bladed rotor. We chose the symmetric NACA 0012 airfoil at the blade root and the 0009 airfoil over the remainder of the span, since this profile has low Reynolds dependency at low speed. Due to the relatively high aerodynamic drag of small airfoils, the design tip speed ratio was low: about 5.5, compared to about 7 in the field. This lower tip speed ratio means that the chord has to be enlarged by a factor of about $(7/5.5)^2$ to get the same lift. And to keep the lift over drag ratio for this low Reynolds numbers situation acceptable we chose $c_l = 0.6$, where $c_l/c_d = 17$. For $c_l = 0.7$, c_l/c_d drops already to 12. Large turbines with Reynolds number above 300.000 reach c_l/c_d-values above 100 for c_l=1.2. This means that we had to extend the chord by another factor 1.2/0.6 = 2. Therefore an aerodynamically correctly scaled model of a large commercial rotor geometrically rather different. The rotors were made of glass fibre reinforced plastic, which was moulded in an aluminium mould. Using this fabrication method guaranteed precisely equal and highly accurate rotors. We produced rotors with 5 pitch different angles varying from -2.5° to +7.5°, step +2.5°. Positive numbers refer to the vane direction. Adjusting the pitch can be carried out by changing the rotor. Rotors with pitch angle 0° were designed to operate close to the Lanchester-Betz limit, the rotors with larger angles were meant to serve 'Heat and Flux'-operation. The -2.5° was meant to enable anti-Heat and Flux experiments. Using these rotors at the upwind side would give more than normal axial induction and should lead to a large decrease of production in the entire farm according the model.

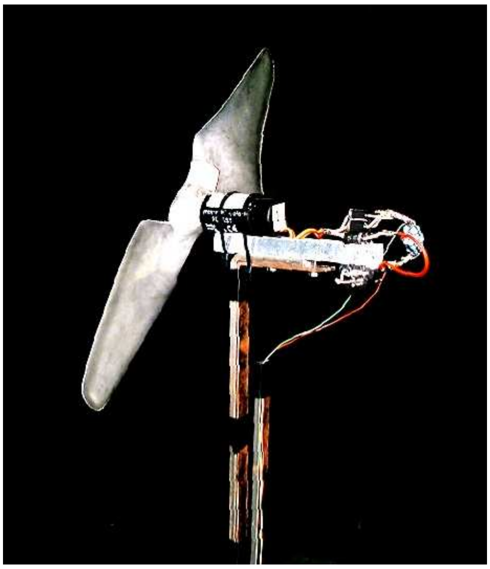

Figure 5. Turbine model, 25cm diameter, 25cm hub height, NACA 0009 airfoils, pitch between -2.5° and +7.5° adjustable. $C_{p,max} \approx 0.30$.

The 30 turbines are all equipped with three phase generators. The output power is rectified with low voltage (0.2 V) Scottky diodes, then it is fed via an adjustable number of diodes to a precisely known load resistor. The diodes are rated 6A, while the maximum operating current is only 2A. As a result they hardly change of operating temperature so that their characteristics are stable during the experiments. This diode control gives us low torque at low generator speed since the diodes are not conducting. Therefore the rotors, with their low starting torque, can easily speed up the generators. When a certain speed is reached, the diodes start conducting and the generator is loaded. We can adjust the number of diodes to obtain the proper rotation speed. From most turbines we measure only a single signal: the voltage over the high precision resistor. It is measured at 16 Hz. From this signal we derive 4 different physical quantities, by applying previously determined calibration curves. The 4 physical quantities are the generator torque, the generator speed, the undisturbed wind speed at the rotor and finally the axial force on the rotor. The last two relations depend on the pitch angle of the rotor so we determined a calibration curve for each of the five different rotors. For 6 turbines we also measure the axial forces by strain gauges at the foot of the towers. The error in the axial force is rather high due to the uncertainty about the vertical position where the aerodynamic force acts on the rotor: wind shear will cause the upper half of the rotor to be loaded more than the lower half. We estimated that this could be maximally 2 cm above the hub. The strain gauges were installed about 20 cm

quantity	unit	accuracy
generator voltage U	[V]	1% ± 0.0024
generator torque Q	[mNm]	3% ± 0.25
generator speed ω	[rpm]	1% ± 50
axial force F_{ax}	[N]	5-10% ± 0.02
wind speed V	[m/s]	2% ± 0.25
axial force (strain) $F_{ax,s}$	[N]	5-10% ± 0.02

Table 1: measurement accuracy's

below the rotor centre and so we find that the axial force will be underestimated by 5-10%. Table 1 gives the resulting signals and the corresponding accuracy's.

rotor	C_t	C_p (Q x ω)
–2.5°	0.95 - 1.05	0.12 - 0.21
+0.0°	0.84 - 0.91	0.18 - 0.30
+2.5°	0.66 - 0.74	0.21 - 0.30
+5.0°	0.47 - 0.59	0.20 - 0.27
+7.5°	0.32 - 0.48	0.07 - 0.24

Table 2: C_t and C_p for λ between 4 and 6: almost every C_t-value between 0.32 and 1.05 can be selected.

Before we started the actual wind farm measurements we characterized the rotors. The results of the 3 mostly applied rotors are shown in figure 6. It can be seen that the rotors reach a good performance ($C_p \approx 0.30$) given their small size. It can also be seen that we can adjust the axial force coefficient C_t we like by choosing a rotor. Table 2 gives an overview of the C_t and C_p values we could choose by installing a certain rotor. The pitch +0.0 rotor behaves as a classical Lanchester-Betz turbine with a C_t of about 8/9.

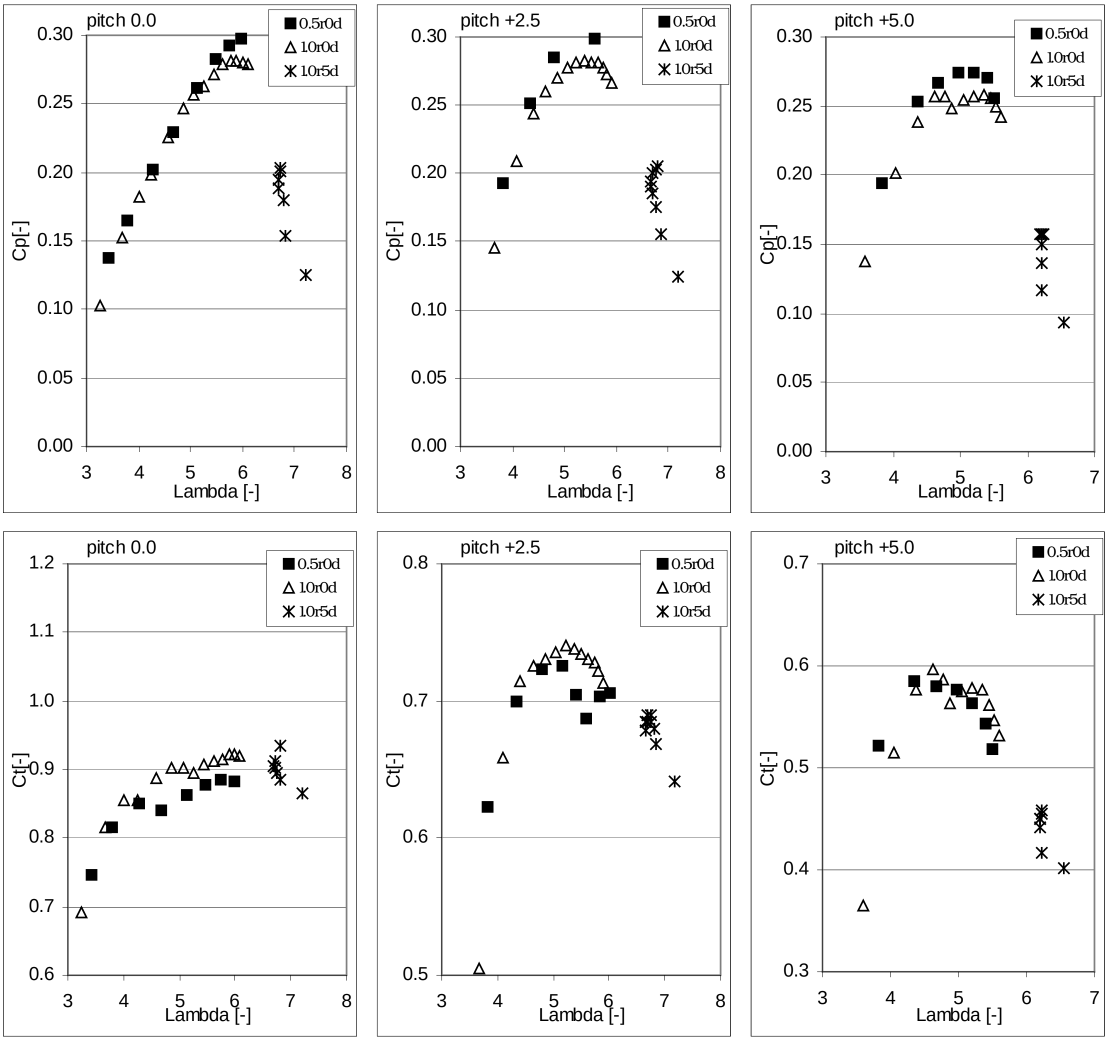

Figure 6: model turbine characteristics. The higher data point in the C_p-graphs are measured at higher wind speed and thus higher Reynolds numbers. Optimum performance $C_p \approx 0.30$ is reached for both the pitch 0 and +2.5 rotors at $\lambda = 5.6$ and $U = 10$ m/s. Under those conditions the chord based Reynolds number at 70%R is 80.000. The accuracy in both the C_p and the C_t values is ± 10%.

3.3 First Farm Measurement

In May 2003 we took measurements from the farm in the classical situation, which means that all turbines were operating at the Betz limit and thus that pitch angle 0° rotors were installed on all turbines. Subsequently the turbines were operated at their individual maximum output like in a classical commercial wind farm. Those measurements were compared to measurements of the same farm in 'Heat and Flux'-operation. This means that we installed rotors of larger pitch angles, and thus lower C_t-values, in the first two rows at the windward side of the farm. We published the results by [2] and showed that the production of the model farm increased by 4.5% and that the maximum axial force on a turbine in the farm decreased by tens of percents. This former publication also noted that the measured reduction of the axial loading could reliably by transferred to the situation of a full scale turbines and that this could not be done for the result on the production. In November 2003 we carried out a second series of measurements with results on the production that are processed in such a way that they should be valid for the on scale situation.

3.4 Increasing the Accuracy

The measurement inaccuracy is hindered by at least three effects physical effects and by the measurement inaccuracy. We will start with the physical effects, which are in short the Reynolds dependency, the tunnel blockage and the pressure gradient in wind direction.

The performance of our small rotors is much dependent on the chord based Reynolds number, which more or less increases proportional to the wind speed. Since the average wind speed in a Heat and Flux-operated farm goes up, part of the production increase might be caused by the higher turbine performance at higher wind. This effect will be much less for full scale turbines, since at high Reynolds numbers the flow around the blades becomes more Reynolds independent. Therefore the power measured in the model farm by multiplying torque and rotor speed (Q x ω) is not suitable as an on scale estimate for the power. Our way out for the next series of measurements will be to base the production increase on measurements of the axial force and the wind speed. The axial force is in fact the real parameter that determines how much power is extracted from the flow [6,7]: the extracted power is $F_{ax}\cdot U$, of which a fraction $F_{ax}\cdot U(1\text{-}a)$ can be transformed into useful power and a fraction $F_{ax}\cdot Ua$ is lost as heat. This is correct for both the scaled wind tunnel situation as for the on scale field situation. So if we calculate the power from the farm by adding $F_{ax}\cdot U(1\text{-}a)$ for all turbines, then we have an estimate for the on scale power. The applied method uses the measured undisturbed wind speed at the turbine and the axial force on the turbine. Then we use the relation $C_t=4a(1\text{-}a)$ so that $a = ½ (1\text{-}\sqrt{(1\text{-}C_t)})$. Now we have all values to calculate $P = F_{ax}\cdot U(1\text{-}a)$ for every turbine. Hereby the physical problem of the unknown dependency of C_p on the Reynolds number is solved. At the same time this solves the not accurately known dependency of C_p on the tip speed ratio λ.

The second and third physical problem is that the flow in the wind tunnel cannot expand due to the fixed cross section of the tunnel and that a negative pressure gradient in flow direction has to be laid up by the tunnel fan. This pressure gradient will have an accelerating effect on the flow. The speed up of slow air is much more than that of fast air when both air flows pass the same pressure gradient. Therefore the pressure gradient will dominantly speed up retarded wake flow. Especially the wakes of the Lanchester Betz turbines in the classic farm will therefore have much upspeeding and thus become less pronounced. The consequence is that the negative effects of wakes will be less in the wind tunnel and therefore the advantage of Heat and Flux will be underestimated. This argument becomes even stronger when we realize that the turbines in a farm in Heat and Flux operation have lower axial forces and thus will cause less tunnel blockage. So the driving (adding power) pressure gradient in the tunnel will be smaller in the case of a Heat and Flux farm. We have a half way out for this second problem: if we install two farms next to each other in the wind tunnel, one in classic operation and one in Heat and Flux-operation. Then the blockage will be averaged over the tunnel and both farms will see the same pressure gradient. Now the pressure gradient is adding the same energy to both farms and this error source has vanished. The problem that the pressure gradient takes away more of the negative wake effects in the classic farm than in the Heat and Flux farm however remains unsolved, so we will get an underestimate of the effect of Heat and Flux operation.

Then the last problem: the measurement inaccuracy. The power increase will be in de order of percents and if we look back to table 1, we can see that just the wind speed reading is already uncertain by 2%. Since the power relates to the cube of the wind speed, we get 6% uncertainty in the power. Overcoming this accuracy problem by improving the equipment will be hard. We think that the above mentioned change in measurement method will give more perspective: using differential measurements between two farms. In that case we still keep the uncertainty in the wind speed and in other parameters, however the uncertainty will be the same for both farms. The next section presents the precise method and the results.

3.5 Differential Measurements with two Wind Farms

We installed two farms of each 2 turbines wide and 7 turbines deep next to each other in the wind tunnel. We measured the performance of the farms at 7, 8 and 9 m/s tunnel speed for 6 modes of operation. Those modes are indicated by a number of Heat and Flux units. The precise meaning is given by table 3.

HF-modus	Farm layout		
	turbine 1	turbine 2	turbines 3-7
–1	–2.5°	+0.0°	+0.0°
0	+0.0°	+0.0°	+0.0°
1	+2.5°	+0.0°	+0.0°
2	+2.5°	+2.5°	+0.0°
3.	+5.0°	+5.0°	+0.0°
4	+7.5°	+7.5°	+0.0°

Table 3: Heat and Flux modes.

All modes were applied to both farms, while the other farm always served as a reference in mode 0. The results of all differential measurements are shown in figure 7. We give one example to explain one point in the figure. The point at 2 Heat and Flux units of the curve average is the average over the three wind speeds of the subtraction of two differential powers divided by 2. In formula form:

([P(farm 1, mode 2) - P(farm 2, mode 0)] - [P(farm 1, mode 0) - P(farm 2, mode 2)])/2.

4. Discussion

Our experiment is realistic regarding the boundary layer profile, the ambient turbulence, the truly rotating wind turbines and their wake characteristics. We have solved the problem of the C_p-dependency on the blade Reynolds number and on the tip speed ratio. We have solved most of the accuracy problems by the differential measurements. The tunnel blockage effects are partly cancelled due to the differential method. For the next series of measurements we have the following issues to improve our knowledge on Heat and Flux and to further improve the experimental results.

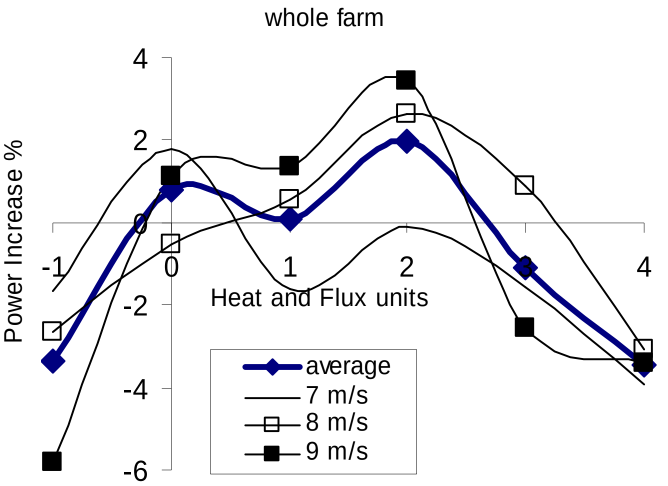

Figure 7: Results of the differential experiments between a farm in classic operation and a Heat and Flux farm. The over all measurements averaged result is that the Heat and Flux farm produces 2.0% ± 0.8% more power than the classic farm for the case of 2 Heat and Flux units. At the same time the axial force on the upwind turbines was reduced by 20%.

1. The pressure gradient in flow direction in the tunnel will lead to an underestimation of the advantage of Heat and Flux.
2. The absence of the lateral pressure gradient and the lateral Coriolis force in the tunnel compared to the field situation. The effect of this small difference is unknown.
3. The loss of power into heat is reduced much by Heat and Flux operation. Can we take more advantage of this in other farm configurations.
4. Many different possible Heat and Flux modes were not tested. One of those modes could have a better results than the
5. present maximum reached in mode 2.
6. Heat and Flux operation has in fact 3 advantages: 1. more farm production, 2. less axial trust on the upwind turbines and 3. less turbulence and thus less fatigue loading for the downwind turbines. The third effect has not yet been quantified.

7. The advantages of Heat and Flux have to be tested in a full scale wind farms. Such a project is under preparation.

5. Conclusion

We conclude that we have three physical arguments stating that the Lanchester-Betz limit of an axial inductioin of 1/3 is not valid for wind turbines in a wind farm. We propose to decrease the axial induction of the wind turbines at the windward side of a wind farm in order to get more power at less loads. We showed by differential measurements in the wind tunnel that a farm can produce in the order of 2% more at about 20% less axial loading. Those numbers are valid below the nominal wind speed and for the situation that the wind is parallel to the turbine rows.

Acknowledgement

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Appendix A. Maximum Energy Extraction for a Cascade of Turbines

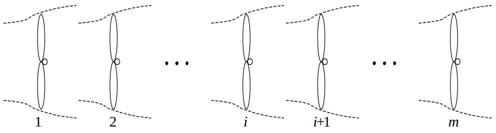

Figure 1: the wind is flowing parallel to a line of m turbines

Consider that the wind flows parallel to the line from turbine 1 to m (see figure 1). We model each turbine by the commonly applied actuator disc. The axial induction factor of turbine i is a_i and the accompanying power coefficient is:

$$C_{\mathrm{p}i} = 4a_i(1-a_i)^2 \tag{1}$$

We assume that the wake of a turbine is developed completely before it reaches the pressure field of the next turbine and that there is no momentum inflow from outside. Then the wind speed in the far wake of turbine i comprises the 'ambient' wind speed of turbine i+1:

$$U_{i+1} = (1-2a_i)U_i \tag{2}$$

in which U_i is the 'ambient' wind speed of turbine i. For the energy extracting body that is formed by turbines i to m the production is proportional to $K_i \cdot U_i^3$, in which power coefficient K_i is given by:

$$K_i \equiv \sum_{j=i}^{m} \frac{P_j}{\frac{1}{2}\rho A U_i^3} = C_{\mathrm{p}i} + K_{i+1}(1-2a_i)^3 \tag{3}$$

The first term on the right hand side comes from the production by turbine i, the last term from the production by the turbines in its wake. The a_i value that maximises the production by turbines i to m follows after filling in (1) in (3) and calculating $\partial K_i / \partial a_i = 0$. Then a_i appears to be related to K_{i+1} conform:

$$K_{i+1} = \frac{2(1-a_i)(\frac{1}{3}-a_i)}{(1-2a_i)^2} \tag{4}$$

Substituting (1) and (4) in (3) gives:

$$K_i = \frac{2}{3}\left(1-a_i^2\right) \tag{5}$$

Since i is an ordinary number, equation (5) is also valid for turbine i+1, so that (5) can also be written as:

$$K_{i+1} = \frac{2}{3}\left(1-a_{i+1}^2\right) \tag{6}$$

Finally, combining equation (4) and (6) gives:

$$a_i = \frac{a_{i+1}}{2a_{i+1}+1} \tag{7}$$

According to Lanchester and Betz, the production of a single turbine is maximized for $a = \frac{1}{3}$. For a line of m turbines, this means that the maximum farm production will require $a_m = \frac{1}{3}$. If we substitute this result for $i = m$-1 in equation (7) we get $a_{m-1} = \frac{1}{5}$. If we continually substitute the last result of equation (7), we get a series that can generally be written as:

$$a_i = \frac{1}{3+2(m-i)} \tag{8}$$

So in the theoretical case that the wake of an actuator disc would be developed completely before it reaches the pressure field of the next actuator disc and that there would be no momentum inflow from outside, maximum farm production would be realised by:

$$\{a_m, a_{m-1}, a_{m-2}, \ldots\} = \{{}^1/_3, {}^1/_5, {}^1/_7, \ldots\} \tag{9}$$

Remark that the a_i value that maximises K_i still follows from the partial derivative of (3) when a_{i+1} does not maximise the production of turbine i+1 to m.