ECN-RX--01-001

Heat Generation by a Wind Turbine

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This paper has been presented at the annual IEA symposium on
the aerodynamics of wind turbines, December 4-5, 2000, NREL, CO, USA

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ABSTRACT It will be shown that an actuator disk operating in wind turbine mode extracts more energy from the fluid than can be transferred into useful energy. At the Lanchester-Betz limit the decrease of the kinetic energy in the wind is converted by $^2/_3$ into useful power and by $^1/_3$ into heat. Behind the wind turbine the outer flow and the flow that has passed the actuator disk will mix. In this process momentum is conserved but part of the kinetic energy will dissipate in heat via viscous shear.

List of Symbols

a	[-]	axial induction factor
A	[m^2]	surface of the actuator disk
C_D	[-]	axial force coefficient
C_H	[-]	tot al pressure head coefficient
C_{heat}	[-]	dissipated heat coefficient
C_P	[-]	power coefficient
D_{ax}	[N]	axial force exerted by the actuator disk
D_N	[N]	normalisation for axial force $½\rho AU^2$
$\dot{m}$	[kg/s]	mass flow of the wind
P	[W]	power
P_{heat}	[W]	power dissipated in heat
P_N	[W]	normalisation power $½\rho AU^3$
p	[N/m^2]	pressure
p_0	[N/m^2]	atmospheric pressure
p^+	[N/m^2]	pressure on upwind site of the actuator disk
p^-	[N/m^2]	pressure on downwind side of the actuator disk
p_d	[N/m^2]	dynamic pressure $½\rho U^2$
U	[m/s]	wind speed
U_D	[m/s]	wind speed at the disk
U_w	[m/s]	wind speed in the far wake
V	[m/s]	wind speed in the very far wake
ΔU	[m/s]	velocity change in very far wake due to actuator disk.
ΔP	[W]	kinetic power extracted from the flow through the stream tube.
ΔP_s	[W]	kinetic power extracted from the flow through the stream tube.
ε	[-]	fraction of the total mass flow $\dot{m}$ through the actuator disk
ρ	[kg/m^3]	air density ≈ 1.25 kg/m^3
η	[-]	efficiency of kinetic energy transfer

The Lanchester-Betz Limit

The theory predicting the maximum useful power that can be extracted from a fluid flow was first published by F.W. Lanchester [1] in 1915. In most cases however, this theory is attributed to A. Betz, who published the same argument in 1920 [2]. To do justice to the first author, we will speak of the 'Lanchester-Betz' limit. Following the text written by Glauert [4], the actual wind turbine will be replaced by a so-called actuator disk which was introduced by Froude (see figure 1). This actuator disk is an abstract theoretical analogue of a wind turbine being used in momentum theory. The disk has a surface A, equal to the swept area of the wind turbine, and it is oriented perpendicular to the wind. The disk does not consist of several rotor blades but has a homogeneous structure. The undisturbed wind speed is U, at the actuator disk it is U_D=(1-a)U and in the far wake it is (1-2a)U. The parameter a is called the induction factor which takes into account the decrease of the wind speed when it passes through the permeable actuator disk. The mass flow through this disk is $\rho A(1-a)U$ and it is driven by the difference in pressure p^+ on the upwind side of the disk and p^- on the downwind side. So the pressure at the disk is discontinuous and the disk is subject to a net axial force $D_{ax} = A(p^+ - p^-)$. This force is also exerted on the fluid and thus it should be equal to the change of the flow of momentum. From conservation of mass it follows that the stream tube just enclosing the flow through the actuator disk has a constant mass flow $\rho A(1-a)U$ at all cross sections from far upstream to far downstream. The figure shows this stream tube and its expansion. Behind the actuator we have a clear slipstream, but in front of it such a boundary does not exist, therefore we dashed the slip stream contour here. As this mass flow is constant, the change of momentum should be attributed to a velocity difference between the flow in the far wake and the undisturbed wind speed far upstream:

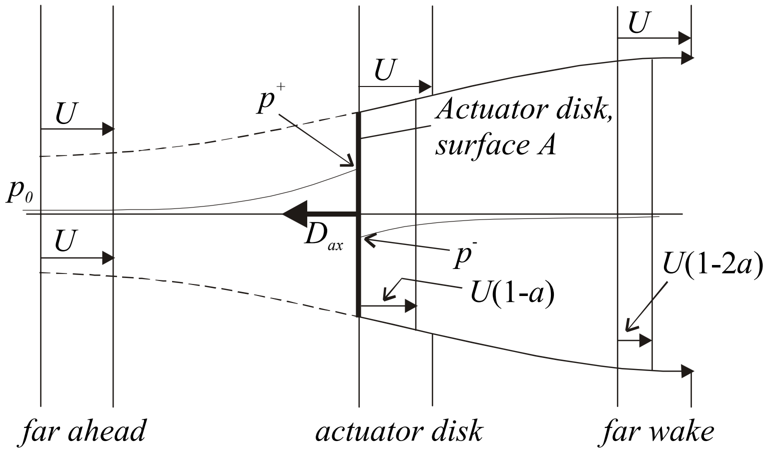

figure 1 *Froude's actuator model. The stream tube consists of a slipstream behind the disk, but has no velocity discontinuity in front of the disk.*

$$p^+ - p^- = \rho(1-a)U(U-U_w). \quad (1)$$

Upwind and downwind of the actuator disk, the kinetic energy in the flow is transferred into 'pressure' energy. So the actuator disk does not directly extract kinetic energy. The disk slows down the flow, which causes a pressure difference over the disk. The extracted energy comes from the product of the pressure difference and the volume flow through the disk. Application of Bernoulli's relation that $p+½\rho U^2$ = constant along a streamline (when no power is extracted), yields for the flow upwind and downwind respectively:

$$\tfrac{1}{2}\rho U^2 + p_o = \tfrac{1}{2}\rho(1-a)^2U^2 + p^+, \quad (2)$$

$$\tfrac{1}{2}\rho U_w{}^2 + p_o = \tfrac{1}{2}\rho(1-a)^2U^2 + p^-, \quad (3)$$

where p_o is the undisturbed atmospheric pressure. By subtracting equations 2 and 3 it follows that:

$$p^+ - p^- = \tfrac{1}{2}\rho(U^2 - U_w^2). \quad (4)$$

The combination of equations 1 and 4 demonstrates that the velocity decrease in front of the disk equals that behind the disk:

$$U_w = (1-2a)U, \quad U_D = (1-a)U. \quad (5)$$

The extracted power is equal to the difference of the kinetic energy in the flow far upstream, minus the kinetic energy in the flow far downstream, multiplied by the mass flow $\rho A(1-a)U$. Far upstream the velocity is U and far downstream it is $(1-2a)U$. Thus we find for the power:

$$P = 4a(1-a)^2\,\tfrac{1}{2}\rho A U^3 = 4a(1-a)^2\,P_N, \quad (6)$$

in which P_N ($½\rho AU^3$) is the flow of kinetic energy through a cross section of size A perpendicular to the undisturbed wind. It follows that only the axial induction factor a determines the fraction of the power extracted by the wind turbine. From dP/da =0 we find that the maximum fraction extracted is $^{16}/_{27}$, which corresponds to a=$^1/_3$. This maximum was derived by Lanchester in 1915. If both the maximum power and the corresponding axial force are normalised with P_N and D_N = $½\rho AU^2$ respectively, then it follows that:

$$C_P = 4a(1-a)^2 = \frac{16}{27}, \tag{7}$$

$$C_D = 4a(1-a) = \frac{8}{9}, \tag{8}$$

for the power and the axial force coefficients respectively.

Equation 7 only gives the fraction of P_N that can be converted into useful power. It should not be confused with the efficiency of the turbine. When we read the literature of almost a century ago we find the following text on efficiency written by Betz, 1920 [2]: *'Eine Flache welche dem Winde einen gewissen Widerstand entgegensetzt, dadurch seine Geschwindigkeit, also seine kinetische Energie, vermindert und diese ihm entzogene kinetische Energie* ***verlustlos*** *in nutzbare Form überführt.'* But in the same paper Betz states that a turbine on an airplane translating with velocity U and axial force D_{ax} has efficiency $P/(U{\cdot}D_{ax})$, which is 1-a. However Betz says about this: *'Diese Definition befriedigt nun zwar das theoretische Bedürfnis, da die Axialkraft eine Größe ist, die für die wirtschaftliche Beurteilung eines Windmotors nur untergeordnete Bedeutung hat'.* Glauert 1934 [4] confirms this by stating that it is necessary to distinguish between a windmill driven by the speed of an airplane and a windmill on the ground driven by the wind. In the first case the efficiency is meaningful, but for the latter only the extracted energy is relevant. So, in classic theory the efficiency of a wind turbine (1-a) is considered unimportant, which probably was one reason for not paying attention to the physical effect which caused the loss.
In recent literature we find that the decrease of the flow of kinetic energy equals the useful power produced by the actuator disk. Spera 1994 [5], Hunt 1981 [6] and Wilson and Lissaman 1974 [7] normalise the power produced by (1-a)P_N instead of P_N since the mass flow *through* the actuator is (1-a)UA and not UA. So they hold that the power in the flow is converted with an *efficiency* (defined as power output/power input) = $C_P/(1-a)=4a(1-a)$, which is $^8/_9$ at the Lanchester-Betz limit. This means that they limit themselves to the wind that flows through the actuator disk. They find that $^1/_9$ of the kinetic energy remained in the flow and thus $^8/_9$ was converted into useful power, where the conversion is assumed to have an efficiency of 100%.

In the next section the power transfer by an actuator disk will be calculated for the case in which the outer flow is included.

Heat Generation

In the actuator disk model, the power extracted by the axial force is $-(1-a)U{\cdot}D_{ax}$. However, if the same actuator disk, exerting a force D_{ax}, is fixed on an airplane moving with speed U, the power required to move the disk would be $-U{\cdot}D_{ax}$. So it takes more power to drive the disk than the maximum power that can be generated by the disk. This difference is understood when the flow around the actuator is also included in the analysis. It then follows that the energy conversion by an actuator disk has an inherent dissipation of kinetic energy into heat.

Kinetic Power Transfer by an Axial Force

Let $\dot{m}$ be the indefinite but large mass flow in the wind, in which an actuator disk is placed perpendicular to the flow direction (see figure 2). Only a fraction ε of $\dot{m}$ flows through the stream tube that just encloses the actuator disk, which exerts a finite axial force D_{ax} on the flow against the flow direction. In the far wake, the momentum and the energy relations will be:

$$D_{ax} = -\varepsilon\,\dot{m}2aU, \tag{9}$$

$$\Delta P_s = \tfrac{1}{2}\varepsilon\dot{m}(U^2 - (1-2a)^2U^2) = -(1-a)U \cdot D_{ax}. \tag{10}$$

Where ΔP_s refers to the change of the kinetic power in the flow in the stream tube when it crosses the actuator disk. We now provide the actuator disk model with a *very far wake*, defined as the location beyond the far wake, where the velocity distribution has become uniform again. The definition of the far wake remains classic, namely the location where the axial force has stopped transferring momentum to the flow, or in other words, where the stream tube is no longer expanding. The velocity

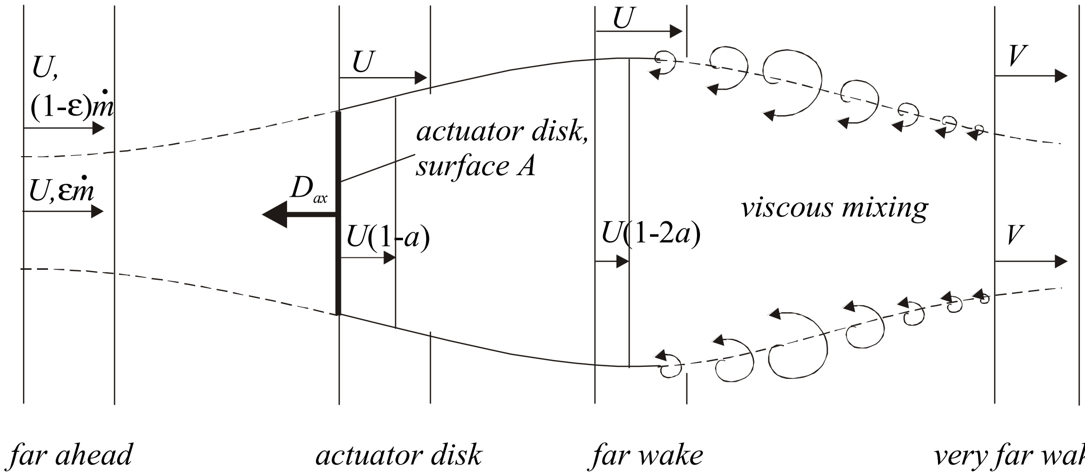

figure 2 *Introduction of the very far wake and viscous dissipation. When the outer flow and that inside the stream tube mix, heat is generated and the slipstream vanishes while it contracts.*

is $(1\text{-}2a)U$ in the far wake and U outside the wake. The smoothing of the velocity profile behind the far wake is due to turbulent mixing and viscous shear, which will eventually make all velocities equal to a common speed V in the *very far wake*. During this process no external force acts on the flow, so momentum is conserved and the flow does not expand further.

Comparing the flow far upwind $\dot{m}\,U$ with that in the very far wake $\dot{m}\,V$, the difference in the flow of momentum should be equal to the axial force.

$$D_{ax} = -\dot{m}(U - V). \tag{11}$$

We can express V in U, a and ε by using the momentum balance between the far wake and the very far wake. The momentum in the outer flow of the far wake is $(1\text{-}\varepsilon)\dot{m}\,U$, and in the stream tube it is $\varepsilon\dot{m}\,(1\text{-}2a)U$, which together should be equal to $\dot{m}\,V$ to conserve momentum, or

$$V = (1-\varepsilon)U + \varepsilon(1-2a)U = (1-2a\varepsilon)U. \tag{12}$$

The velocity change obtained from the momentum relation 11 is connected to the change of the kinetic power in the wind, by

$$\Delta P = \tfrac{1}{2}\dot{m}(U^2 - V^2) = -(1 - a\varepsilon)U \cdot D_{ax}. \tag{13}$$

To clarify: this is the change of the kinetic power in the flow due to the axial force when the outer flow is included, whereas equation 12 expresses that change when the outer flow is excluded. In practice the mass flow $\dot{m}$ is large but finite, so that the fraction of $\dot{m}$ going through the disk, ε, is much smaller than 1 and ΔP is close to $-D_{ax}\cdot U$. So, the decrease of flow of kinetic energy by a force D_{ax} approaches the scalar product of the undisturbed wind speed U and $-D_{ax}$ and *not* the often used product of the local velocity $(1-a)U$ and the force $-D_{ax}$. The latter corresponds to the power extracted from the flow.

Dissipation into Heat

In the process of mixing between the far wake and the very far wake, the kinetic power in the flow will not be conserved, but it will be partially converted into heat. This heat is generated by the viscous force that accelerates the flow in the stream tube to the velocity V in the very far wake. In this process the flow inside the stream tube gains less kinetic energy than the outer flow loses. In the far wake the kinetic power inside the stream tube is $\frac{1}{2}\varepsilon\dot{m}\,(1-2a)^2U^2$ and in the outer flow it is $\frac{1}{2}(1-\varepsilon)\,\dot{m}\,U^2$. In the very far wake the kinetic power is $\frac{1}{2}\dot{m}\,V^2$. The difference has to be the heat generated;

$$P_{heat} = \tfrac{1}{2}\dot{m}\left\{[\varepsilon(1-2a)^2U^2 + (1-\varepsilon)U^2] - V^2\right\} = -(1-\varepsilon)aU \cdot D_{ax}. \tag{14}$$

Of course, this is also equal to $\Delta P - \Delta P_s$.

If we want to normalise to $P_N = \frac{1}{2}\rho A U^3$, as in the previous section, the mass flow through the actuator disk $(1-a)\rho AU$ has to be replaced by $\varepsilon\dot{m}$. So we use $P_N = \frac{1}{2}\varepsilon\dot{m}\,U^2/(1-a) = -D_{ax}\cdot U/(4a(1-a))$. Since the mass flow through the actuator disk is much smaller than the flow outside the wake, we take the limit $\varepsilon\rightarrow 0$, and find the following power coefficients,

$$C_H = \frac{\Delta P}{P_N} \approx 4a(1-a), \tag{15}$$

$$C_P = \frac{\Delta P_s}{P_N} = 4a(1-a)^2\,, \tag{16}$$

$$C_{heat} = \frac{P_{heat}}{P_N} \approx 4a^2(1-a). \tag{17}$$

Here C_H refers to the transferred kinetic power, C_P to the kinetic power actually extracted and C_{heat} to the power in the viscous heating. C_P is the commonly used (classic) power coefficient.

It follows that the maximum efficiency for the process of transfer of kinetic energy into useful power by an actuator disk η is:

$$\eta = \frac{C_P}{C_H} \approx 1 - a\,, \tag{18}$$

which is in agreement with Betz's result [7]. Our calculation makes clear that an actuator disk does not convert all transferred kinetic energy into useful energy. The energy balance reads:

$$C_H = C_P + C_{heat} \,. \tag{19}$$

As mentioned before, the maximum extractable useful power from the flow is obtained for $a = {}^1/_3$. In that case a fraction $C_H = {}^{24}/_{27}$ of the flow of kinetic energy P_N is transferred. From this, ${}^2/_3$ is extracted as useful power, and ${}^1/_3$ is dissipated as heat. Figure 3 shows schematically the power transfer by an actuator disk representing a wind turbine. We introduced $U_i = -aU$ for the induction velocity in order to make the model more general, so that the situation for an actuator disk representing a propeller is also included.

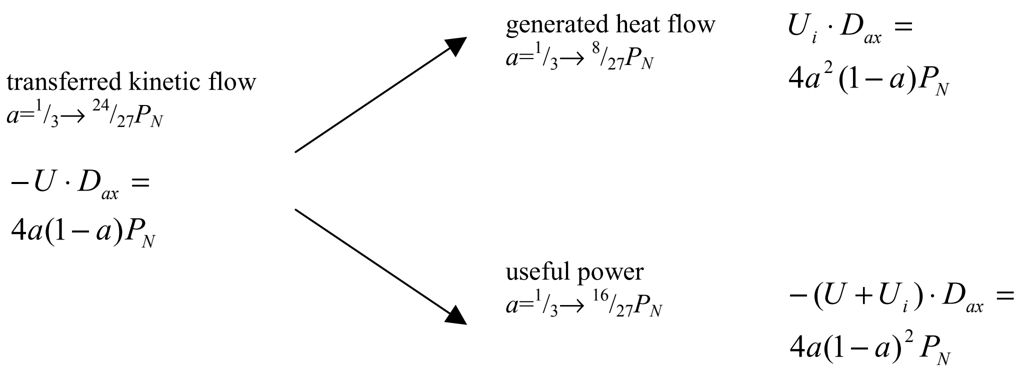

figure 3 *Schematic view of the kinetic energy transfer by an actuator disk.*

For an actuator disk representing a rotor in hover (U=0) it follows from equations 15 to 17 and from figure 3 that the power required to yield D_{ax} is $U_i \cdot D_{ax}$ and that this power is entirely converted into heat. It first turns up as kinetic energy, which is eventually dissipated via turbulent mixing and viscous shear as heat. For an actuator in propeller state ($U \geq 0$ and $U_i \geq 0$), the engine power is $(U+U_i) \cdot D_{ax}$, the heat produced is $U_i \cdot D_{ax}$ and the kinetic power in the flow is increased by $U \cdot D_{ax}$ (The co-ordinate system is still attached to the actuator).

Conclusion

We conclude that the inherent limitation to the efficiency of energy extraction by an actuator disk is determined by dissipation as heat. This dissipation is $a/(1-a)$ times the extracted useful energy. The heat capacity of the mass flow through a wind turbine is so large that the heat generated will hardly affect the temperature. To give an example: a wind turbine operating at 10 m/s at the Lanchester-Betz limit will transfer 44.4J of kinetic energy per unit of air mass into 29.6J of useful work and 14.8J of heat. This heat raises the temperature by only 0.015°C. In practice it will be even less since the heat generated is not limited to the flow inside the stream tube.

<u>Practice</u>
The above analysis does not put the Lanchester-Betz limit in a different light, since the maximum extractable useful energy of a wind turbine remains unchanged. But for a wind turbine park as a whole (present park optimisation studies are based on momentum balances and thus deal correctly with the dissipated heat), our model clarifies what determines the loss. And we conclude that the maximum extractable useful energy shall not occur when all turbines operate individually at maximum output. By choosing the induction factor 10% below the optimum, the power coefficient

decreases less than 1%, while the efficiency rises more than 3%. In the turbulent wake state in particular, when *a* is approximately 0.4-0.5, the efficiency (1-*a*) becomes rather low, thus other wind turbines in the wake get a lower power input. This could be reason to operate turbines at the upwind side of a park below the optimum for *a*, and certainly not in the turbulent wake state, so that the production of the park as a whole increases.

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Acknowledgement

The wishes to express his gratitude to prof. dr. ir, C.D. Andriesse of the University Utrecht for his detailed checks of this analysis.