ECN-R–01-001

Optical measurement on transparent sheet- and sandwich samples

Determining the optical properties n(λ) and k(λ) from reflection and transmission measurements

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February 2001

Abstract

This report describes a method to determine the refractive index and absorption coefficient of transparent materials from reflection and transmission measurements.

CONTENTS

1 INTRODUCTION

Crystalline silicon solar cells are sealed in a laminate. In current laminates the cells are glued to a hardened glass sheet with a polymer, in most cases EVA (Ethylene Vinyl Acetate). For conversion efficiency reasons the lamination materials at the illuminated side of the cell should be transparent in the wavelength region that a silicon solar cell can convert to electricity

Another important optical property is the UV transmission for wavelengths shorter than those which the cell can convert to electricity. This is important because of the aging effects of UV radiation.

Also interesting is the refractive index of the materials used, as the refractive index influences the reflection of light from the module. ECN has an integrating sphere setup with which reflection and transmission of samples can be measured, for instance of a sheet of glass.

Reflection and transmission curves however have a limited usefulness. In order to calculate the optical properties of a module, one needs the real- and imaginary parts $n(\lambda)$ and $k(\lambda)$ of the refractive index as a functions of the wavelength λ. Another problem is that for some materials such as EVA it is difficult to measure reflection and transmission curves of the materials directly. One can measure however sandwich samples consisting of EVA laminated between two glass sheets. We would like to be able to determine the optical properties of this layer from the reflection and transmission curves of the sandwich.

2 SINGLE SHEET SAMPLES

In this section we show how to determine the $n(\lambda)$ and $k(\lambda)$ of a sheet of material from reflection and transmission measurements. A good reference on optics is the book by Hecht and Zajac [2]. The following equation relates the absorption coefficient α to k:

$$\alpha = \frac{4\pi k}{\lambda} = \frac{2\omega k}{c} = \frac{4\pi f k}{c} \tag{1}$$

For the reflection coefficient r on the interface between two non-absorbing media with refractive indices n_1 and n_2 for normal incidence we have:

$$r = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \tag{2}$$

For instance when we have an interface between air (n_{air}=1) and glass ($n_{\text{glass}}\approx$ 1.5) as on the front side of a typical module we have $r = 0.04$.

Equation (2) can be readily generalised ([2, 4.47 and 4.83]) to the case of materials with complex refractive indices $n_1 + \imath k_1$ and $n_2 + \imath k_2$ respectively:

$$r = \left(\frac{(n_1 - n_2)^2 + (k_1 - k_2)^2}{(n_1 + n_2)^2 + (k_1 + k_2)^2}\right) \tag{3}$$

We now have expressions for the reflection coefficients at the interfaces. Note that the reflection coefficient in both equations (2) and (3) is independent of from which side the radiation impinges on the interface.

However the reflection and transmission of a sheet or a sandwich of sheets is not only determined by reflection and transmission at the interfaces, but also by repeated reflections within the sheet.

Consider figure 1. The figure indicates the different relevant radiative fluxes. In a reflection and transmission measurement we measure the ratios R/I and T/I. We assume a wavelength λ. Suppose the sheet has complex refractive index $n_s + \imath k_s$ at this wavelength. With t_s we denote the transmission coefficient for light for a single pass through the sheet. For t_s we have:

$$t_s = \exp(-\alpha_s w) = \exp\left(\frac{-4\pi w k_s}{\lambda}\right) \tag{4}$$

For the reflection coefficient r_s at the sheet-air interfaces we have from equation (3):

$$r_s = \left(\frac{(n_s - 1)^2 + k_s^2}{(n_s + 1)^2 + k_s^2}\right) \tag{5}$$

In the sequel we set $I = 1$. From conservation of energy we can deduce the following four relations for the internal fluxes F_1, F_2 and for the total reflected and transmitted fluxes R and T.

$$\begin{aligned} F_2 &= F_1 t_s r_s \\ F_1 &= (1 - r_s) + F_2 t_s r_s \\ R &= r_s + F_2 t_s (1 - r_s) \\ T &= F_1 t_s (1 - r_s) \end{aligned}$$

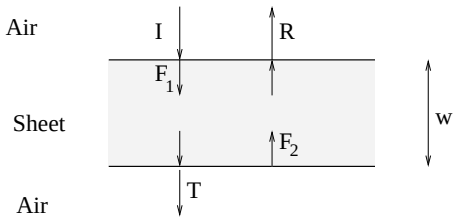

Figure 1: Radiative fluxes for a sheet of thickness w. I is the incoming radiation. R and T are the radiation reflected and transmitted by the sheet. F_1 and F_2 are internal radiative fluxes used in the calculations.

On substitution of the expression for F_2 in the remaining three equations we obtain

$$\begin{aligned} F_1 &= (1 - r_s) + F_1 t_s^2 r_s^2 \\ R &= r_s + F_1 t_s^2 r_s (1 - r_s) \\ T &= F_1 t_s (1 - r_s) \end{aligned}$$

From this system we can easily find F_1 and hence R and T:

$$\begin{aligned} F_1 &= \frac{(1 - r_s)}{1 - t_s^2 r_s^2} \\ R &= r_s + \frac{(1 - r_s)^2 t_s^2 r_s}{1 - t_s^2 r_s^2} = \frac{r_s(1 + t_s^2) - 2t_s^2 r_s^2}{1 - t_s^2 r_s^2} \qquad (6) \\ T &= \frac{t_s (1 - r_s)^2}{1 - t_s^2 r_s^2} \qquad (7) \end{aligned}$$

Now we go back to problem of determining n and k from the measured reflection R and the measured transmission T. From equations (6) and (7) we see that we have two equations for the unknowns r_s and t_s. We solve the system of equations for r_s and t_s from the reflection and transmission through equations (6) and (7) in an iterative way. To that end we first rewrite equations (6) and (7) as quadratic equations for r_s and t_s respectively:

$$r_s^2(2t_s^2 - Rt_s^2) - r(1 + t_s^2) + R = 0 \quad (8)$$

$$t_s^2 T r_s^2 + t_s(1 - r_s)^2 - T = 0 \quad (9)$$

These equations have the solutions ([1]):

$$r_s = \frac{1 + t_s^2 - \sqrt{(1 + t_s^2)^2 - 4t_s^2 R(2 - R))}}{2t_s^2(2 - R)} \quad (10)$$

$$t_s = \frac{-(1 - r_s)^2 + \sqrt{(1 - r_s)^4 + 4T^2 r_s^2}}{2Tr_s^2} \quad (11)$$

For improved numerical accuracy these equations can be rewritten as:

$$r_s = \frac{2R}{1 + t_s^2 + \sqrt{(1 + t_s^2)^2 - 4t_s^2 R(2 - R))}} \quad (12)$$

$$t_s = \frac{2T}{(1 - r_s)^2 + \sqrt{(1 - r_s)^4 + 4T^2 r_s^2}} \quad (13)$$

The iterative solution proceeds as follows. We start by setting t_s to 1. Then we can calculate a value for r_s from (12). With r_s known we can calculate an improved value for t_s from equation (13). We can then start the next iteration. This iterative procedure converges in a few steps.

When the sample does not absorb any light ($t_s = 1$), the procedure converges in a single step.

Once we know r_s and t_s, the refractive index can be determined by solving equations (4) and (5) provided we know the thickness of the sheet. We have:

$$k_s = \frac{\alpha_s \lambda}{4\pi} = -\frac{\log(t_s)\lambda}{4\pi w} \quad (14)$$

$$n_s = -\frac{r_s + 1 + \sqrt{(1 + r_s)^2 - (1 - r_s)^2(k_s^2 + 1)}}{r_s - 1} \quad (15)$$

$$= -\frac{r_s + 1 + \sqrt{2r_s(k_s^2 + 2) - r_s^2 k_s^2}}{r_s - 1} \quad (16)$$

When the imaginary part k_s of the index is 0 or small compared to 1 we can write for n_s:

$$n_s = -\frac{\sqrt{r_s} + 1}{\sqrt{r_s} - 1}$$

When the sample does not transmit any light ($T = t_s = 0$) equations (12) and (13) reduce to $r_s = R$ and $t_s = 0$. The iterative procedure converges in two steps. In that case we cannot determine n_s and k_s, and we need additional information. For instance if we know one of n_s and k_s we can determine the other one.

3 SANDWICH SAMPLES

The basic single sheet theory from section 2 can be used for many sandwich cases too. We proceed as follows. Say we make a sandwich of a polymer layer of thickness w_p between two identical glass sheets of thickness w_g. First we measure reflection and transmission of a single glass sheet and hence we can determine n_g and k_g of the glass sheet as described in section 2.

Now we observe that most polymers have refractive indices quite close to those of glass in the range of [1.4-1.6]. Suppose the refractive index of the glass is 1.5 and that of the polymer 1.6. We get for the reflection coefficient at the interface between glass and the polymer:

$r = \left(\frac{.1}{3.1}\right)^2 \approx 0.001$

So in a lot of cases we can neglect the reflection at the internal interfaces. This also shows that a sandwich is not a suitable sample setup to determine the refractive index of the intermediate layer.

We treat the sandwich as a single sheet and determine the surface reflection r_s and the transmission coefficient from front- to rear side t_s as in equations (12) and (13). The surface reflection r_s as explained before is determined by the refractive index of the glass.

Comparing to the transmission coefficient for a single sheet (equation (4)) we can write for the transmission coefficient of the sandwich:

$$t_s = \exp(-2\alpha_g w_g - \alpha_p w_p) \tag{17}$$

So from t_s we can determine α_g:

$$\alpha_g = (-\log(t_s) - 2\alpha_g w_g)/w_p \tag{18}$$

So from the thicknesses of the different sheets, the absorption coefficient of the glass and t_s we can determine the absorption coefficient of the intermediate layer and from the absorption coefficient we can in turn determine the k_p of the intermediate layer.

4 EXAMPLES

In this section we present some examples of our method.

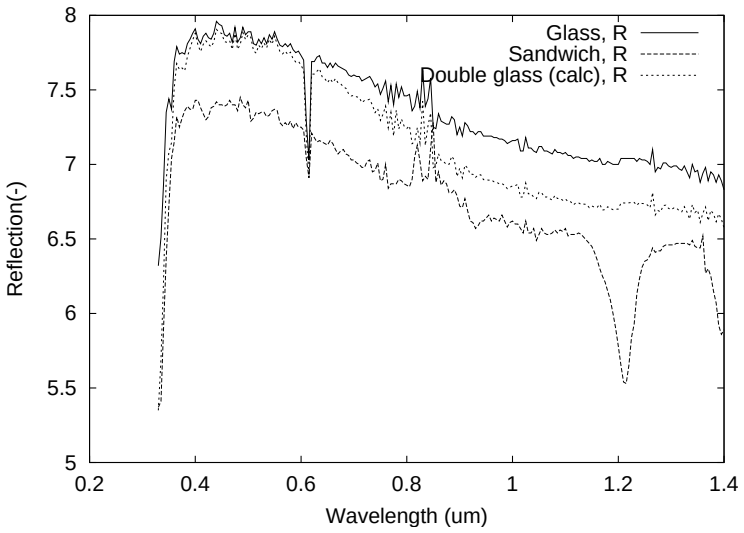

Figure 2: Reflection of glass R_g and glass-surlyn sandwich R_s

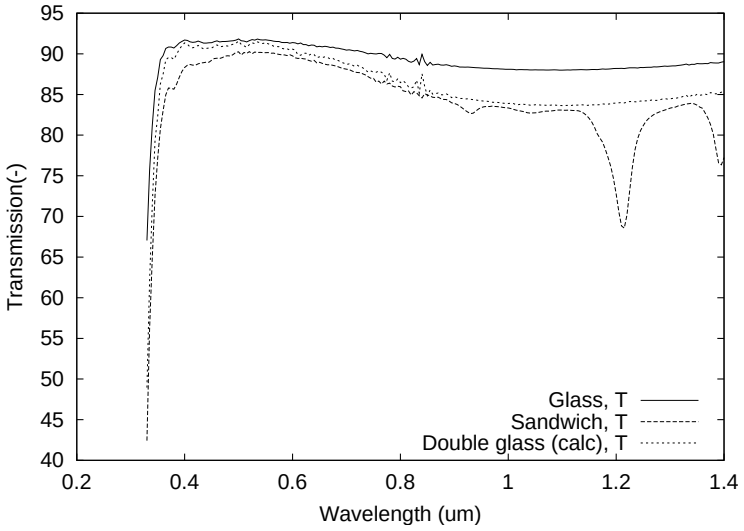

Figure 3: Transmission of glass T_g and glass-surlyn sandwich T_s

Figures 2 and 3 show measured reflection and transmission curves of a glass sheet and a layer of surlyn laminated between two glass sheets. The thickness of the glass was .94mm and of the surlyn 1.0mm. The absorption of the surlyn laminate is determined by two passes of the light through the glass as compared to only one pass for the single glass sheet. For comparison the plot shows the calculated reflection and transmission of a glass sheet twice as thick as the sheet used, but with the same optical properties. We see that the surlyn laminate has additional absorption for wavelengths smaller than 600nm, and around 1200nm and 1400nm.

Figure 4 shows the refractive index of the glass found by the method described in section 2. We see that the refractive index is around 1.5 and decreases slightly with wavelength.

Figure 5 shows the imaginary part of the refractive index of the glass and the surlyn by the

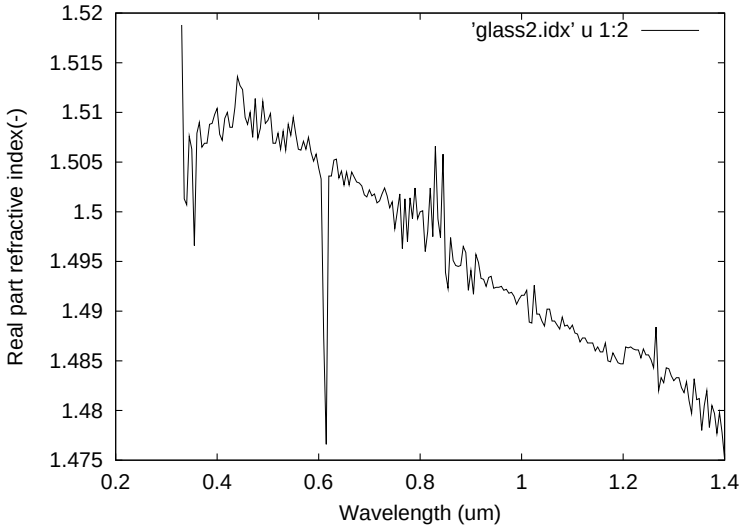

Figure 4: Real part of refractive index of glass n_g

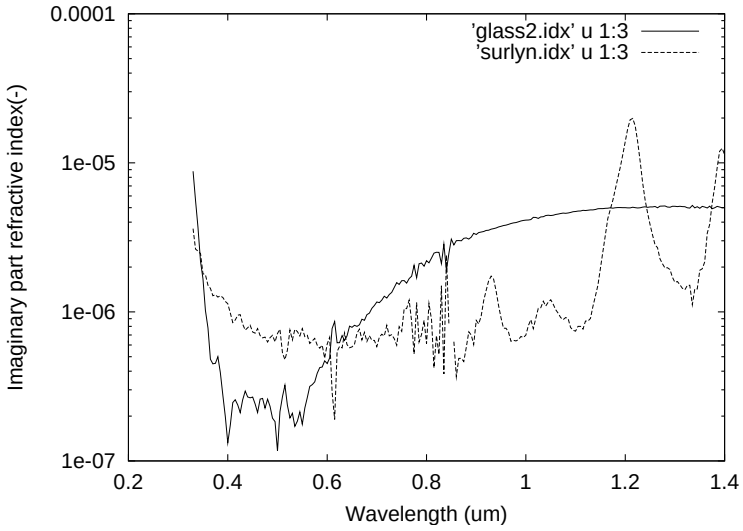

Figure 5: Imaginary part of refractive index of glass k_g and surlyn k_s

method described in section 3. We see the peaks in the imaginary part around 1200 and 1400 nm. We also see that the surlyn is more transparent than the glass for wavelengths between 600 and 1180nm, but less transparent for wavelengths between 350 and 600nm. Below 600nm the refractive index is around 1.5 and decreases slightly with wavelength.

We can also conclude that the glass (regular greenish window glass) used in this experiment is not ideal for determining the absorption of the intermediate layer, as the glass has a significant absorption itself.

5 CONCLUSIONS AND RECOMMENDATIONS

The n- and k of transparent sheets can be determined from reflection- and transmission measurements of the sheet. The sheet must be thick enough to have a measurable absorption, but not so thick that it absorbs the light completely.

In the case where the material cannot be prepared as a sheet, we can determine the absorption coefficient by laminating the material between to glass sheets with known refractive index. This sample setup is not suitable to determine the refractive of the material. The glass used in the laminate should have a low absorption coefficient for maximum sensitivity to the absorption coefficient of the intermediate layer.

REFERENCES

[1] H-J. Bartsch. *Handbook of mathematical formulas*. Academic Press, 1973.

[2] E. Hecht and A. Zajac. *Optics*. Addison-Wesley, second edition, 1987.