ECN-C–95-074

# Fatigue Equivalent Load Cycle Method

## A General Method to Compare the Fatigue Loading of Different Load Spectrums

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October 1995

**Abstract**

A method is presented with which the fatigue loading of two or more load spectrums may be compared on a quantitative basis taking into account both the range and the mean of the load cycles.

## Keywords

fatigue, wind, turbine , wind energy

# CONTENTS

# 1 INTRODUCTION

Fatigue is the main design driver for the calculation of the structural integrity of wind turbine components. In the JOULE II project "Load and Power Measurement Programme on Wind Turbines Operating in Complex Mountaineous Regions" and in several other research projects there is a need to compare different fatigue load spectrums on a quantitative basis.

A common way to compare two or more fatigue load spectrums is the use of an equivalent load range, see [2]. The calculation of the equivalent load range is easy to perform. The fatigue behaviour of the material is formulated with a straight S-N curve on log-log scale. Different material behaviour may be characterised with different slopes of the S-N curve.

A disadvantage of the above method is the neglection of the mean level of a load cycle. In case of glass-polyester, glass-epoxy, cast steel, carbon epoxy, or wood laminates the mean level of the cycle effects the fatigue life. This could be avoided by calculating the fatigue stress reserve factor. This factor is defined as the factor by which the prevailing fatigue stress has to be multiplied in order that the calculated fatigue lifetime equals the design lifetime ([1]). The disadvantages of the fatigue stress reserve method are the need of detailed cross sectional data, the need of the specific fatigue formulae of the materials, the iterative calculation of the factor, and the fact that the results are not easy to generalise for other materials than considered.

In this document an extension to the equivalent load range method is defined. With the extension the mean level of the load cycles is taken into account. The method is easy to apply and fully consistent with the equivalent load range method.

In chapter 2 the formulae for calculating the equivalent load range are given. In chapter 3 the formulae for the equivalent load cycle method are given. An example is presented in chapter 4. Some conclusions are given in chapter 5.

## 2 FATIGUE EQUIVALENT LOAD RANGE

The allowable number of cycles N for a straight S-N line on log-log scale is given by:

$$N = k \cdot S_r^{-m} \tag{1}$$
$$\tag{2}$$

In which $S_r$ is the range of a load cycle and -1/m is the slope of the S-N line on log-log scale. The damage caused by a load spectrum of $n$ cycles with ranges $S_{r,i}$:

$$D = \sum_{i=1}^{n} \frac{1}{k \cdot S_{r,i}^{-m}} \tag{3}$$

The damage caused by $N_{eq}$ (constant amplitude) cycles with range $S_{r,eq}$ has to equal the damage of the above load spectrum:

$$\frac{N_{eq}}{k \cdot S_{r,eq}^{-m}} = \sum_{i=1}^{n} \frac{1}{k \cdot S_{r,i}^{-m}} \tag{4}$$

$$S_{r,eq} = \left( \sum_{i=1}^{n} \frac{S_{r,i}^{m}}{N_{eq}} \right)^{1/m} \tag{5}$$

From equation 2.5 it can be shown that the ratio of two equivalent load ranges of two spectra is independent of the chosen $N_{eq}$.

## 3 FATIGUE EQUIVALENT LOAD CYCLE

To take into account both the mean level and the range of a cycle different S-N curves have to be used for different R values (R = $\frac{S_{min}}{S_{max}}$). The fatigue behaviour of the several materials used in wind turbine design is very complex. It is proposed to use a simplified fatigue formulation. A symmetric Goodman diagram with straight constant life lines:

$$N = k \cdot (\frac{S_r}{S_u - |S_m|})^{-m} \tag{6}$$

In which $S_u$ is the ultimate load.

The damage caused by a load spectrum of $n$ (constant amplitude) cycles with ranges $S_{r,i}$ and means $S_{m,i}$:

$$D = \sum_{i=1}^{n} \frac{1}{k \cdot \left(\frac{S_{r,i}}{S_u - |S_{m,i}|}\right)^{-m}} \tag{7}$$

$$= \sum_{i=1}^{n} \frac{1}{k} \cdot \left(\frac{S_u - |S_{m,i}|}{S_{r,i}}\right)^{-m} \tag{8}$$

The damage caused by $N_{eq}$ cycles with range $S_{r,eq}$ and mean $S_{m,eq}$ has to equal the damage of the above load spectrum:

$$\frac{N_{eq}}{k} \cdot \left(\frac{S_u - |S_{m,eq}|}{S_{r,eq}}\right)^{-m} = \sum_{i=1}^{n} \frac{1}{k} \cdot \left(\frac{S_u - |S_{m,i}|}{S_{r,i}}\right)^{-m} \tag{9}$$

or

$$S_{r,eq} = \left(\sum_{i=1}^{n} \frac{\left(S_{r,i} \cdot \frac{S_u - |S_{m,eq}|}{S_u - |S_{m,i}|}\right)^{m}}{N_{eq}}\right)^{1/m} \tag{10}$$

From equation 3.5 it can be shown that the ratio of two equivalent load ranges of two spectra is independent of the chosen $N_{eq}$ and independent of the chosen $S_{m,eq}$. So similar to the choice of $N_{eq}$ an arbitrary choice for $S_{m,eq}$ may be given. It is proposed to evaluate the equivalent load cycle range with $S_{m,eq} = 0$.

Note that equation 3.5 equals equation 2.5 for the limit $S_u \rightarrow \infty$ or for $S_{m,eq} = S_{m,i} = 0, \forall_{i=1,n}$

Similar to evaluating the equivalent load range with different slopes m of the S-N curves the equivalent load cycle may be evaluated with different slopes m and different ultimate strength values $S_u$. Both the parameters m and $S_u$ represent the material behaviour. The parameters $S_u$ is however not dimensionless but has the dimension of the load under consideration. It is preferable to relate the property to the level of the load spectrum and to use a dimensionless parameter. The ratio of the maximum occurring load $S_{max}$ over $S_u$ can be used to determine some valid choices for $S_u$. In every design this ratio will be smaller than 1 and larger than 0 (note that a ratio 0 stands for $S_u \rightarrow \infty$). It is proposed to evaluate 4 different $S_u$ over $S_{max}$ rates: 0.2, 0.4, 0.6, and 0.8. In case for reason of comparison more than one spectrum is analysed, all spectrums should be analysed with the same value for $S_u$.

# 4 EXAMPLE

## 4.1 Measured time series

With the proposed equivalent load cycle method the influence of different material behaviour with respect to the mean level may be evaluated. As an example a measured time series (labelled "str.crs") used in a benchmark exercise for the Mounturb project is analysed ([3]. The 1 Hz equivalent load ranges calculated for different slopes of the S-N curve are listed in table 1. The equivalent mean $S_m, eq$ equals 0. The maximum occurring load in the time series is equal to 29.76, for practicale reasons the value 30 has been used.

Table 1: Example of ranges of equivalent cycles

| | m=4 | m=6 | m=8 | m=10 |
|---|---|---|---|---|
| equivalent range method: $\frac{30}{S_u} = 0.0$ | 12.95 | 14.40 | 16.10 | 17.66 |
| $\frac{30}{S_u} = 0.2$ | 14.46 | 16.07 | 17.92 | 19.63 |
| $\frac{30}{S_u} = 0.4$ | 16.41 | 18.21 | 20.25 | 22.13 |
| $\frac{30}{S_u} = 0.6$ | 19.02 | 21.07 | 23.35 | 25.42 |
| $\frac{30}{S_u} = 0.8$ | 22.72 | 25.14 | 27.74 | 30.04 |

Because the equivalent load cycle method is meant to be used for the comparison of different load spectrums the absolute value is of less importance then the ratio of the equivalent load range of two spectrums. To get an idea of the influence of the mean level on this ratio the above mentioned time series have been split into two parts of equal length. The ratio of the equivalent load cycle of the first half over the last half is shown in table 2. The choice of $S_u$ influences the ratio up to 6% (even as the choice of m).

Table 2: Example of the ratio of two ranges of equivalent cycles, measured time series

| | m=4 | m=6 | m=8 | m=10 |
|---|---|---|---|---|
| equivalent range method: $\frac{30}{S_u} = 0.0$ | 1.02 | 0.95 | 0.90 | 0.87 |
| $\frac{30}{S_u} = 0.2$ | 1.01 | 0.95 | 0.90 | 0.87 |
| $\frac{30}{S_u} = 0.4$ | 1.00 | 0.95 | 0.91 | 0.88 |
| $\frac{30}{S_u} = 0.6$ | 0.99 | 0.94 | 0.91 | 0.89 |
| $\frac{30}{S_u} = 0.8$ | 0.98 | 0.94 | 0.92 | 0.90 |

Two examples of the Goodman or constant life diagrams are given in the figures 1 and 2.

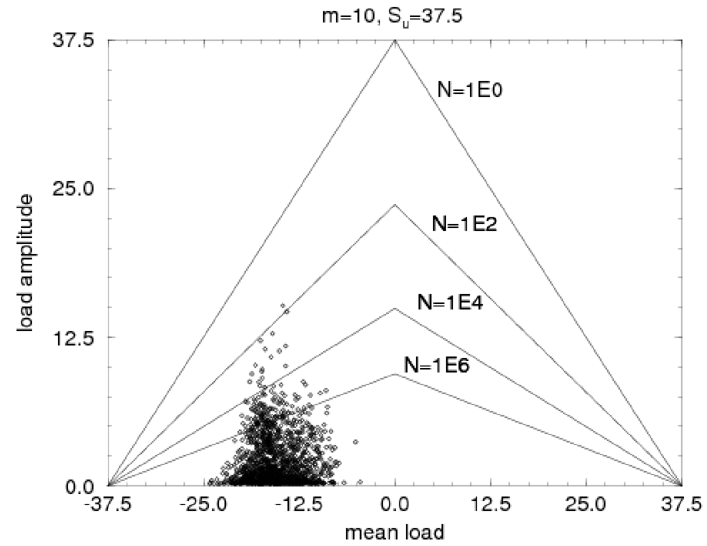


Figure 1: Goodman or constant life diagram for m10, high $\frac{S_{max}}{S_u}$

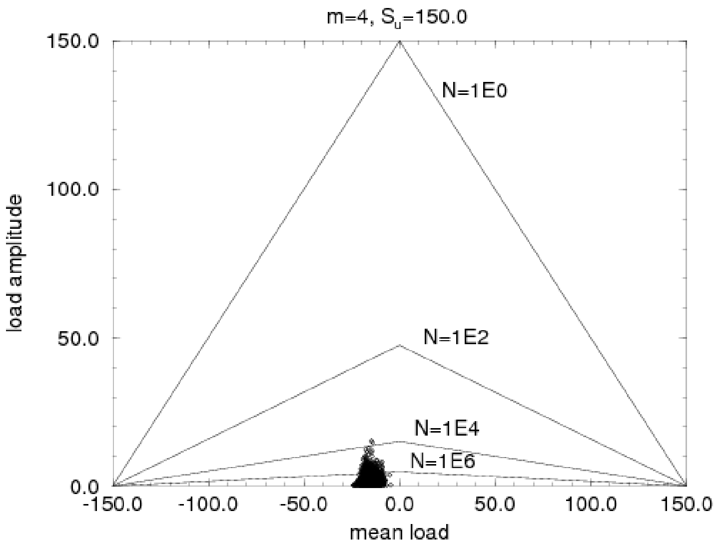


Figure 2: Goodman or constant life diagram for m4, low $\frac{S_{max}}{S_u}$

## 4.2 Simulated time series

As an example the WINCON 110XT wind turbine has been simulated. Three different simulations of 320 seconds each have been performed. Only the turbulent length scale of the u-component has been varied.

The following wind input parameters have been used:

| | |
|---|---|
| standard deviation of u-component $\sigma_u$ | : 2.000 |
| standard deviation of v-component $\sigma_v$ | : 2.000 |
| standard deviation of w-component $\sigma_w$ | : 1.000 |
| turbulent length scale of u-component ${}^xL_u$ | : 100.0/200.0/400.0 |
| turbulent length scale of v-component ${}^xL_v$ | : 100.0 |
| turbulent length scale of w-component ${}^xL_w$ | : 10.0 |
| decay factor for u-component $A_u$ | : 8.5 |
| decay factor for v-component $A_v$ | : 8.5 |
| decay factor for w-component $A_w$ | : 8.5 |
| the angle $\phi$ between the mean wind speed and the horizontal | : 0.0 |

Table 3: Ratio of the equivalent cycles for ${}^xL_u$=200 and ${}^xL_u$=100

| | m=4 | m=6 | m=8 | m=10 |
|---|---|---|---|---|
| equivalent range method: $\frac{30}{S_u} = 0.0$ | 0.94 | 0.94 | 0.93 | 0.93 |
| $\frac{30}{S_u} = 0.2$ | 0.95 | 0.94 | 0.94 | 0.94 |
| $\frac{30}{S_u} = 0.4$ | 0.95 | 0.95 | 0.94 | 0.94 |
| $\frac{30}{S_u} = 0.6$ | 0.95 | 0.95 | 0.95 | 0.95 |
| $\frac{30}{S_u} = 0.8$ | 0.96 | 0.96 | 0.96 | 0.96 |
| $\frac{30}{S_u} = 1.0$ | 0.96 | 0.96 | 0.97 | 0.97 |

Table 4: Ratio of the equivalent cycles for ${}^xL_u$=400 and ${}^xL_u$=100

| | m=4 | m=6 | m=8 | m=10 |
|---|---|---|---|---|
| equivalent range method: $\frac{30}{S_u} = 0.0$ | 0.90 | 0.89 | 0.88 | 0.88 |
| $\frac{30}{S_u} = 0.2$ | 0.90 | 0.89 | 0.89 | 0.89 |
| $\frac{30}{S_u} = 0.4$ | 0.91 | 0.90 | 0.90 | 0.90 |
| $\frac{30}{S_u} = 0.6$ | 0.91 | 0.91 | 0.91 | 0.92 |
| $\frac{30}{S_u} = 0.8$ | 0.92 | 0.92 | 0.93 | 0.94 |
| $\frac{30}{S_u} = 1.0$ | 0.93 | 0.94 | 0.95 | 0.95 |

The choice of $S_u$ influences the ratio in a larger extent than the choice of m.

# 5 CONCLUSIONS

A simple extension to the equivalent load range method is presented by which different load spectrums may be compared taking the into account both the range and mean of each cycle. The fatigue behaviour of the material is characterised by straight S-N curves for given mean value or a symmetric Goodman diagram with straight lines.

The two material parameters are the slope of the S-N lines and the ultimate load $S_u$. Three examples of the ratio of the equivalent load cycle of two load spectrums are given. The choice of the two material parameters influences this ratio in the same magnitude.

# REFERENCES

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| | Date: | October 1995 | Number of report: ECN-C–95-074 |
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| **Title** | Fatigue Equivalent Load Cycle Method |
| **Subtitle** | A General Method to Compare the Fatigue Loading of Different Load Spectrums |
| **Author(s)** | H.B. Hendriks B.H. Bulder |
| **Principals(s)** | European Commission DGXII<br>NOVEM |
| **ECN project number** | 1.4352 |
| **Principals(s)project number** | JOU2-CT93-0378<br>NOVEM 224.720-9445 |
| **Programme(s)** | JOULETWIN |


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