JOINT INVESTIGATION OF DYNAMIC INFLOW EFFECTS AND IMPLEMENTATION OF AN ENGINEERING METHOD

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Abstract

This report describes the work performed within the CEC JOULE 1 project 'Joint Investigation of Dynamic Inflow effects and Implementation of an Engineering Method'

The following organisations (and persons) cooperated in the projects:

- Netherlands Energy Research Foundation (ECN) (coordinator) together with Stork Product Engineering (SPE) (both NL): H. Snel, J.G. Schepers, B. Montgomery, Th. van Holten;
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- Tekniskgruppen AB (TA, SE): H. Garander (on voluntary basis)
- National Technical University of Athens (NTUA, GR), together with Groupe d’Energetique et de Mecanique du Havre (GEMH, FR): S. Voutsinas, S. Huberson

The aim of the projects was to develop one or more engineering models, for the wake induced unsteadiness and non-uniformity in the rotor inflow, which occur at pitching variations (full as well as partial span), coherent wind gusts and yawed conditions.

An engineering method means a computationally effective model which can be implemented in state of the art aerelastic computer programs for wind turbine design.

Validation of the engineering methods was based on full scale measurements, wind tunnel measurements and comparison of results with advanced free wake methods. It is shown that the models which have been developed lead to a definite improvement to standard blade element momentum theory. This is of particular importance for the correct prediction of yaw loads and loads on turbines which are exposed to pitching variations.

Keywords

Aerodynamics
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Distribution

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Joint investigation of dynamic inflow effects and implementation of an engineering method
SUMMARY

This report describes the work and the results of the project 'Joint investigation of Dynamic Inflow effects and implementation of an engineering method for response calculations', in short DYNAMIC INFLOW, which is performed within the framework of the JOULE 1 project. The work was sponsored by the European Commission under contract and on a 50% reimbursement of the estimated total cost.

The objective of the project was the development, implementation and validation of engineering model(s) of dynamic inflow. The model(s) must be easily implemented within existing wind turbine aerelastic codes. Therefore the model(s) should be economic in computer usage, comparable to the present generation of (equilibrium) inflow models. The model(s) should provide an appreciable improvement in load prediction when compared with those presently in use. To this end the project had three constituent parts;

1. Implementation of dynamic inflow models within existing aerelastic codes
2. Use of free vortex wake models to obtain phenomenological insight into flow features to aid in the validation of the engineering models.
3. Full scale and wind tunnel measurements of wind turbine response during dynamic inflow events to validate the models.

The project was contracted in two parts. The original contract contemplated the modelling of axisymmetric conditions (pitching transients and coherent wind gusts) and validation through measurements on the Tjaereborg wind turbine. Work under this contract commenced in October of 1990. In July of 1992 the contract was extended by the Commission. Modelling of yawed flow conditions was added, together with yawed flow load measurements on the Tjaereborg turbine and wind tunnel measurements both under axisymmetric and under yawed conditions. The National Technical University of Athens also joined the project.

In the project 8 different models have been applied, which are abbreviated in the following way:

- ECN, i.e.: ECN integral wake model, a prescribed wake model;
- ECN, d.e.: ECN differential equation model, engineering model: A model with a time derivative added to the axial momentum equation and Glaucrt terms for the yaw modelling;
- DUT: Delft University of Technology, acceleration potential model;
- GH: Garnad and Hassan Ltd., engineering model based on Pit and Peters;
- NTUA: National Technical University of Athens, free wake Vortex Particle Method;
- TA: Teknikgruppen AB, engineering model for yawed conditions, based on Glaucrt type of modelling;
- TUDK: Technical University of Denmark, engineering model. A model with a time derivative added to the momentum equation and Glaucrt terms for the yaw modelling;
- Univ (or UBM): University of Stuttgart, free wake panel method. In the first phase of the project, this model was applied by the University of the Bundeswehr.

The most important results from the project are:
• A clear dynamic inflow effect was observed in measured data obtained with the Tjæreborg turbine at pitching steps. It is shown that the models which are developed in the present project predict the response of the turbine much better. The conventional methods underpredict the load fluctuations at pitching variations.

• The dynamic inflow effects for coherent wind gusts appeared to be very limited. This was derived from theoretical considerations, and confirmed by wind tunnel measurements.

• It is shown that the conventional models do not correctly predict the loads on a wind turbine under yawed conditions. This is in particular true for the yawing moment. This moment has negligible values when calculated with the conventional methods. The newly developed engineering models, calculate at least the sign and the order of magnitude correctly (with a limited computational time), which is very important for the design of yawed controlled turbines.

• The engineering models were not suited to predict the details of the flow field under yawed conditions. This could be concluded from the comparison with wind tunnel measurements. The free wake methods predicted these details better. It is obvious however that these methods require more computational effort.

Apart from ad-hoc presentations and presentations at JOULE contractor’s meetings, intermediate results were presented at the:
• EWEC conference in October 1991 in Amsterdam [1];
• ECWEC conference in March 1993 in Travemünde [2];
• European Rotorcraft Forum in October 1994 in Amsterdam [3];
• EWEC conference in October 1994 in Thessaloniki [4];
• Journal of Wind Engineering and Industrial Aerodynamics [5]
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1. INTRODUCTION

Wind turbines operate in a very instationary environment. The aerodynamic loads on blades and structures consequently are of an instationary nature. Nevertheless, most of the operational computer codes for the prediction of these loads and of the dynamic behaviour of turbines, make use of quasi steady aerodynamics. Comparison of calculational results with measurements has shown that, under certain circumstances, important and systematic differences are obtained which can be attributed to the unsteadiness.

Instationary effects may be divided into two parts, viz:

- instationary profile aerodynamics and
- dynamic inflow.

The first part accounts for the dependence of sectional aerodynamical forces on the time varying angle of attack. This includes the effect of the shed vorticity in the immediate near wake on the shedding blade itself. The shed vorticity results from circulation changes in time for a section, and the vortex lines emanate parallel to the blade trailing edge. The characteristic time scale is the ratio of the chord to the effective velocity seen by the blade section, approximately $\Omega r/\Omega r$. More or less representative values for this time scale are then approximately 0.2 s (at blade root) and 0.01 s (at the tip).

The second part, dynamic inflow, accounts for the influence of the time varying trailing wake vorticity on the inflow velocity in the rotor plane. Trailing wake vorticity is the result of varying circulation strength along the blade axis; it also exists in the stationary case. It is especially strong where the changes are important, i.e. near the blade tip and the blade root. Trailing vorticity lines emanate perpendicular to the blade axis. The characteristic time scale for this phenomenon is $D/V_\infty$. For large turbines, this will be of the order of 5 to 10 seconds, hence one or two orders of magnitude larger than the profile time scale. Phenomena having a time scale large compared to this may be regarded as quasi steady.

Conditions for which dynamic inflow is of importance include:
- coherent wind gusts (typical time scale of a few seconds);
- blade pitching actions (time scale 0.5 to 2 seconds);
- 1P phenomena such as yawed operation (time scale 1 to 2 seconds).

Within the EU JOULE program, a project was carried out with the aim of improving the aerodynamic and aeroelastic response programs, by implementation of a verified "engineering" model for dynamic inflow effects. The qualification "engineering" is meant to indicate that the model can be implemented in PC or workstation based programs, without unduly increasing the run time. This condition makes it almost imperative that the engineering models be modifications of the blade element-momentum (bem) method that has traditionally been used by wind turbine designers. Instationary profile aerodynamics is usually accounted for by changing the stationary profile characteristics for unsteady relations in the blade element force calculations. Dynamic inflow effects can be taken into account by changing the equilibrium momentum part of the equations into a dynamic relation, including azimuthal and radial dependancy of the induction terms.
As is true for all design tools, it is essential that the models developed are verified and validated. This is done in the present project by mutual comparison, by comparison with more sophisticated aerodynamic models and by comparison with measured results. For the sophisticated models, use is made of free vortex wake methods as developed by the University of the Bundeswehr (in a later stage of the project: the Stuttgart University in Germany) and by the National Technical University of Athens, Greece. The acceleration potential method of the Delft University of Technology has been useful in providing flow field details. In fact, the use of the above methods makes it possible to compare models on the basis of the resulting flow field, which aids to the understanding of the flow phenomena and is an important asset in the process of modelling. It should be stated that these models have important advantages in the sense of representing the basic physical phenomena (although not free from approximation, see section 7.2.2). However, the computer time needed for calculations with this type of model is still such that inclusion in an aerelastic design code can not (yet) be considered as sensible.

For experimental results, dedicated load measurements were carried out at the Tjæreborg 2MW turbine (60 m diameter) and the DUT open jet wind tunnel on a 1.2 m diameter model. At this utility, some flow field measurements are also performed. In a JOULE II sequel to the project, more validation measurements will be carried out on other instrumented wind turbines. The combination of modelling on different levels, full scale measurements and wind tunnel measurements has been very advantageous for the project.

With regard to technical contents, the project can be divided into two parts:
- An investigation of dynamic inflow under axisymmetric conditions (pitching transients and coherent wind gusts);
- An investigation of dynamic inflow under asymmetric conditions (yaw misalignment).

Due to sequence in which components of the project were contracted, the chronological order in which the different activities are performed does not always follow this partition. In fact, the Tjæreborg axially symmetric measurements (pitching transients) were carried out at the start of the project, while those in the DeH3 wind tunnel (wind gusts) were executed in the end. In the intermediate period the yawed flow measurements were carried out both on the Tjæreborg turbine and on the wind tunnel model.
2. OBJECTIVE OF THE PROJECT

For a good understanding of the work, the statement of the objective of the investigations, forming part of the project description, is reproduced here and somewhat elaborated.

The main objective of the project is the definition and implementation of one or more simple theoretical models for dynamic inflow (axisymmetric and asymmetric). These models must be able to calculate the wake induced velocities along the rotor blade as a function of time and azimuth.

The models must be such that they can easily be implemented into available computer codes for aeroelastic loads and the dynamic behaviour of wind turbines. For that reason it should be economic in computer time usage, comparable to the present generation of (equilibrium) inflow models. The model must result in an appreciable improvement in comparison with the stationary models commonly used.

The models are to be verified, on one hand by comparison with complex instationary vortex wake models, on the other with experiments on wind tunnel model and full scale rotors.
3. WORK PROCEDURE

The project was contracted in two parts. The original contract contemplated the modelling of axisymmetric conditions (pitching transients and coherent wind gusts) and validation through measurements on the Tjæreborg 2MW wind turbine. Work under this contract commenced in October of 1990. In July of 1992 the contract was extended by the Commission. Modelling of yawed flow conditions was added, together with yawed flow load measurements on the Tjæreborg turbine and wind tunnel measurements both under axisymmetric and under yawed conditions. The National Technical University of Athens also joined the project.

Chapter 4 lists the participants in the project.

The project includes three different types of work:
1. Development, implementation and validation of engineering model(s) for dynamic inflow. This is the core of the project.
2. Use of the free vortex models (Universities of Stuttgart and Athens), in order to obtain phenomenological insight in flow features and to (partially) validate the engineering models.
3. Dedicated full scale and wind tunnel measurements, for the validation of the models.

The methodology followed in the project can be characterized as follows:

- On the basis of existing methods and physical reasoning, engineering models have been developed and implemented into computer programs by the different participants. These models are described and explained in chapter 7 for axisymmetric conditions and chapter 11 for yawed asymmetric conditions.
- Testcases were defined in which results of the engineering models were compared mutually and with results of free vortex wake calculations. For these cases, purely aerodynamic quantities were calculated, i.e. induced velocities and aerodynamic loads. Insight gained in this way was instrumental in further model development.
- Specific measurements were requested from the Tjæreborg turbine (pitching transients and yawed flow measurements) and from the wind tunnel environment (wind gusts and yawed flow). The engineering models were implemented in the aerelastic codes used by the different participants, in order to compare calculated structural loads with measured values. This formed the final validation of the models and their implementation.

The experimental configurations (i.e. the Tjæreborg turbine and the wind tunnel model) are detailed in chapter 5. The Tjæreborg pitching transient measurements are described in chapter 8.1. This is followed by a description and evaluation of the different calculational cases carried out on the Tjæreborg geometry in chapter 8.2 and 8.3. Four Tjæreborg cases are distinguished, viz:
- Case I, a preliminary test case, consisting of a simple windstep and a simple pitching transient. This case does not refer to a measurement campaign.
- Case II, consisting of a number of pitching transients, under comparable conditions with measured campaigns. Blade and shaft loads are compared with measurements. Also, the time history of induced velocities was calculated in order to aid in the interpretation of the results.
- Case III, consisting of the same pitching transients, but now structural dynamics was included in the calculations. A more complete comparison with
measurements was made possible in this way.
- Case IV, taken in accordance with measured safety stops, using blade feathering.

The axisymmetric wind tunnel case (consisting of a wind speed step) is described in section 9. This case is denoted by case 'tungust'.

The Tjørebøg measurements under yawed conditions are described in section 12. The description and analysis of these cases are given in section 12.2 and 12.3. The following cases have been performed:
- Case V, a preliminary test case, which was used to gain first experiences with the yaw models;
- Case VII: The simulation of the measurements which had become available. Four measurement series were reproduced at yaw angles, ranging from -51° to +54°. Azimuthal binned averaged values have been compared. In addition, the calculation of so-called 'summary data' was performed, which consist of the prediction of rotor averaged values as function of free stream wind speed and yaw angle. Verification took place by mutual comparison of calculated results.

The wind tunnel measurements under yawed conditions are described in section 13.1. Four different measurement series were available at yaw angles, ranging from 0 to 30 degrees. These cases are denoted by case VI.1 to VI.4. In addition to the mechanical loads, details of the flow field were measured as well. The reproduction of these measurements is described in section 13.2 and 13.3.

In the course of the execution of these cases, some of the models have undergone changes, as a result of improved understanding. Also, in many cases simple coding errors (bugs) were removed from the implementations. In general, the cases II, III, IV, VI, and VII show the results of the matured models and implementations.

The calculational cases considered by the participants in the original contract, were also carried out by the NTUA when this organization joined the project in 1992. Furthermore Teknikgruppen AB has performed the simulation of case VII on a voluntary basis, although they officially did not participate.

The group of participants meted with a frequency of approximately 3 times per year. During these plenary meetings, model features were presented and discussed. Measurement campaigns were defined, discussed and results thereof were presented. Calculational results were compared with measurements and interpreted. The main items of each meeting are recorded in the minutes, by the coordinator.

Within the course of the project, it appeared to be very difficult but also very useful to get consensus about sign conventions and definitions. Agreement was reached on the conventions given in Appendix A.
4. PARTICIPANTS

The following parties participated in the project:

- Netherlands Energy Research Foundation (ECN), NL (coordinator) together with Stork Products Engineering (SPE) NL;
- University of the Bundeswehr (UBM), Munich, later the University of Stuttgart (Unist.), FRG;
- National Technical University of Athens (NTUA), Gr. together with Laboratoire de Mecanique des Fluides, University of Le Havre (F);
- Garrad Hassan and Partners (GH), UK;
- Delft University of Technology (DUT), NL;
- Technical University of Denmark (TUDk), Fluid Mechanics Department Lyngby, together with Elsamprojekt A/S, Fredericia, DK;

Teknikgruppen, AB (S) participated on a voluntary basis.
5. CHARACTERISTICS OF THE TJÆREBERG TURBINE AND THE WIND TUNNEL MODEL

Both full scale measurements and wind tunnel measurement are used in the project. The present chapter summarizes the characteristics of the two facilities used, viz. the Tjæreborg 2MW wind turbine and the DUT open jet wind tunnel and model turbine.

The Tjæreborg turbine was used for measurements of pitching transient (including safety stops) and for measurements in yawed flow conditions, (yaw angles between -50 degrees and +50 degrees approximately).

The DUT open jet wind tunnel has been used for measurements of the wind gust and for measurement in yawed flow (both of rotor loads and of the flow field).

The global characteristics of the Tjæreborg turbine and of the wind tunnel model are listed in the following subsections. Detailed data, as supplied by the operators of the facilities are given in Appendix T, Appendix U and Appendix S. These data have been used by the participants to compose the aeroelastic model of the turbines, and should be sufficient for any party who would like to compare his method with the results as reported here. The aeroelastic turbine data can be obtained from the coordinator on floppy disc, upon request.

5.1 Tjæreborg turbine

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>61 m</td>
</tr>
<tr>
<td>Hub height</td>
<td>60 m</td>
</tr>
<tr>
<td>No. of blades</td>
<td>3</td>
</tr>
<tr>
<td>Tilt angle</td>
<td>3°</td>
</tr>
<tr>
<td>Cone angle</td>
<td>0°</td>
</tr>
<tr>
<td>( V_{rated} )</td>
<td>15 m/s</td>
</tr>
<tr>
<td>( P_{rated} )</td>
<td>2 MW</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>22.3 rpm</td>
</tr>
<tr>
<td>Generator</td>
<td>2% slip</td>
</tr>
</tbody>
</table>

The blade eigenfrequencies are:

- Flap: 3.3 P (rotational)
- Lead-lag: 6.3 P
- Torsional: very stiff

Drive train eigenfrequency: approximately 2P.

Tower bending eigenfrequency: 2.18P (0.81 Hz)
Blade bending moments (flatwise and edgewise) are measured at $r = 2.75$ m. Furthermore, the torque on the low speed shaft is measured, but the torque which is presented in this report is calculated from the measured torsion of the high speed shaft by multiplication with the gear box ratio, as the real main shaft torque sensor is unreliable.

The flow field is measured with a (fixed) meteorological mast with cup anemometers and wind vanes at 5 different heights. In all calculational cases, this meteorological mast was upstream of the turbine.

5.2 Wind tunnel model

<table>
<thead>
<tr>
<th>Diameter</th>
<th>1.2 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades</td>
<td>2</td>
</tr>
<tr>
<td>Root cut out</td>
<td>30%</td>
</tr>
<tr>
<td>Aerofoil sections</td>
<td>NACA 0012</td>
</tr>
<tr>
<td>Chord</td>
<td>0.08 m (no taper)</td>
</tr>
<tr>
<td>Total twist</td>
<td>6 degrees from $0.3 &lt; r/R &lt; 0.9$ outer 10% of blade: untwisted</td>
</tr>
<tr>
<td>Rotor speed</td>
<td>0.25 - 16 Hz ($\pm 0.05$ Hz)</td>
</tr>
</tbody>
</table>

OPEN-JET WIND TUNNEL data:

<table>
<thead>
<tr>
<th>Outlet diameter</th>
<th>2.24 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum wind speed</td>
<td>14.5 m/s</td>
</tr>
<tr>
<td>Turbulence intensity</td>
<td>0.8%</td>
</tr>
<tr>
<td>Uniformity over section</td>
<td>within 2.5%</td>
</tr>
</tbody>
</table>

Measured quantities:

- Flap and lead lag blade root bending moments at $r = 0.129$ m radial position (at different blades)
- Rotor torque and thrust (not for all cases).
- Azimuth angle and rotor speed;
- Undisturbed tunnel wind velocity by means of pilot tube and Betz manometer;
- Additional flow velocities are measured with a hot wire system, for specific cases.
6. REVIEW OF DYNAMIC INFLOW MODELLING IN HELICOPTER AERODYNAMICS.

6.1 Introduction

Before describing the dynamic inflow models developed and used in the present project, it is useful to roughly sketch the inflow models that have been applied in helicopter aerodynamics.

There is a long history of inflow modelling in this field of application, even more so for 'yawed flow conditions' (forward flight) than for axisymmetric conditions (hover and vertical flight). Therefore both yawed flow conditions as well as axisymmetric conditions are discussed in this section. Wind turbine aerodynamics can benefit much from the experience gained in helicopter applications, although it should be kept in mind that the flow situations are not always comparable. Helicopter rotors are designed for high thrust at the lowest possible power consumption, which leads to low values for the induction factor $a = u_r/V_{\infty}$. Wind turbine rotors are designed for maximum power extraction, which leads to a highly loaded disk and values of the induction factor around $1/3$. Linearization applied in helicopter aerodynamics must often be questioned for wind turbine applications.

The present chapter intends to provide a review of helicopter dynamic inflow theory with a view to wind turbine applicability. In the main, the nomenclature that has become usual in wind turbine aerodynamics will be adhered to, with an occasional reference to helicopter notation, which is quite different. The present review draws heavily on two recent review articles from the helicopter literature, [1] and [2]. An attempt is made to connect the modelling aspects to a physical description of the phenomena.

6.2 Inflow models

Dynamic inflow phenomena have an influence on fluctuating rotor loads. In the case of helicopters, these are of importance for the motion of the craft. Hence, much of the early and present interest is concerned with flight dynamics. In the wind turbine application, the load levels and fatigue loads are influenced, both for the individual blades and for the yaw mechanism and the hub, together with the power quality. The first inflow phenomena to be studied were connected with forward flight, the equivalence of yaw misalignment. Unsteadiness arising from 'collective pitch' motions of the blades, (e.g. for ascending and descending flight), comparable to pitching transients for the wind turbine, were only studied at a later moment. Modern dynamic inflow models include both aspects.

In describing the axial (normal to the plane of rotation) induced velocity in the rotorplane, the following aspects can be discerned:

---

1Terms between brackets indicate the equivalent helicopter cases.

2In helicopter literature, the usual definition and notation for the induced velocity coefficient is $\lambda = V_{\infty} \cos \phi - u_r/\lambda_r$, if $\phi_r$ is the yaw misalignment angle. Keep in mind that the sign of $u_r$ in helicopter applications is opposite to the sign for wind turbines.
Joint investigation of dynamic inflow effects and implementation of an engineering method

- its disk averaged value, which will be described by \( u_{i,0} \)
- its distribution over the rotorplane, a function of azimuth and radial position.
  Due to \( 2\pi \) interval of the azimuth, this can be expanded in a Fourier series at each radial position.

Hence \( u_i \) can be written as:

\[
u_i = u_{i,0} \cdot f_0 \left( \frac{r}{R} \right) + \sum_{n=1}^{\infty} \left( v_{c,n} \left( \frac{r}{R}, \phi \right) \cos(n\phi) + v_{s,n} \left( r/R, \phi_\lambda \right) \sin(n\phi) \right) \tag{6.1}\]

In this formula, \( f_0 (r/R) \) must have a disk average equal to unity:

\[
\frac{2}{R^2} \int_0^R f_0 \left( \frac{r}{R} \right) dr = 1 \tag{6.2}\]

while all Fourier coefficients (which depend on \( \phi \)) and on the yaw misalignment angle \( \phi_\lambda \) must be equal to zero at \( r=0 \).

6.3 The disk averaged value of the induced velocity.

The first model of the type of equation 6.2 is due to Glaubert [3], see also the discussion in Johnson [4], pp 126 and following. In this model, the disk averaged induced velocity \( u_{i,0} \) is related to the disk loading through a formula which is valid both for axially symmetric flow (\( \phi_\lambda = 0 \)) and for a circular wing, i.e. a loaded disk aligned with the outer flow (\( \phi_\lambda = 90 \) degrees), viz:

\[
T = \rho S |\tilde{V}_\infty + \tilde{u}_{i,0}| \cdot 2 \tilde{u}_{i,0} \tag{6.3}\]

Here both the thrust force \( T \) and the induced velocity \( u_{i,0} \) are normal to the plane of rotation, along the rotor shaft direction. It was next assumed by Glaubert that the same expression holds for all misalignment angles between 0 degrees and 90 degrees. However, the forces in the two limiting cases are of a different nature.

In the axially symmetric case, the term \( \rho S |\tilde{V}_\infty + \tilde{u}_{i,0}| \) expresses the mass flow through the rotor disk, which undergoes a momentum change equal to \( 2 \tilde{u}_{i,0} \) in the direction of the undisturbed flow velocity. The thrust force \( T \) is of drag type and can only be introduced in the (potential) flow model through the action of an actuator disk. There is no net momentum interchange between the flow across the actuator disk and the flow passing outside of it.

In the case of the wing, the expression does not involve the mass flow across the rotor, but a 'representative' mass flowing around the wing which would undergo a momentum change of magnitude \( 2 \tilde{u}_{i,0} \) perpendicular to the undisturbed flow velocity. Hence the thrust force \( T \) is now of lift type. There is a momentum exchange between the flow across the disk and the exterior flow.

Glaubert's formula for the disk averaged induced velocity is still widely used in helicopter aerodynamics in the form:

\[
\nu_{i,0} = \frac{C_T}{2(\mu^2 + \lambda^2)^2} \tag{6.4}\]
where $\mu$ is the advance ratio, defined as $V_{\infty}\sin(\phi_y)/\Omega r$, and $\lambda$ is the inflow ratio defined before. $C_T$ is the thrust force coefficient, in helicopter practice defined as: $C_T = T/[(\rho S(\Omega R)^2]$. The induced velocity $v_{i,0}$ in 6.4 is also nondimensionalized by $\Omega R$: $v_{i,0} = u_{i,0} / \Omega R$. Equation 6.4 also forms part of the widely used Pitt and Peters model which will be discussed in the next section.

It is difficult to find an experimental validation for the theoretical value of the disk averaged induced velocity, for values of the yaw misalignment angle that are of interest for wind turbines, i.e. $\phi_y \approx 30$ degrees. For higher values, very close to the circular wing limit ($\phi_y$ approximately 90 degrees), [11] (table 4) tabulates some comparison with measurements. The theoretical values are higher than those inferred from the experiments, by 11% to 35% (!)

6.4 The distribution of the induced velocity over the disk.

For the distribution of the induced velocity over the disk, Glauert only maintained the first sine term, proposing:

$$u_i = u_{i,0}(1 - \frac{r}{R} K_c \sin(\phi_y))$$

(6.5)

with $u_{i,0}$ from 6.3

This term is clearly needed to express the influence of the skewed wake position, which causes a larger induced velocity in the 'downwind' half of the rotorplane compared to the 'upwind' part, see figure 6.1.

![Figure 6.1 Skewed wake](image)

It is also the term which is responsible for the stabilizing yaw moment, as the blades in the upwind part of the rotorplane experience larger angles of attack and
consequently larger aerodynamical forces (assuming linear aerodynamics), than those in the downwind part. Coleman, Feingold and Stemplin [5] found a relation for the factor $K_c$ in Glauert’s expression as a function of the wake skew angle $\chi$ (also defined in Figure 6.1, assuming a slanted cylindrical vortex wake:

$$K_c = \tan \chi / 2 \quad (6.6)$$

This expression was later refined by Meyer Dreves [6], who included a cosine like variation of the bound vorticity, to:

$$K_c = \frac{4}{3} [1 - 1.8 \left( \frac{\sin \phi_y}{\lambda} \right)^2] \tan \chi / 2 \quad (6.7)$$

where $\lambda$ is the tip speed ratio, and $\sin \phi_y / \lambda$ is quotient of the inplane velocity component and the tip speed, i.e. the advance ratio $\mu$ used in the helicopter literature. Because of the cosine like variation of the bound vorticity, Meyer Dreves also obtained a value for the first cosine like harmonic in the induced velocity distribution, equal to $-2\mu\nu/R$.

For small values of the wake skew angle $\chi$, 6.7 gives a larger value for the induced velocity gradient along the line joining $\phi_y = 90$ degrees and $\phi_y = 270$ degrees than does 6.6. From measurements (see the results, given in Table 3 of [1]), both values appear too low. Better results, although still slightly low as compared to measurements, are obtained with the so called Pitt and Peters model [7]. This model moreover adds more detail to the modelling which must be included from a physical point of view. It is presently considered as state of the art in helicopter applications. The general model will be discussed in a qualitative sense and in more detail in a simplified form.

Before discussing the model, a few observations should be made about some shortcomings of the models discussed above. In Glauert’s formulation, the sinusoidal term in the induced velocity distribution of the induced velocity is a fraction of the disk averaged value. This distribution gives rise to a moment of momentum about the yawing axis in the rotorplane, equal to:

$$M_{yaw} = -\frac{1}{2} RK_c \omega_0 n \sin \theta \quad (6.8)$$

if $\dot{m}$ is the mass flow over the disk, $\rho(V_{\infty}\cos \phi_y - u_{1,0}) A R$.

Hence, its value should not only depend on the thrust force and the wake skew angle, as it does in Glauert’s model, (these two factors determine the average vortex strength and position of the wake) but also on the yawing moment exerted by the turbine on the air, or the negative value of the turbine yawing moment (which determines the sine like distribution of the vortex strength in the wake). This is accounted for in Pitt and Peters’ model by making use of the first few terms of the solution of the flow over an actuator disk with load distributions giving rise to what would be yaw and tilt moments for a wind turbine (pitch and roll moments respectively for a helicopter). In [7] the resulting relation is written

---

Many references can be given to the Pitt and Peters model, some of them containing inaccuracies or ‘typing’ errors, also in the formulae employed. The reference given here seems to be the most adequate.
between on one hand the average induced velocity and it's first two harmonics (hence expression 6.1 truncated at \( n=2 \)), and on the other hand the thrust force, the rolling and pitching moments and the second moments about the same axis. These moments are force moments acting on the actuator disk.

The expressions of [7] are however incremental relations, i.e. they relate increments of the various terms linearly to increments in forces and moments; these relations are used in finding stability derivatives such as damping terms, in which case the linearization is needed. However for the calculation of time histories of loads, one would prefer the nonlinear relations between said quantites. A reduced set of these can be found in [1], where reference is made to [8] as source of the expressions. For the present use, i.e. application to wind turbine systems, the relations are shown below first in the form as given in [1], and secondly translated to wind turbine notation.

\[
\begin{pmatrix}
v_0 \\ v_r \\ v_c
\end{pmatrix} =
\begin{pmatrix}
\frac{1}{3V_T} & 0 & \frac{\lambda_2}{\rho V_T^2} \tan 0.5 \chi \\
0 & \frac{4}{\rho v_m(1 + \cos \chi)} & 0 \\
\frac{15 \pi}{64 \rho V_T} \tan 0.5 \chi & 0 & \frac{-3}{\rho v_m(1 + \cos \chi)}
\end{pmatrix}
\begin{pmatrix}
C_T \\ C_{L\gamma} \\ C_{M\gamma}
\end{pmatrix}.
\tag{6.9}
\]

with

\[
V_T = (\mu^2 + \lambda^3)^{0.5} \tag{6.10}
\]

\[
V_m = \frac{\mu^2 + \lambda(\lambda + V_0)}{V_T} \tag{6.11}
\]

where \( \mu \) and \( \lambda \) are the advance ratio and inflow ratio defined before. The definition

![Diagram](image)

Figure 6.2 Helicopter notations

of the moment directions, and of the rotor azimuth in the preceding relation are shown in figure 6.2, in accordance with common practice in helicopter literature.
Translated to wind turbine applications, this expression reads, (in scalar form):

\[
2u_{i,0} [V_{\infty}^2 \sin^2 \phi_y] + (V_{\infty} \cos \phi_y - u_{i,0})^2 \right]^0.5 =
\]

\[
\frac{T}{\rho S} + \frac{15\pi}{64} \frac{V_{\infty}^2 \sin^2 \phi_y + (V_{\infty} \cos \phi_y - u_{i,0})^2}{V_{\infty}^2 \sin^2 \phi_y + V_{\infty} \cos \phi_y (V_{\infty} \cos \phi_y - u_{i,0})} \frac{M_{yaw}}{\rho SR} \tag{6.12}
\]

\[-v_{s,1} [V_{\infty}^2 \sin^2 \phi_y + (V_{\infty} \cos \phi_y - u_{i,0})^2]^{0.5} =
\]

\[
\frac{15\pi \tan(0.5\chi)}{64} \frac{4 \cos \chi V_{\infty}^2 \sin^2 \phi_y + (V_{\infty} \cos \phi_y - u_{i,0})^2}{(1 + \cos \chi V_{\infty}^2 \sin^2 \phi_y + V_{\infty} \cos \phi_y (V_{\infty} \cos \phi_y - u_{i,0})} \frac{M_{yaw}}{\rho SR} \tag{6.13}
\]

\[v_{c,1} = \frac{4}{1 + \cos \chi V_{\infty}^2 \sin^2 \phi_y + V_{\infty} \cos \phi_y (V_{\infty} \cos \phi_y - u_{i,0})} \frac{M_{yaw}}{\rho SR} \tag{6.14}
\]

In the first place, it should be noted that the sine and cosine components of the helicopter and wind turbine notation are exchanged and inversely of sign, due to their definitions, shown in figure 6.2, already referred to. Note further that the above result is valid for the situation in which the only in-plane component of the wind velocity is along the direction of \(\phi_0 = 90\) degrees to 270 degrees, hence only yaw- but no 'tilt' misalignment. Finally the formula refer clearly to total disk values of the loads (thrust, yawing and tilt moment) and not to annular contributions as are usually considered in the annular blade element momentum method. It is clear that an annular treatment of the loading is no longer appropriate as axial symmetry, is no longer present under yaw misalignment.

### 6.5 Time dependent inflow determinations.

As mentioned in the introduction, section 6.1, changes in time in the inflow (distribution) that result from time varying loads or external conditions, were studied in the helicopter literature only some time after the initial work in yaw misalignment (forward flight). The first notice of the importance of such effects was given by Carpentar and Fridovich in the '50's, [9]. A transient behaviour was modelled by accounting for a mass of air that had to be accelerated or decelerated in order to reach a new equilibrium state. The work of Pitt and Peters (with a large and ever growing number of associates) includes the formulation of this type of time dependence, in a so called Mass Matrix,

\[
\begin{pmatrix}
\tau \\
V_0 \\
V_s \\
V_c
\end{pmatrix}
\begin{pmatrix}
V_0 \\
V_s \\
V_c
\end{pmatrix}
+
\begin{pmatrix}
V_0 \\
V_s \\
V_c
\end{pmatrix}
=
\begin{pmatrix}
I_R \\
C_{T} \\
C_{M,y} \\
C_{M,t}
\end{pmatrix}
\tag{6.15}
\]

which forms part of the linear system of relations they propose:

with \(\tau\), the time constant matrix, equal to:

\[
\begin{pmatrix}
\tau \\
\end{pmatrix}
=
\begin{pmatrix}
\frac{128}{5\pi} & 0 & 0 \\
0 & \frac{-16}{45\pi} & 0 \\
0 & 0 & \frac{-16}{45\pi}
\end{pmatrix}
\tag{6.16}
\]
In order to facilitate comparison with the models developed and used in the present project, described in the next chapter, the equations are reduced to a single ordinary differential equation for the axisymmetric case ($\phi_y = 0$) and translated to wind turbine terminology. This gives rise to the following form:

$$\frac{256}{75\pi} \frac{\rho S}{\Omega} (V_w - u_i) \frac{d}{dt} u_i + 2\rho S(V_w - u_i)u_i = T$$ (6.17)

This formula is an expression for the disk averaged induced velocity. It has not been used in this form by any of the participants in the project, all of which have used annular rather than disk averaged expressions (see chapter 7).
7. SHORT DESCRIPTION OF MODELS FOR THE AXIAL SYMMETRIC CASES

7.1 Introduction

The present chapter gives a short and comparative overview of the different models used in the project. A detailed description of these can be found in the appendices Appendix J through Appendix Q. The short descriptions and the appendices have been supplied by the respective participants. In the present subsection, a physical description of the phenomena to be modeled will be given.

In the equilibrium situation, the axial flow velocity (inflow) in the rotor plane depends on the wind speed and on the degree of loading (axial force) of the turbine. For instance, for a turbine with zero loading, the speed in the rotor plane is equal to the windspeed, while an operating, loaded wind turbine slows down the wind speed to a lower value. The difference between (the axial component of) the wind speed and the axial flow velocity in the rotor plane is usually called the "induced" velocity, the velocity induced by the presence of the turbine. Figure 7.1 shows this principle for a non loaded and a loaded wind turbine. When the load situations changes, the change in the induced velocity will lag behind, since an appreciable amount of air must be accelerated or decelerated.

![Figure 7.1 Wind turbine with and without loading](image)

In the actual operation of the wind turbine, its load situation is changing continuously, either because of wind speed fluctuations or through blade angle variations (pitch control). Nevertheless, most of the operational computer codes for the prediction of these loads and of the dynamic behaviour of turbines, make use of quasi steady aerodynamic models, assuming at any instant in time an equilibrium.
between the load situation and the induced velocity. In the sequel it will be shown from a comparison of computational results with measurements has shown that, under certain circumstances, notably pitching transients, important and systematic differences occur which can be attributed to unsteadiness and the time lag in the adjustment in the induced velocity.

This phenomenon can be described by taking into account the unsteady formation of the rotor wake, and its effect on the 'inflow' into the rotor plane of the turbine. This is done by the so called free vortex wake models that are able to calculate the wake formation and subsequent development, and its feedback on the inflow in the rotor plane. In the project such methods are contributed by the University of Stuttgart and the National Technical University of Athens. However, these methods require considerably more computer time than the simpler aerodynamic models. For that reason, they are not usually incorporated in the aeroelastic programs (calculating loads as the combined result of aerodynamics and structural dynamics) used by wind turbine designers. Moreover, these methods are of a nature that limits their validity to attached flow around the blades and to a few diameters downstream of the turbine (no viscous effects are accounted for). In the project, this type of model is used to obtain essential information and physical insight into the most important phenomena, to be fed into the engineering methods.

From a more global point of view, the blade element momentum relations which equate instantaneous forces (on the blade) to the momentum difference in upstream and downstream sections, can be converted in a non-equilibrium relation by adding a time lag term. This is the approach taken in the engineering models that are developed and used within this project. This changes the algebraic blade element momentum relations into ordinary differential equations in time, which are easily implemented in the existing aeroelastic codes. In fact, in these codes a set of differential equations is already solved in the time domain, for the structural degrees of freedom.

7.2 Models used in the project

In the project seven models (including the free wake models) are applied. These will shortly be described below.

ECN, cylindrical wake model

The induced velocities in both axial and tangential direction are calculated with a cylindrical vortex sheet model. The vortex distribution on the cylindrical wake is obtained by the time history of the trailed tip-vorticity which is related to the axial force on the blade. With the Biot-Savart law the induced velocities can then be found. It is assumed that the wake extends up to three rotor diameters downstream. Inputs that may be semi-empirically adjusted are the axial convection velocity and the effective diameter of the cylindrical wake. A detailed description of the model is given in Appendix I. Although the method demands a relatively long computer time it is more flexible in its application than the second ECN model.
ECN, differential equation

The induced velocity in axial direction is calculated with the blade element momentum equation with the addition of a time derivative of the induced velocity:

$$4Rf_0 \frac{\partial}{\partial t}(u_i) + 4u_i(V - u_i) = \sigma V^2 c_n$$  \hspace{1cm} (7.1)

The value of the time constant is derived from the equations for the cylindrical wake mentioned above (ECN, i.w.).

$$f_0 \left(\frac{r}{R}\right) = 2\pi \int_0^{2\pi} \frac{[1 - \tau/R \cos \phi_i]}{[1 + (\tau/R)^2 - 2\tau/R \cos \phi_i]^{3/2}} d\phi_i$$ \hspace{1cm} (7.2)

In a similar way the tangential induced velocity is calculated from the tangential momentum equation with time constant \(\tau R\). A detailed description of this model is given in Appendix L.

The DUT model

Within the Institute for Wind Energy of T.U. Delft an aerodynamic model has been developed a number of years ago based upon the asymptotic acceleration potential theory [10], [11]. Under the assumption of incompressible, inviscid and irrotational flow, in situations where the velocity perturbations are small with respect to the undisturbed value, it can be shown that the pressure perturbation in the complete flow field is given by a Laplace equation:

$$\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = 0$$ \hspace{1cm} (7.3)

The pressure perturbation \(p\) then acts as an acceleration potential function. Integration of the accelerations experienced by particles of air, which travel from far upstream to the rotor blade, determines the velocities in the rotor plane. With these velocities the aerodynamic loads can be calculated.

In the model the rotor blades are represented as discrete surfaces on which a pressure discontinuity is present. The model implies the presence of spanwise and chordwise pressure distributions, which are composed of analytical asymptotic solutions for the Laplace equation. This makes the approach equivalent to a lifting surface model. A similar approach for the determination of loads on helicopter blades was described by Van Holten [12]. In the first order approximation used for the present purpose, the chordwise pressure distribution is restricted to a flat plate type analogon.

Asymptotic expansion techniques are often used in finding solutions of Laplace equations. The lifting line approach is such an example: Often the span of a lifting surface is large with respect to the chord (e.g., an aircraft wing or a wind turbine rotor blade). At some distance of such a surface (in the far field) the experienced accelerations will be equivalent to those felt by a line on which the load is concentrated. So in this area the wing can be modelled with a pressure dipole line. Close to the surface the experienced accelerations will be dominated by the chordwise load distribution. So in the near field the experienced accelerations will be almost two dimensional, and can be modelled with two dimensional pressure
distributions.

Application of asymptotic expansion techniques to the pressure distribution representing the rotor blade eventually leads to:

\[
\frac{p}{0.5\rho V^2} = -\frac{1}{\pi} \frac{l(y, t)}{0.5\rho V^2 c(y)} \frac{\sin(\Phi)}{(\cosh\eta + \cos(\Phi))} + \frac{1}{2\pi} \frac{l(y, t)}{0.5\rho V^2 c(y)} \frac{c(y)}{\sin(\chi)} + \frac{1}{\pi} \sum_{n=1}^{\infty} A_n(t) P_n^1(\cos\theta) Q_n^1(\cosh\nu) \sin\chi
\]

(7.4)

In 7.4 the expression \(c(y)\) yields the chord distribution.

The first expression on the right hand side of equation 7.4 is the near field term written in local elliptical coordinates \(\Phi\) and \(\eta\). The third expression is the far field term, written in prolate spheroidal coordinates \(\theta, \nu\) and \(\chi\); and the middle expression in the right hand side is the common field expression written in circular cylinder coordinates \(y, r_0\) and \(\chi\). The \(P_n^1\) and \(Q_n^1\) functions represent associate Legendre functions of the first and second kind. Legendre functions are the natural solutions for problems written in prolate spheroidal coordinates. In equation 7.4 it can be seen that close to the blade the common field expression exhibits a singular behaviour (caused by \(r_0^{-1}\)). This behaviour is also found in the far field term (when the \(Q_n^1\) term is evaluated), but with the opposite sign. Thus the total expression does not have this essential singularity.

In expression 7.4 the function \(l(y, t)\) is used. This is the lift distribution over the blade. It can also be expressed in terms of associate Legendre functions of the first kind:

\[
l(y, b) = \frac{1}{2} \rho V^2 b \sqrt{1 - \frac{y}{b/2}} \sum_{n=1}^{\infty} A_n(t) P_n^1\left(\frac{y}{b/2}\right)
\]

(7.5)

The coefficients \(A_n(t)\) are determined by application of the tangential flow condition using a collocation point method.

In its simplest implementation (code PREDICCHAT 1) an iterative procedure is developed for the calculation of the stationary coefficients \(A_n\). It starts from an assumed straight, unperturbed path, which is travelled by the particles of air with an imposed constant speed. For the speed the value 0.6657 \(\sqrt{V}\) is used, the optimum value in the rotorplane determined by axial momentum theory. The accelerations experienced during the travel are integrated in order to obtain the velocities at the collocation points. This yields a first guess of the stationary coefficients \(A_n\).

With the resulting pressure field the perturbed axial velocities of the particles travelling to the collocation points can be calculated. Within the code PREDICCHAT 1 the velocity is kept constant along the straight unperturbed path, although its value is collocation point dependent. In repeating this iterative procedure the ultimate stationary coefficients are determined. In a more elaborated version (PREDICCHAT 2) the option of a perturbed path with time dependent velocity can be chosen.

The induced angle is determined in the present method by integration of the common field and the far field terms (2nd and 3rd term of expression 7.4).

The time dependent (dynamic inflow) solutions are obtained according to the following procedure:
First a stationary calculation is carried out using PREDICHAT. This calculation uses the initial values of the relevant parameters (such as pitch angle and wind speed at \( t = 0 \)).

With the now well determined stationary pressure field the unsteady paths of the particles (and their time and position dependent velocities) are calculated using a step by step variation procedure. Every next step the whole process of determination of the accelerations (now time dependent), the velocities and the paths is repeated, thus calculating the dynamic inflow velocities in the rotor plane. The latter process is equivalent to a vortex wake calculation with a dynamically varying wake (a process called dynamic wake adaptation). The code in which this calculation is implemented is designated PREDICDYN.

A detailed description of this model is given in Appendix P.

The GH model

The axial induction is calculated using a method based on the Pitt and Peters model [7] but GH applies the method to each blade element assuming independent annuli. This is consistent with the combined blade element and momentum method used in the GH aeroelastic code BLADED. The Pitt and Peters model was originally developed in terms of induction factors. GH used this approach initially but later in the project developed the model to work in terms of induced velocity.

A detailed description of this model is given in Appendix M.

The TUDk model

The induced velocities in axial direction are calculated with the blade element -momentum equations with the addition of a time derivative of the induced velocity. This is similar to "ECN, d.e."

TUDk however uses two differential equations. These have the following form:

\[
\begin{align*}
y + \tau_1 \frac{dy}{dt} &= x + k \tau_1 \frac{dx}{dt} \\
z + \tau_2 \frac{dz}{dt} &= y
\end{align*}
\]

with \( k = 0.6 \) and:

\[
\begin{align*}
\tau_1 &= \frac{1.1}{(1 - 1.3a)} \frac{R}{V} \\
\tau_2 &= [0.39 - 0.26(\frac{r}{R})^2] \tau_1
\end{align*}
\]

This amounts to one short and one longer time scale for the decay. The time constants follow from an actuator disk-vortex ring program which includes the effect of wake expansion. A detailed description of the model is given in Appendix N.

The Unist. method

The Unist. free wake method is based on the solution of the Laplace equation. The rotor blades are covered with panels having constant doublet strength. Using the equivalence of a doublet panel and a vortex ring the induced velocities are calculated by Biot-Savart law where the vortex strength is gained by doublet differences. The kinematic boundary condition on the blade and the Kutta condition
are fulfilled automatically by the numerical method. Due to the free stream velocity, rotation and induced velocities at the blades, points from the separation edges move downstream and build up the panels of the free wake which is deformed in such a way that the forces on the wake vanish. The results of the method are:

- Doublet distribution along the blade,
- Wake geometry,
- Representation of the flow field.

Comparison of Unist. method with other methods. The main differences to "normal" BET methods are:

- shedding of the vorticity into the wake is automatically included,
- roll-up of tip and inboard vortex sheet,
- no empirical input required (just core size of the vortices, time step),
- arbitrary inflow conditions can be simply modelled by adding the according velocities (earth shear layer, gusts, yaw etc.)
- flow field can be generated at arbitrary points during post processing,
- geometry of the wake is a result and can help to interpret other results.

A detailed description of this model is given in Appendix O.

The NTUA model
The flow around a horizontal axis wind turbine is an unsteady three-dimensional vortical flow with moving boundaries. More specifically:

- The flow is three-dimensional because wind turbines operate in the atmospheric boundary layer. For the same reason the flow is also unsteady (periodic) even if the inflow is assumed steady in time (Unsteadiness can also originate from transient operational controls).
- The flow is vortical (or rotational) due to the spatially distributed vorticity. On the blades bound vorticity appears as a result of their shape. Due to the three-dimensional character of the flow and according to Kelvin's theorem vorticity will be shed in the undisturbed flow.

For detailed computations of vortical flows of this kind, Vortex Methods (VM) seem to be a good choice, mainly because of their grid-free structure. In fact vortex methods were constructed so as to approximate numerically complex flows at low computational cost.

Within this context, a free-wake aerodynamic model has been defined and incorporated in a computational environment by the name GENUVP (GENeral Unsteady Vortex Particle code). GENUVP is a time-marching code based on a consistent combination of the Boundary Element Method (BEM) and the Vortex Particle Method (VPM). The BEM is used to reproduce numerically the irrotational part of the velocity field, i.e. the flow induced by the blades. This is accomplished by distributing sources and/or dipoles on the solid surfaces of the flow. The boundary conditions of the flow, determine the intensities of these distributions. On the other hand, the VPM is used to simulate the generation and evolution of the free-vorticity contained in the wakes of the blades. Particularly, the vorticity shed along the edges of the blades, is locally integrated and assigned to point vortices (or equivalently to fluid particles carrying vorticity). Next the evolution of these vortex particles is followed by integrating the vorticity transport equations in Lagrangian coordinates.
During the progress of the present project but within another JOULE project, entitled "Development of a new generation of design tools for horizontal axis wind turbines" (JOU2-CT92-0113), GENUVP was improved and completed. First, the tower was included as a non-lifting body represented by a distribution of sources over its surface. Second, GENUVP was coupled with a beam-type structural model, to give a non-linear aeroelastic numerical model. The full model was used in the cases of yawed operation where the aeroelastic coupling plays a dominant role.

A detailed description of the model can be found in Appendix Q.

7.2.1 Qualitative discussion of the differences between models

The three 'engineering models' have in common that they are adaptations of the blade element momentum (bem) method. In case of steady circumstances, they will reduce to the steady blade element momentum theory. There are however some differences in the way the bem expressions are adapted, which will be discussed here.

The ECN d.e. and the TUDk models add time derivative terms of the dimensional quantity $u_\tau$. Physically this is the correct way, since the time varying wake vorticity changes the induced velocity. In the original Pitt and Peters model (see chapter 6), the time derivative is of a nondimensional form of $u_\tau$, where the tipspeed $\Omega_t$ is used to non-dimensionalize. In the original GH implementation of the Pitt and Peters model, a time derivative of the nondimensional induced velocity factor \( 'a' \) (i.e., $u_\tau$ divided by the wind speed $V_{\infty}$) is used. All these methods would be equivalent if $V_{\infty}$ or $\Omega_t$ were constant. In wind turbine applications, $\Omega_t$ is frequently constant (constant rpm operation) but of course the wind speed never is. The GH model was changed during the course of the project so that the induced velocity is calculated directly. Then, there is very little difference between the approaches from ECN, GH and TUDk.

All of the 'engineering methods' include dependence of the time constants on rotor size but approach the radial dependence of the time constants in different ways. The GH implementation of the Pitt and Peters model, because it works at the annulus level, has a radial dependence which has an inverse relationship with radial position. The ECN d.e. model uses a physical argument which comes directly from the cylindrical wake model which causes the time constant to reduce near the tip. This is related to the dominant behaviour of the tip vortex in determining the inflow distribution. The TUDk model has a similar radial dependence of the shorter of the two time scales applied. The absolute magnitude of the time constants is not so easy to estimate and can only be judged on the basis of the results presented in Chapter 8.

A final difference in implementation is between the TUDk model on one side and the ECN d.e. and GH models on the other side. The first model has a set of two equations describing the changes of the induced velocity in time, modelling basically two time scales: one for the initial phase (e.g., directly following the pitch step) and one for the remaining phase. This mimics the behaviour observed in numerical results from a vortex ring wake model.

The remaining models used in the project are the acceleration potential model of the DUT and the free vortex wake models of the Unit. and the NTUA. All three methods are basically flow field solvers for inviscid flow. In the near field of the
flow about the rotor blades, this restricts the applicability to attached flow. In the far field (the global rotor wake) the applicability is restricted to the region where viscous diffusion and/or dissipation of vorticity is not important. This is most likely true for that part of the wake which is important for the induced velocity in the rotor plane, i.e. a few rotor diameters behind the rotor plane.

The DUT model is linearized in the sense that a small perturbation equation is solved, the other two solve the complete equations, making use of surface singularity distributions on the blade (vortex lattice) and vorticity distributions in the wake. The difference between the Unist. and NTUA models is mainly in the wake resolution technique applied. The Unist. uses vortex filaments as a natural extension of the blade vortex lattices, while the NTUA uses a vortex particle representation in the wake. The reader is referred to the relevant appendices (Appendix O and Appendix Q respectively) for the details of the implementation.

One important aspect of the free vortex wake models is the desingularization of the vortex particles or filaments. This has been the subject of a more detailed numerical study which is discussed in the next subchapter. In conclusion no fundamental differences are present between the NTUA and Unist. models.

7.2.2 Numerical study of convergence and desingularization of the free vortex wake models

The free vortex wake models are based on an exact formulation of the inviscid flow equations. This means that these models can in principle be very accurate, on condition that there are no important viscous effects in the real flow. The long experience with non-viscous flow models (both analytical as well as numerical models) has provided sufficient background to know the limits of application of such models. In the dynamic inflow project there are no special conditions encountered which would give rise to more than the usual caution in this respect.

The next question is, whether the actual numerical implementation chosen could introduce errors of such magnitude that appreciable differences with the "exact" inviscid flow occurs.

In this respect the discretization procedure warrants special attention. For practical purposes it is obviously necessary to discretize the continuous formulation of the exact theory. In the present case the continuous vortex distributions are discretized into either vortex lattices or into a distribution of discrete vortex particles.

Such a discretization artificially introduces singularities in the description of the flowfield. These singularities take the form of points or lines where the velocities become infinite, which is obviously a large departure from the actual physical flow.

Nevertheless, in a case where the position of the singularities is fixed in space (linearized theory) or is prescribed beforehand as a function of time on the basis of experiments ("prescribed wake methods"), it can be shown that the correct inviscid limit is approached when the number of discrete vortex elements is increased indefinitely.

One of the purposes of the "free wake" analyses used in the dynamic inflow project is, to be able to determine by computation the shape and configuration of the free vortex sheets. The position of the singularities in the wake is in that case not fixed, but is instead determined during the computation by letting the vortex elements
float freely through the field in such a way that a "force free" condition is satisfied, as it should be.

In the case of freely convecting vortex elements ("free wake") there may occur a computational stability problem due to the very irregular flow field associated with the discretized singularities.

For this reason artificial desingularization (or: "regularization") is used in both free-wake methods used in the dynamic inflow project. This is done by the introduction of a "cut-off" length: close to the singular points a regular flowfield is substituted in place of the almost singular flowfield near the discrete singularities. Very roughly speaking, one can compare this artifice physically with a kind of viscous vortex core.

Although the mathematical form of the desingularization and the numerical constants chosen are different between the vortex lattice method and the vortex particle method, in both these methods the goal has been achieved that numerical instability is avoided.

What has to be investigated next is, whether the convergence characteristics are still maintained, i.e. whether the numerical solution despite the regularization still converges to the exact inviscid solution when the number of discrete singularities increases indefinitely.

A simple testcase has been used to obtain some insight in this question. The testcase consisted of a two-dimensional strip on which vorticity is continuously distributed in such a way that the selfinduction is constant along the strip.

Although the testcase is mathematically very simple and possesses an exact solution in closed form, in some respects it is a very a demanding testcase. What is in fact tested is, how accurately a numerical calculation will predict the free convection of vorticity under rather difficult circumstances, viz. in the edge region of the free vortex sheets in the wake.

Some of the test results are shown in the figures 7.2 and 7.3.

Figure 7.2. Influence of discretization, 2D vortex strip

Figure 7.2 shows first of all the influence of the number of discrete vortex elements (N). It is seen that the numerical results converge towards the exact solution. When
the number of discrete vortices is chosen at $N = 80$, the difference with the exact result is probably negligible for all practical purposes.

Figure 7.3 shows the influence of the "cut-off length ($R_0$)". It is found that one should be careful with the regularization procedure, because the exact result is only recovered when small cut-off lengths are used.

One may conclude that a free wake analysis will be stable as well as convergent, as long as the number of discrete vortices and cut-off length are carefully chosen, in a mutually consistent way. In both the free wake methods of the project this has been done, by choosing only certain combinations of grid size, time step and cut-off length. When the grid size and time steps are decreased, at the same time the cut-off lengths are made smaller.

The finally remaining question is then, how accurate the numerical free wake analyses are in the case of a practical choice of grid size and time step.

In practice the number of discrete vortex elements is necessarily much smaller (relatively) than in the test case considered above. This would mean that the finer details of the flow near the edges of vortex sheets, and the details of the rolling up of vortex sheets cannot be represented very accurately by the free wake analyses.

In the remaining parts of the flowfield where the velocity gradients are much smaller, the situation will be more favourable.

A test that has been done for both the numerical methods was, how sensitive the resulting load distributions on the blades of the rotor are with respect to grid size and time step. In both methods the grid size was chosen such that a refinement of the grid size did not add additional accuracy to these load distributions.

Loads however constitute a relatively insensitive criterion. It is well known that the vortex configuration often does not influence very critically the resulting loads. This is the very reason why linearized methods where the vortex sheets are assumed to lie in flat planes without deformations by self induction effects, are often very successful.

In conclusion it can be said that in those flow regions where the deformations of the free vortex sheets are really of importance, and for which free wake methods
are essentially meant, the grid size would probably have to be made much smaller than is practically feasible at present. However, such refinements would not make much sense as long as there is a lack of experimental data to validate the numerical methods. What would be needed first of all are flow field measurements, revealing considerable detail of the kinematics of the wake deformations and the formation of "tip vortices" of rotors.

In order to gain more insight into the accuracy of free wake methods, it was decided to compare calculational results of these methods with DUT wind tunnel measurements in the near wake, which had already been made in the past, see section 9.
8. AXIAL SYMMETRIC CASES, TJÆREBOG

8.1 Available measurements

A number of measurement series have been taken on the Tjærebøg turbine which are aimed to validate the model under axisymmetric conditions.

For a global description of the Tjærebøg turbine, see section 5.1.

8.1.1 Measurement accuracy

The measurements on the Tjærebøg turbine which are described in this section have been carefully selected in order to have a good validation base for the project. It is believed that the recordings of the loads has been very accurate, both in terms of static and dynamic loads. Some uncertainty existed about the exact value of the initial value of the pitch angle for case II.1 (see section 8.1.2). A more detailed description of the measurement series can be found in the references [13], [14], [15], [16] and [17].

8.1.2 Pitching transients (case II and III)

Six different cases with pitching transients have been measured on the Tjærebøg turbine. In these transients, after an initial period, the blade pitch angle is first increased at a fast rate over 2 or 3°, next maintained constant for about 30 seconds and then decreased to its initial value at a fast rate. The measurement period extends over a total of 60 seconds. Measured values for blade and shaft loads have been averaged over a number of realizations, and over the three blades in order to filter out stochastic wind influences and deterministic effects as (average) windshear and tower shadow. The averaging process is described in [13] and is repeated below:

First the time trace for the pitch angle is searched for the times where it crosses upwards through the level of the initial pitch angle. The complete time series is then divided into a number (say N) segments of 30 seconds duration, each starting 5 sec. before the level crossing. A new time series is constructed as the average of these N time series in the sense that the value of each sensor signal at a given time is the average of the N individual values from the N time series at the same time relative to the pitch angle level crossing. A new sensor signal is calculated as the mean value of the 3 averaged flatwise moments (of all three blades) to further reduce the influence of turbulence, wind shear and tower shadow. This procedure is repeated for the decreasing pitch steps determined as crossing down through the second value of the pitch angle. The resulting averaged time series is then appended to the first, starting at t=30 seconds.

Table 8.1 shows the conditions pertinent to the measurements in order of decreasing λ. The 'case' number in the first column refers to the chronological order in which the cases were analysed, and is also used in section 8.2.2, where the cases II.1 to II.4 have been simulated. The windspeed is the average value for the case, θi is the initial and final blade pitch angle, θ, the intermediate one after the pitch step. The value of Δt gives the time in seconds used for the both pitching changes. Also indicated in this table are calculated values of the equilibrium induction factors a1 and a2 at 70 % span, to give an indication of the loading situation.
The indication 'long' for the $\Delta t$ of case II.3 refers to the fact that the first

Table 8.1 *Conditions of the measurements*

<table>
<thead>
<tr>
<th>case</th>
<th>$\lambda$</th>
<th>$U$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\Delta t$</th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>II.1</td>
<td>9.5</td>
<td>7.4</td>
<td>1.0</td>
<td>3.0</td>
<td>2.0</td>
<td>0.35</td>
<td>0.27</td>
</tr>
<tr>
<td>II.4</td>
<td>8.1</td>
<td>8.7</td>
<td>0.1</td>
<td>3.7</td>
<td>0.5</td>
<td>0.34</td>
<td>0.23</td>
</tr>
<tr>
<td>II.3</td>
<td>7.8</td>
<td>9.0</td>
<td>0.2</td>
<td>3.4</td>
<td>long/0.6</td>
<td>0.33</td>
<td>0.22</td>
</tr>
<tr>
<td>II.5</td>
<td>7.8</td>
<td>9.1</td>
<td>-0.4</td>
<td>2.5</td>
<td>2.07</td>
<td>0.4</td>
<td>0.26</td>
</tr>
<tr>
<td>II.6</td>
<td>6.6</td>
<td>10.5</td>
<td>0.2</td>
<td>3.9</td>
<td>0.74</td>
<td>0.31</td>
<td>0.21</td>
</tr>
<tr>
<td>II.2</td>
<td>5.6</td>
<td>12.5</td>
<td>1.2</td>
<td>3.2</td>
<td>1.3</td>
<td>0.22</td>
<td>0.19</td>
</tr>
</tbody>
</table>

pitch change was done with an exponential rate, with a rather steep initial value, but slowing down in the end, so that the entire pitch change takes about 3 seconds, in contrast to the second step (back to initial) which is done in 0.6 seconds.

For the lowest $\lambda$ case (case II.2, $\lambda$=5.6) the inner part of the blade is in stall. This is reflected in the low values of the induction factor, which does not undergo a major change. For case II.1 (highest $\lambda$, 9.5) the rotor is partly in the turbulent wake state. This is also true for case II.5 ($\lambda$ = 7.8), due to the negative pitch angle. The other cases have a $\lambda$ near to the design value. All these cases are high loading cases with induction factor values of 0.33 and above, changing by approximately 30%. Case II.4 has the highest pitching rate. The values of the pitching rate covered, vary between 1 °/s (case II.1) and 7.2 °/s (case II.4).

The fast pitch changes result in overshoots with respect to the equilibrium values of the rotor loads, as exemplified in figure 8.1 and 8.2 for the measured blade root bending moment and rotor shaft torque for case II.4.

![Figure 8.1](image)

*Figure 8.1 Tjørnø, measured lift moment at blade root for step on the pitch angle, case II.4*

A normal equilibrium wake calculation would just show the equilibrium values as reached at the end of the transient, but the dynamic inflow models should show the measured behaviour, see section 8.3. Table 8.2 quantifies the measured overshoots for the blade root flingwise bending moment, table 8.3 for rotor shaft torque.
Figure 8.2 Tjæreborg; Measured rotorshaft torque for step on the pitch angle, case II.4

In these tables the quantities with subscript 1 list the values at the first and final equilibrium state (pitch angle \( \theta_1 \)), while subscript 2 refers to the equilibrium state at pitch angle \( \theta_2 \), as reached some time before the second pitch step. The subscript 'min' refers to the minimum value of the load, reached immediately after the first pitch step, 'max' to the maximum value immediately after the second pitch step. The tables also show the load ranges, as would occur with equilibrium wake (the column headed by '1-2') and the dynamic range (indicated by 'max-min'). The same order as in table 8.1 is used, i.e. in order of decreasing \( \lambda \).

In estimating the values from the measurements, it has been attempted to remove effects of structural flexibility by not taking the max and min values at the very peaks, but shave the signal of the sharp peaks with the blade natural frequency.

<table>
<thead>
<tr>
<th>case</th>
<th>( M_1 ) [kNm]</th>
<th>( M_2 ) [kNm]</th>
<th>( M_{\text{max}} ) [kNm]</th>
<th>( M_{\text{min}} ) [kNm]</th>
<th>( 1 - 2 ) [kNm]</th>
<th>max – min [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>II.1</td>
<td>530</td>
<td>420</td>
<td>500</td>
<td>380</td>
<td>110</td>
<td>210</td>
</tr>
<tr>
<td>II.4</td>
<td>670</td>
<td>460</td>
<td>750</td>
<td>375</td>
<td>210</td>
<td>375</td>
</tr>
<tr>
<td>II.3</td>
<td>675</td>
<td>470</td>
<td>735</td>
<td>500</td>
<td>175</td>
<td>265</td>
</tr>
<tr>
<td>II.5</td>
<td>770</td>
<td>590</td>
<td>820</td>
<td>550</td>
<td>180</td>
<td>270</td>
</tr>
<tr>
<td>II.6</td>
<td>860</td>
<td>635</td>
<td>940</td>
<td>550</td>
<td>225</td>
<td>390</td>
</tr>
<tr>
<td>II.2</td>
<td>920</td>
<td>810</td>
<td>930</td>
<td>770</td>
<td>110</td>
<td>160</td>
</tr>
</tbody>
</table>

The results show the strongest dynamic inflow effects for case II.4, which has the highest pitching rate and a high loading. For case II.2 (high windspeed, low \( \lambda \)) the overshoots are much less although still appreciable. It is also clear that the slow pitching change at the first step of case II.3 reduces the overshoot appreciably, compared to the second change, or to case II.4 which has comparable loading conditions.
Table 8.3 Values of measured rotorshaft torque and ranges

<table>
<thead>
<tr>
<th>case</th>
<th>$Q_1$ [kN.m]</th>
<th>$Q_{\text{max}}$ [kN.m]</th>
<th>$Q_2$ [kN.m]</th>
<th>$Q_{\text{min}}$ [kN.m]</th>
<th>$1 - 2$ [kN.m]</th>
<th>$\text{max} - \text{min}$ [kN.m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>II.1</td>
<td>150</td>
<td>135</td>
<td>190</td>
<td>110</td>
<td>15</td>
<td>80</td>
</tr>
<tr>
<td>II.4</td>
<td>265</td>
<td>220</td>
<td>350</td>
<td>140</td>
<td>45</td>
<td>210</td>
</tr>
<tr>
<td>II.3</td>
<td>260</td>
<td>220</td>
<td>335</td>
<td>195</td>
<td>40</td>
<td>140</td>
</tr>
<tr>
<td>II.5</td>
<td>310</td>
<td>360</td>
<td>280</td>
<td>240</td>
<td>30</td>
<td>120</td>
</tr>
<tr>
<td>II.6</td>
<td>442</td>
<td>530</td>
<td>385</td>
<td>300</td>
<td>57</td>
<td>230</td>
</tr>
<tr>
<td>II.2</td>
<td>560</td>
<td>530</td>
<td>590</td>
<td>495</td>
<td>30</td>
<td>100</td>
</tr>
</tbody>
</table>

8.1.3 Safety stops (Case IV)

Measurements of two safety stops have been supplied by TUDk, at two different wind conditions, viz. 7 m/s and 14 m/s. In both cases, the blade is feathered to 90° pitch angle at a high pitching rate of about 12°/s and the signal for generator loss is given when a certain negative value of the electrical power is exceeded. In the process the blade goes through stall at negative angles of attack. After loss of generator power, the drive train exhibits a strong vibration in free drive train mode. The measured values for rotorspeed, blade root flat moment and rotorshaft torque are shown and discussed in section 8.3.5, where they are compared with model calculations.

8.2 Definition of calculation cases

Four different cases on the Tjæreborg turbine at (approximately) axisymmetric conditions have been performed, which are denoted by case I to case IV. All these cases have been subdivided in more specific subcases.

The description of the Tjæreborg turbine is given in Appendix T and Appendix U.

8.2.1 Preliminary calculations (Case I)

In order to study the behaviour of the aerodynamic models without the complicating factor of structural dynamics, first calculations were performed for aerodynamic loads of the Tjæreborg turbine, under constant axisymmetric wind conditions, i.e. without windshear, tilting angle or towershadow effects. In fact two calculation rounds of case I have been performed. It appeared that the output properties and the conditions chosen for the first round were not ideal for the purpose of the project. Therefore only the second round of case I is described in this report. A step on the wind speed (denoted by case I.1) has been calculated as well as a step on pitch angle (denoted by case I.2). The wind speeds were chosen such that the turbine is not in the turbulent wake state while the loading is high. The rotor speed is 22 rpm.

The conditions for case I.1 are:
- $\theta = 0^\circ$;
- $V_{\infty}$ (see figure 8.3):
The conditions for case 1.2 are:

- $V_\infty = 10 \text{ m/s}$;
- \( \theta \) (see figure 8.4);
- \( t(s) \theta(^\circ) \)

<table>
<thead>
<tr>
<th>$t(s)$</th>
<th>$V_\infty (\text{m/s})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>13.0</td>
</tr>
<tr>
<td>6.0</td>
<td>13.0</td>
</tr>
<tr>
<td>6.5</td>
<td>10.0</td>
</tr>
<tr>
<td>40.0</td>
<td>10.0</td>
</tr>
<tr>
<td>40.5</td>
<td>13.0</td>
</tr>
<tr>
<td>60.0</td>
<td>13.0</td>
</tr>
</tbody>
</table>

Required as function of time:

- Axial induced velocities at $r = 9 \text{ m}$, $r = 15 \text{ m}$, ($\approx 50\%$ span) and $r = 21 \text{ m}$ ($\approx 70\%$ span);
- Aerodynamic flatwise moments at $r = 2.75 \text{ m}$, $r = 15 \text{ m}$ ($\approx 50\%$ span) and $r = 21 \text{ m}$ ($\approx 70\%$ span);
- Axial force and rotorshaft torque.

It was agreed that the project should focus on dynamic inflow effects and not on comparison of steady state results. Therefore the quantities are presented in a non-dimensional way, i.e.:

\[ \Delta q \text{ as } f(t) \text{ and } \Delta q_s , \text{ with:} \]

\[ \Delta q_i = \frac{q - q_{s1}}{q_{s2} - q_{s1}} \]

\[ \Delta q_s = \frac{q_{s2} - q_{s1}}{q_{s1}} \]

$q$ = actual value

$q_{s1}$ = stationary value at begin conditions
8.2.2 Pitching transients (Case II)

This case consisted of a simulation of the pitching steps, which are described in section 8.1.2. Only the cases II.1 to II.4 have been simulated. The calculations were performed under constant wind conditions matching the average wind speed of the measured cases, and without windshear, tilting angle or towershadow effects. The time history of the pitch angle was prescribed and fitted to the measured time history. The rotor speed was 22 rpm for all cases.

The conditions for case II.1 are:
- $V_{\infty} = 7.4 \text{ m/s}$
- Pitch angle: The pitch angle step is approximated by:
  1) step up: \( t(s) \quad \theta(\text{o}) \)
    - 0.00  1.00
    - 4.38  1.00
    - 6.48  3.00
  2) step down: \( t(s) \quad \theta(\text{o}) \)
    - 34.40 3.00
    - 36.50 1.00
    - 60.00 1.00

(see figure 8.5)

The conditions for case II.2 are:
- $V_{\infty} = 12.5 \text{ m/s}$
- Pitch angle: The pitch angle step is approximated by:
  1) step up: \( t(s) \quad \theta(\text{o}) \)
    - 0.00  1.164
    - 4.70  1.164
    - 6.00  3.190

![Figure 8.4 Tjæreborg: Step on pitch angle for case I.2; $V_{\infty} = 10 \text{ m/s}$](image)
Figure 8.5 Tjæreborg: Measured and approximated step on pitch angle for case II.1; $V_\infty = 7.4$ m/s

2) step down:  
<table>
<thead>
<tr>
<th>t (s)</th>
<th>$\theta$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>34.58</td>
<td>3.190</td>
</tr>
<tr>
<td>35.70</td>
<td>1.164</td>
</tr>
<tr>
<td>60.00</td>
<td>1.164</td>
</tr>
</tbody>
</table>

(see figure 8.6)

Figure 8.6 Tjæreborg: Measured and approximated step on pitch angle for case II.2; $V_\infty = 12.5$ m/s

The conditions for case II.3 are:
- $V_\infty = 9.0$ m/s;
- Pitch angle: The pitch angle step is approximated by:
  1) step up: For $t < 0.92$ s: $\theta = 0.210°$  
    For $t > 0.92$ s: $\theta = 3.380 - 3.170e^{1.41(1.92-t)}$  
  2) step down:  
    | t (s) | $\theta$ (°) |
    |-------|---------------|
    | 32.04 | 3.380         |
    | 32.62 | 0.210         |
    | 60.00 | 0.210         |

(see figure 8.7)
The conditions for case II.4 are:

- \( V_\infty = 8.7 \text{ m/s} \);
- Pitch angle: The pitch angle step is approximated by:

1) step up: \[
\begin{array}{c|c}
\text{t(s)} & \theta (\degree) \\
0.00 & 0.070 \\
2.00 & 0.070 \\
2.50 & 3.716 \\
\end{array}
\]
2) step down: \[
\begin{array}{c|c}
\text{t(s)} & \theta (\degree) \\
32.00 & 3.716 \\
32.70 & 0.070 \\
60.00 & 0.070 \\
\end{array}
\]

(see figure 8.8)

Required:
- As case I but for this case dimensional values are requested. Only the blade root moment and the rotor shaft torque can be directly compared to measurement.
There to the aerodynamic blade flatwise moment are reduced with 9% to account for centrifugal stiffening, see [13]. Also the aerodynamic blade flatwise moment at \( r = 15 \text{ m} \) and \( r = 21 \text{ m} \) are reduced with 9%.

8.2.3 Pitching transients including structural dynamics (Case III)

The real behaviour of the loads for a stepwise pitch angle change is not only affected by dynamic wake effects, but also by structural dynamics and possibly instationary profile aerodynamics. In order to study the results of these in the comparison, a simulation was done of one single time period of 60 seconds from which the case II.4 measurement is composed. Windshear is included. The rotor speed is 22 rpm. This case is denoted by case III.1

![Figure 8.9 Tjæreborg: Measured pitch angle for case III.1; \( V_{\infty} = 8.74 \text{ m/s} \)]

Case III.2 consists of a calculation similar to the case II.4 calculation, but including blade dynamics. Gravity, windshear and tower shadow is excluded, so that comparison with the case II.4 calculations and with the filtered measurements is possible.

The conditions for case III.1 are:
- \( V_{\infty} = 8.74 \text{ m/s} \) (constant in time);
- Wind shear according to the power law with exponent 0.110;
- The measured pitch angle behaviour is prescribed, see figure 8.9.

The wind conditions are on the average similar to those of the measured time serie. The pitch angle behaviour is slightly different from the one of case II.4 (case II.4 is the average of a large number of measurement series, one of them is the present case). The required output properties are (as function of time):
- Flapwise bending moment at \( r = 2.75 \text{ m} \) for all blades;
- Chordwise bending moment at \( r = 2.75 \text{ m} \) for all blades;
- Rotorshaft torque.

The conditions for case III.2 are similar to the conditions of case II.4. The required output properties are as function of time:
- Flapwise bending moment at \( r = 2.75 \text{ m} \) for one blade;
- Chordwise bending moment at \( r = 2.75 \text{ m} \) for one blade;
- Rotorshaft torque.
8.2.4 Safety stops (Case IV)

These calculations were taken in accordance with the measurements from the safety stops, see section 8.1.3. There are two cases, one at low wind speed (indicated by case IV.1) and one at high wind speed (indicated by case IV.2).

\[
\begin{align*}
\theta & \quad | \quad t \\
0 & \quad | \quad 0 \\
10 & \quad | \quad 10 \\
20 & \quad | \quad 20 \\
30 & \quad | \quad 30 \\
40 & \quad | \quad 40 \\
50 & \quad | \quad 50 \\
60 & \quad | \quad 60 \\
90 & \quad | \quad 90 \\
\end{align*}
\]

Figure 8.10  Tjæreborg: Measured pitch angle for case IV.1; \( V_{\infty} = 7.09 \, \text{m/s} \)

The conditions for case IV.1 are:
- \( V_{\infty, 60 \text{m}} = 7.09 \, \text{m/s}, \) constant in time;
- Wind shear according to power law with exponent 0.402;
- Yaw misalignment: The average yaw misalignment in the measured time series is about 11.5 °. It was recommended to include this yaw angle, if possible.
- Pitch angle: The measured pitch angle is prescribed, see figure 8.10.

Required as function of time:
- Flapwise bending moment at \( r = 2.75 \, \text{m} \) for all blades;
- Chordwise bending moment at \( r = 2.75 \, \text{m} \) for all blades;
- Rotorshaft torque;
- Rotor speed.

The conditions for case IV.2 are:
- \( V_{\infty, 60 \text{m}} = 14.55 \, \text{m/s}, \) constant in time;
- Wind shear according to power law with exponent 0.158.
- Yaw misalignment: The average yaw misalignment is very small (2.0 °) and is therefore neglected.
- Pitch angle: The measured pitch angle is prescribed, see figure 8.11.

For both cases, structural flexibility, wind shear and yaw misalignment have been taken into account by those participants who were able to. Participants who did not take into account structural flexibility have reduced their flapwise moments with 9% to account for centrifugal stiffening (although this is of course unrealistic at the end of the transient when the rotor is at standstill).
8.3 Results

8.3.1 General

The aim of this section is to discuss the results for the axial symmetric cases. In the previous section (8.2), the description of the four cases is given:

- Case I: Preliminary calculations for a step on the wind speed and a step on the pitch angle; Artificial cases, no comparison with measured data.
- Case II: Calculations for a step on the pitch angle which are compared with measurements on the Tjæreborg turbine. No attention is paid to structural dynamic effects.
- Case III: Calculations for a step on the pitch angle including structural dynamic effects. The calculations are compared with measurements on the Tjæreborg turbine.
- Case IV: Calculations of a safety step. The calculations are compared with measurements on the Tjæreborg turbine.

8.3.2 Preliminary calculations (Case I)

Discussion of results

As already described in section 8.2.1, the required results for the preliminary case I were non-dimensionalised, because the main interest of the project is on the relative changes which are caused by dynamic inflow instead of an accurate prediction of stationary values.

In Appendix B the results are presented. The figures B.1 to B.8 give the results for the step on the wind speed and the figures B.9 to B.16 give the results for the step on the pitch angle. The differences in equilibrium values are presented in table B.1 and B.2. The free wake calculations of Unist, for the entire period would take some hundred hours on the IBM-RISC/6000 used. Therefore the time domain was split in two parts, each starting with the appropriate steady state. In the figures, the two parts are connected by a straight line. However for the determination of the \( \alpha_{eq} \) value the average was taken of the state resulting a few seconds after the first step and the steady state before the second step. This explains why the results of Unist, do not always depart from 1.0 at the second step. The ECNde calculations are obscured by a bug, which leads to a time scale which is too large.
Analysis of results
The main observations on the results are:

- An important dynamic wake effect is visible for the step in pitch angle: Most calculations predict a change in rotorshaft torque just after the step in pitch angle which is more than 5 times the stationary change. (Note from table B.2 that the stationary change in rotorshaft torque is rather small.) The predicted change in flatwise moment is about 1.5 times the stationary change. The dynamic wake effect appears to be rather limited for a step on the wind speed. The reason is that the changes in $u_l$ between the equilibrium states are rather small although this does not hold for the inner blade elements. This also explains the high values of $\Delta u_l$ in the figures B.7 and B.8: Since $u_{42} - u_{41}$ is very small, a limited change in $u_l$ yields a very large value of $\Delta u_l$. In order to explain this small change in induced velocity, a linearized expression for the change in induced velocity as function of the change in wind speed is derived in Appendix R.

It is found that the change in induced velocity is proportional to the value of the factor $a_1 - \sigma \lambda s_{1,41}/4$ which is shown to be proportional to the value of $\theta + \alpha_0$ with $\alpha_0$ the zero lift angle of attack. This factor appears to be decisive for the change in induced velocity. For the Tjæreborg turbine this factor is very small leading to very small changes in induced velocity. In Appendix R it is shown that the factor is somewhat larger for other turbines, leading to larger changes in induced velocity.

- Generally speaking the agreement between results is at least qualitatively good although there are differences in time scale. The most striking deviations are:
  1. The relative peak loads predicted by the Unist. just after the step in pitch change are much lower than predicted by most other participants. This may partly be due to the larger stationary changes predicted by Unist. in particular for the rotorshaft torque (see table B.2).
  2. Unist. also predicts a large peak in axial induced velocity just after the step in pitch angle and to a lesser degree just after the step in wind speed. This is possibly caused by instantaneous profile aerodynamics, i.e. the tilting of the lift vector due to the shed vortex following the pitch step. This effect is automatically included in this method.
  3. The axial induced velocity predicted by 'ECN, i.w.' in the case of a wind speed step first changes in a direction further away from the new equilibrium situation.
  4. The results predicted by 'ECN, d.e.' in the case of a pitch angle step have not reached equilibrium at the end of the calculations due to a larger time scale. However this appeared to be a bug in the program which was eliminated for the next cases.
  5. The results of TUD and Unist. show higher frequency fluctuations which are a result of the vorticity shed by the previous blade. For this reason TUD finds in general a shorter time constant than most of the other participants where the wake is "averaged" over one revolution.

Conclusions and recommendations
This section has shown that the newly developed models predict substantial dynamic wake effects on the flatwise moments and the rotorshaft torque at fast pitching changes, which are not found with standard quasi-stationary blade element momentum theory methods. In the following section, calculations will be compared with measurement of pitching steps on the Tjæreborg turbine. The dynamic wake effect for coherent wind gusts is much less. This is explained by the very small change in induced velocities for the gusts which are considered. The
change in induced velocity was shown to be proportional to the value of $\theta + \alpha_0$
with $\alpha_0$ the zero lift angle of attack. This factor is very low for the Tjæreborg
 turbine. However for other turbines this factor can be larger.

8.3.3 Pitching transients (Case II)

Discussion of results
The definition of the case is given in section 8.2.2. Four measurement series of
pitching steps on the Tjæreborg turbine are simulated. The rotorshaft torque and
the flatwise moment at blade root are compared with measurements. Effects of
structural flexibility, windshear, tower, turbulence etc. are not taken into account
in this section. Therefor the measurement series were averaged in such a way that
these influences were eliminated as far as possible, see section 8.1.2. The results
are presented in Appendix C.

Analysis of results
In general good agreement between the calculations and the measurements and
between the calculations mutually is found. The overshoots in loads which are
discussed in section 8.1.2 are clearly present in the calculated results. It must be
noted that calculations with standard quasi-stationary blade element momentum
theory would not yield any overshoot at all. In the following the results are com-
pared in terms of overshoots and time scales.

OVERSHOTS

Tables 8.4 and 8.5 summarize the results for blade root moment and shaft torque
in the same manner as these of the measurements, see section 8.1.2, but only in
terms of the ranges for equilibrium and dynamic wake conditions. The first 2
columns give the averaged values from the differential equation models used by
TUDk, GH and ECN, together with the variation (absolute) between the models,
while the second two columns show the same for the wake type models used by
DUT, Unist. and NTUA.

The reason for this grouping is twofold. Firstly, the first three models are en-
geineering models, to be applied to aeroelastic codes and are similar in structure.
Secondly, the mutual differences between these models is relatively small, as
reflected by the (absolute) variation in the results, while the dynamic ranges
(max-min) of the free wake models are consistently lower than those of the first
three models. The results from the 'integral wake' model of ECN (ECN,i.w.) are
not used in this summary, in some cases these results are close to the engineering
models, in other cases close to the free wake results. Also the equilibrium range
of the rotorshaft torque predicted by NTUA are excluded in table 8.5. They differ
from both the engineering models as well as from the other free wake models.

For the purpose of the present study the dynamic ranges are of interest. By
comparing tables 8.2 and 8.3 with 8.4 and 8.5 the following conclusions can be
drawn:

- the dynamic ranges calculated by the engineering models are in good agreement
  with the measured values, although in general slightly higher;
- the dynamic ranges calculated with the free wake models are consistently lower
  than those of the measurements and those calculated by the engineering models.
Table 8.4  Averaged values and variations of calculated ranges of root bending moment for two types of models

<table>
<thead>
<tr>
<th>case</th>
<th>1 − 2 [kNm]</th>
<th>max − min [kNm]</th>
<th>1 − 2 [kNm]</th>
<th>max − min [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>II.1</td>
<td>95 ± 5</td>
<td>290 ± 10</td>
<td>106 ± 7</td>
<td>172 ± 7</td>
</tr>
<tr>
<td>II.4</td>
<td>165 ± 5</td>
<td>400 ± 5</td>
<td>208 ± 8</td>
<td>334 ± 20</td>
</tr>
<tr>
<td>II.3</td>
<td>175 ± 8</td>
<td>292 ± 8</td>
<td>190</td>
<td>243 ± 28</td>
</tr>
<tr>
<td>II.2</td>
<td>113 ± 3</td>
<td>163 ± 3</td>
<td>115 ± 5</td>
<td>145</td>
</tr>
</tbody>
</table>

Table 8.5  Averaged values and variations of calculated ranges of rotor shaft torque for two types of models

<table>
<thead>
<tr>
<th>case</th>
<th>1 − 2 [kNm]</th>
<th>max − min [kNm]</th>
<th>1 − 2 [kNm]</th>
<th>max − min [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>II.1</td>
<td>2 ± 1</td>
<td>88 ± 2</td>
<td>15 ± 5</td>
<td>63</td>
</tr>
<tr>
<td>II.4</td>
<td>24 ± 10</td>
<td>231 ± 4</td>
<td>44</td>
<td>165 ± 5</td>
</tr>
<tr>
<td>II.3</td>
<td>26 ± 8</td>
<td>174 ± 6</td>
<td>47 ± 3</td>
<td>110 ± 15</td>
</tr>
<tr>
<td>II.2</td>
<td>38 ± 3</td>
<td>126 ± 8</td>
<td>42 ± 3</td>
<td>92 ± 3</td>
</tr>
</tbody>
</table>

Figure 8.12  Equilibrium range for blade root flat moment as function of loading, calculated and measured

The situation is graphically shown in the figures 8.12, to 8.15 with the tip speed ratio along the horizontal axis. Calculations are only made for 4 cases, while 6 measured cases are presented. Furthermore the reader should keep in mind that the results do not depend on the tip speed ratio alone, but also on the pitching rate and the pitch angle increment.

Although not directly related to the present study, it is of interest to note that the equilibrium ranges predicted with the differential equation models are consistently lower than those of the measurements, while these of the wake models compare well with the measurements. In effect, agreement is better than for the dynamic ranges.

TIME SCALES
Figure 8.13  Dynamic range for blade root flat moment as function of loading, calculated and measured

Figure 8.14  Equilibrium range for rotorshaft torque as function of loading, calculated and measured

For the calculations of the time scales an exponential behaviour is assumed according to:

$$ F(t) = F_1 + \Delta F \ast (1 - \exp^{-(t-t_1) / \Gamma(t)}) $$  \hspace{1cm} (8.1)

(see figure 8.16).

Then the time scale can be derived from:

$$ f(t) = \frac{t - t_1}{\ln((F_2 - F) / \Delta F)} $$  \hspace{1cm} (8.2)

The definition of $t_1$, $F_1$ and $F_2$ (and consequently $t_2$) has been a point of concern for the calculations and in particular for the measurements:

- Calculations:
  - Induced velocities; See as an example figure 8.17.
    
  In section 8.1.2 and 8.2.2, the pitch angle step is performed within a certain ramp period. For $t_1$ the time step at the end of the ramp period is taken.
Joint investigation of dynamic inflow effects and implementation of an engineering method

Figure 8.15 Dynamic range for rotor shaft torque as function of loading, calculated and measured

Figure 8.16 Exponential behaviour for calculation of time scale

- Loads: See as an example figure 8.18. For \( t_1 \) the time step when the load is minimum or maximum is taken.

For \( t_2 \) the time at the end of the pitching step is taken.

- Measurements: In most cases the same definition is used as for the calculations. However in some cases, rather large variations in the measured time series occurred at the end of the pitching step, see as an example the behaviour of the rotor shaft torque at \( t = 60 \text{ s} \) in figure C.12. This resulted in unrealistic values for \( F_2 \). Then the value of \( t_2 \) had to be manipulated to own insight (see below). The fluctuations in the measured time series resulted in large fluctuations of the time scales. Therefore the time scales were approximated with a least square fit.

In figure 8.19, the time scale of the measured flatwise moment of case II.1 is derived with equation 8.2 and the resulting values are approximated with a least square fit. In figure 8.20, the least square fit for the time scale is substituted in equation 8.1.
A compilation and discussion of all time scale results is given in [18]. Examples of the calculated time scales of the induced velocity at two radial positions are given in the figures D.1 and D.2 in Appendix D. (Note that Unist. is not presented in these figures). Examples of calculated and measured time scales of the flatwise moment and the rotor shaft torque are given in the figures D.3 to D.6 in Appendix D. The most important conclusions are:

- ECN iw shows some radial dependence. Decreasing towards the tip.
- ECN dc shows large radial dependence. Decreasing towards the tip.
- DUT shows some radial dependence also decreasing towards the tip.
- TUDk shows some radial dependence reduced slightly at the tip initially but increased as time passes.
- GH shows largest radial dependence increasing towards the tip.
- There is a large spread in calculated time scales.
- Most calculated time scales show an increase with time (df/dt > 0).
- Although there is a large variety in character of measured time scales, the gradient df/dt is in most measurements (much) larger than calculated. A possible explanation might be the structural dynamics effect. In the figures D.5 and
Figure 8.19 Tjæreborg: Least square fit for time scale derived from measured flatwise moment; $V_\infty = 7.4$ m/s; $\theta = 1.0^\circ$ to $3.0^\circ$; (CASE H.1)

Figure 8.20 Tjæreborg: Measured flatwise moment and approximated flatwise moment with least square fit of time scale; $V_\infty = 7.4$ m/s; $\theta = 1.0^\circ$ to $3.0^\circ$; (CASE H.1)

D.6 the calculations which are denoted by "calc.flex" stands for "ECN.d.e." calculations in which structural flexibility has been taken into account. It is clear that the gradient $df/dt$ for the flatwise moment is increased by the structural dynamics effect. The increase in $df/dt$ for the rotorshaft torque is much less.

- In table Appendix D.1 the results of all cases are quantified: The time scales are given at (a somewhat arbitrary) fixed time step: $t = t_i + 0.5D/V_\infty$. In the table the values of the time scale for the cases II.5 and II.6, which are not simulated (see section 8.1.2), are added also. It can be concluded that at this time step the agreement between the measured and the calculated time scales is reasonably good, while the time scale appears to be between 0.9 and 0.5 $D/V_\infty$.
- Although in the period not too long after $t_i$ (say $t < t_i + D/V_\infty$) most measurements fall within the band of calculations it must be realized that the spread in calculations is large ($\approx \pm 0.15 D/V_\infty$, which is $\approx \pm 50\%$).
• At later time steps the measurements fall outside the band of calculations. This is due to the larger measured value of $d\bar{d}/dt$. The load time series is hardly affected by this discrepancy, because the timescale is in the order of 0.3 to 0.5 $D/V_\infty$ and consequently the equilibrium values are almost reached. So from a pragmatic point of view this discrepancy might not be very important since the effect on the time series is rather limited.

• Above, the exact equilibrium values in the measurements have been manipulated in some cases. This of course gives an uncertainty in the value of the measured timescale. The effect of a small change in the second equilibrium value on the timescale is shown in figure D.7. In this figure the timescale is calculated with the second equilibrium value taken as the measured value at $t = 60$ seconds and at $t = 59.44$ seconds. If the resulting timescales are substituted in equation 8.1, the effect on the time series appears to be rather limited, see figure D.8. However, the different equilibrium values result in a maximum difference in timescale of 0.2 $D/V_\infty$ while the gradient $df/dt$ changes sign.

EFFECT OF INSTATIONARY PROFILE AERODYNAMICS

The aerodynamic loads on a turbine which is exposed to pitch angle variations will not only be influenced by instationary wake effects but also by instationary profile aerodynamics.

![Graph](image)

Figure 8.21 Effect of instationary profile aerodynamics at $V_\infty = 12.5 \text{ m/s}$

ECN investigated the influence of instationary profile aerodynamics on the rotor-shaft torque results of case II.2. Thereto calculations with the Onera model, see [19], were performed at a wind speed of 12.5 m/s (case II.2) and a wind speed of 15 m/s, see the figures 8.21 and 8.22. The influence of instationary profile aerodynamics at case II.2 appears to be very limited. At 15 m/s a clear effect can be seen in the rotor-shaft torque: A larger peak due to stall occurs, while the peak due to dynamic wake effect is decreased. It must be realized however that for a well designed pitch control, no pitching changes in stall will occur.

Conclusions and recommendations

The conclusions from the preliminary case 1 calculations (section 8.3.2) is now confirmed by measured data: The dynamic inflow effect appears to be important
when calculating mechanical loads on turbines which are exposed to rapid changes in the pitch angle. The comparison with measurements revealed that the new models developed in this project predict the measurements much better than quasi-stationary blade element momentum theory. A good agreement between calculated and measured overshoots is found. Both calculations and measurements show that the overshoots in loads increase with loading and pitch rate.

The agreement between measured and calculated time scale is reasonable. The time scale in the dynamic wake process appears to be between 0.3 and 0.5 \( \tau / V_\infty \). It must be noted however that there is a large uncertainty in the measured time scale.

8.3.4 Pitching transients including flexibility (Case III)

Discussion of results
The definition of the case is given in section 8.2.3. The goal of this calculational case is to assess the influence of structural dynamics in conjunction with dynamic wake effects. Therein two pitching steps were simulated. Case III.1 consists of one of the measurement series of 60 seconds from which case II.4 is composed. Case III.2 is similar to case II.4, the only difference is the inclusion of blade flexibility. In Appendix E the results are presented. The calculations have only been performed by those participants who are able to include structural dynamics. These are ECN.de, GH, and TUDk.

Analysis of results
In assessing the effect of structural dynamics it is sufficient to compare the case III.2 and II.4 results. The only difference between the two sets of calculations is the inclusion of structural dynamics. When comparing the figures E.7 with C.28, it can be seen that adding structural flexibility to the calculations hardly affects the dynamic range of the shaft torque, due to the relative stiffness of the drive train.

Furthermore it is known from section 8.3.3, and figure D.6, that the influence of structural dynamics on the time scale of the rotorshaft torque is also very small.

However, the dynamic range of the blade root flat moment is increased (compare figure E.8 with C.25), more so in the calculations than can be appreciated in the
measurements. In fact the calculated dynamic range of this case increases from 400 kNm to 450 kNm, while the peak to peak value of the measurements is 405 kNm, an increase of 30 kNm over the estimate of the aerodynamic range from table 8.2. In section 8.3.3 it has been made clear that the effect of structural dynamics on the time scale of the flatwise moment is an increase of the gradient df/dt, which brings it generally more in agreement with the measured slope of the time scale.

Conclusions and recommendations
In this section it has been found that the influence of structural dynamics on the overshoots (and time scale) of the rotor shaft torque is limited. The time scale of the flatwise moment appears to agree better with measured values when structural dynamics is taken into account. The overshoots are somewhat overpredicted when structural dynamics is included.

8.3.5 Safety stops (Case IV)

Discussion of results
The definition of the case is given in section 8.2.4. The goal of this calculational case is to assess the influence of dynamic wake effects on transients. There two safety stops were simulated and compared with measurements. The stop is achieved by feathering the blade at a high pitching rate. Case IV.1 is a safety stop at a low wind speed of 7.04 m/s, case IV.2 is a safety stop at a wind speed of 14 m/s. In Appendix F the results are presented. Some participants took the effect of structural dynamics and wind shear into account, other participants did not.

Analysis of results
There is a reasonable agreement between measured and calculated values of rotor speed, rotor shaft torque, flatwise moment and edgewise moment. The rotor shaft torque predicted by DUT, Unist. and NTUA only represents the aerodynamic torque. The flatwise moment predicted by Unist. does not contain the weight component, which becomes dominant when the pitch angle is feathered. Both the flatwise moments and the shaft torque show a strong vibration after the release of the generator, in their respective natural modes.

In the case of the safety stops, the influence of dynamic inflow on the maximum load level is not very large, as can be seen from figure F.17, comparing the flatwise moments calculated with a dynamic inflow model and equilibrium wake. There is a small overshoot but not as much as in the case of the former pitching transients. This can be explained as follows.

The essential difference between dynamic inflow calculations and equilibrium wake calculations is in the angles of attack experienced by the blade. Those for the dynamic wake lag behind the equilibrium value, while the wake is developing. This effect can be seen in the calculations, where the dynamic inflow loads lag behind the equilibrium wake values. When the stop is initiated, there is an appreciable difference between the two curves, but this is obscured by the steepness of the curves. Maximum loads are attained when the angle of attack is such that the lift coefficient takes up its minimum value (negative stall). This happens for the two types of calculations at different moments, but the magnitude of the load is determined primarily by the minimum lift coefficient, and only two a small degree
by the remaining conditions (blade angle) which are slightly different due to the different time values.

Larger influence of dynamic inflow effects may occur at lower pitching rates, such that for the equilibrium wake calculations the minimum lift coefficient is not reached, but through dynamic inflow effects it would be reached in reality.

**Conclusions and recommendations**
The overshoots by dynamic inflow at safety stops where the blade is feathered with a high pitching rate are not very large. Possibly larger overshoots are found for a stop procedure with a lower pitching rate.
9. AXISYMMETRIC CASES, WINDTUNNEL

9.1 Available measurements

Three different types of measurements have been used in the present project which were taken in the DUT wind tunnel under axisymmetric conditions:

- Measurements at coherent wind gusts;
- Rotor characteristics;
- Near wake measurements.

The core of the investigations was on the coherent wind gust measurements. The second type of measurements served as a check for the validity of the standard acrodynamic models, in particular the profile data used in the third type of measurements resulted additional flow field information to validate the free wake models.

A global description of the wind tunnel and its model is given in section 5.2 and Appendix U.

9.1.1 Equipment

Wind tunnel

The wind tunnel measurements are performed in the open-jet wind tunnel of the Institute for Wind Energy. This tunnel consists of a flow channel of circular cross section (diameter 2.24 m), with a fan at the inlet, a flow straightener and gauzes. The tunnel axis is at 2.33 m from the floor.

Gust generation

As stated in section 9.1 one type of wind tunnel measurements has been made at coherent wind gusts. The coherent wind gusts were realized by means of two gauzes, which could manually be opened or closed by which porosity, and therefore the wind speed, was modified.

Instrumentation (coherent wind gust cases)

The following quantities have been measured (for the coherent wind gust cases):

- Undisturbed tunnel wind velocity with a hot wire at the tunnel exit and next to the rotor, in the rotoplace at 1.5R from the rotor centre.
- Rotational speed and azimuth angle by means of a pulse counter;
- Rotor thrust by means of a strain gauge bridge
- Flap and lead-lag moments by means of strain gauge bridges
- Rotor torque. Commercially available instrument.

The hot wire probe position was set by a traversing system. In radial direction this can be controlled by a computer. In axial direction the adjustment was done by hand.

Data acquisition system

At the start of the project, a HP-200 series HP-BASIC computer with a HP3852 Data Acquisition and Control Unit has been used. The 13-bit high speed voltmeter has a maximum sample frequency of 100 kHz, which is enough to keep track of the 720 samples per revolution for a maximum of eight signals simultaneously at f =16 Hz.
During the project, the HP system was replaced by a PC based system for versatile use and easier transfer and processing of data files, with the same technical specifications and accuracy.

9.1.2 Measurement procedure (coherent wind gust cases)

The measurements at wind gusts which have been selected and which are described in the sequel are averaged over about 30 realisations, synchronised at the drop or increase in wind speed. It was difficult to find the best estimate of the 'free stream' wind speed. Hot wire measurements were available at the tunnel exit and next to the rotor at 1.5R from the rotor centre. The hot wire signal at the tunnel exit appeared to be heavily disturbed by the gauzes and could not act as a reliable measure for the free stream wind speed. Also the hot wire signal next to the rotor was disturbed. Therefore the wind speeds had been estimated from the axial force signal. Due to the fact that the gauzes were opened and closed manually, and the weight of the gauzes, the time which was needed to open or close the gauze varied and appeared to be between 0.14 and 0.2 seconds. The time for the velocity to reach equilibrium has been estimated at 0.2 seconds, from the hot wire signals. The resulting load signals have further been smoothed, by filtering out the most prominent eigenfrequencies, using a band stop filter.

9.1.3 Measurement accuracy

Non-uniformity of the tunnel flow field

The uniformity of the flow field for the coherent wind gust cases is sketched in the figures 9.1 to 9.3:

![Wind Tunnel, uniformity check](image)

Min.: 0.958
Max.: 1.026
St.dev.: 0.015

Figure 9.1 Uniformity of DUT wind tunnel, situation without gauzes
Cross-talk
DUT investigated the cross talk of the wind tunnel measurements. Thereto the maximum load was applied and then the percentage of loading in the perpendicular direction was determined. The cross-talk appeared to be less than 1%.

Axial force and blade root moments
Repeated calibration of axial force meter and blade root strain gauges showed that these had an accuracy of about 5%.

Hot wires
The hot wire probes have been calibrated before every measurement section, because of overnight temperature changes in the laboratory. By doing so, the accuracy of the measured velocity is about 0.1 m/s, which is about 2% at a medium tunnel speed of 5 m/s.

9.2 Definition of calculation cases
The description of the wind tunnel model is given in section Appendix S.
Three types of axisymmetric wind tunnel cases have been analysed in the present project:
- Rotor characteristics (case tun-char)
- Coherent wind gusts (case tungust);
- Flow field at rotor plane (case tun-wake)
9.2.1 Prediction of rotor characteristics (case tun-char)

A test case for the prediction of rotor characteristics \( C_p(\lambda) \) and \( C_{D_{st}}(\lambda) \) under axisymmetric conditions was performed. The pitch angles were 0°, 2° and 4°. This case, denoted by tun-char led to an indication about the accuracy of the standard aerodynamic models, without the complicating dynamic inflow effects. This was of particular importance because some estimates had to be made of the profile data with which the model blade is equipped (NACA0012 data at low Reynolds numbers). Effects of structural flexibility are not taken into account.

9.2.2 Flow field at near wake (case tun-wake)

As already stated in section 7.2.2, near wake measurements which have been taken in the past by DUT, were reproduced by the free wake methods, to get insight into the dependancy of these methods on discretisation and viscous effects.

Hot wire measurements were available at four radial axial positions:
- \( x/R = 0.033 \)
- \( x/R = 0.07 \)
- \( x/R = 0.103 \)
- \( x/R = 0.137 \)

Horizontal traverses were made at fixed positions: \( \phi_z = 90^\circ \) for \( -z/R = 0.4 \) to \( -z/R = 1.1 \) with steps \( \Delta z/R = 0.1 \). The hot wire signals are averaged over a revolution, i.e. the effect of the blade passage (bound vortex) on the signals was eliminated. The pitch angle was 2 degrees. Measurements at two tip speed ratios were made: \( \lambda = 8 \) and \( \lambda = 6 \), corresponding to \( V_\infty = 7.32 \text{ m/s} \) and \( V_\infty = 5.59 \text{ m/s} \). The rotorspeed was constant at 11.65 Hz.

From the raw velocity measurements the bound vortex strength has also been...
determined and compared with calculations.

The required quantities are the total horizontal velocities at the positions described above and the bound vortex strength along the blade.

The case is denoted by case tun-wake.

9.2.3 Coherent wind gusts (case tungust)

The case which concerns the coherent wind gust is denoted by case 'tungust'. Two calculations have been performed, one for the wind speed step going up (case tungust-up) and one for the wind speed step going down (case tungust-down). Axisymmetric conditions have been assumed and the effects of structural flexibility have been neglected. The conditions for case tungust-up are:

- \( 0 < t < 0.8 \text{s} \): \( V_\infty = 4.9 \text{ m/s} \)
- \( 0.8 < t < 2.0 \text{s} \): \( V_\infty = 4.9 + 0.8(1-e^{-(t-0.8)/0.109}) \text{ m/s} \)

The conditions for case tungust-down are:

- \( 0 < t < 0.5 \text{s} \): \( V_\infty = 5.7 \text{ m/s} \)
- \( 0.5 < t < 0.9 \text{s} \): \( V_\infty = 5.7 - 2.4(t-0.5) \text{ m/s} \)
- \( 0.9 < t < 2.0 \text{s} \): \( V_\infty = 4.9 \text{ m/s} \)

The rotor speed is 12 Hz and the tip angle is 2 deg.

The following quantities are required as function of time:

- Axial induced velocities at 50% span, 70% span and 90% span.
- Axial force on the rotor shaft.
- Flatwise moment at \( r = 0.129 \text{ m} \).

9.3 Results

9.3.1 Rotor characteristics (case tun-char)

Discussion of results

The measured and calculated rotor characteristics are compared in Appendix G, the figures G.1 to G.6: Both \( C_{D,ax} \) as well as \( C_p \) are given as function of the tip speed ratio \( \lambda \) for three values of the pitch angle: \( \theta = 0^\circ, 2^\circ, 4^\circ \). Axisymmetric, stationary conditions have been assumed, effects of structural flexibility are neglected.

Analysis of results

In general there is a reasonable agreement between the calculations and measurements. The agreement in \( C_{D,ax} \) is in a relative sense somewhat better than in \( C_p \). This is as expected since the driving force in the power is the \( c_p \sin \phi - c_t \cos \phi \) term which is the small difference of two relative big numbers. In general the measured \( C_{D,ax} \) is higher than most calculations. The ECN,iw model results deviate. Since this method is only used to deliver a qualitative model of the dynamic wake processes, no efforts were made to improve the accuracy (the ECN,de model is used for more accurate calculations). Furthermore the ECN,iw model is not suited for larger tip speed ratios, at the turbulent wake. This leads to reverse flow in the wake, which is not modelled properly and which gives numerical instabilities.

Conclusions and recommendations

The main reason to perform this case was to find whether the aerodynamic profile data which have been used, are good enough to perform the more complicated
dynamic inflow cases. Although there is rather large spread in calculated results, the agreement with the measured results is considered acceptable.

9.3.2 Near wake (case tun-wake)

Discussion of results
The velocities calculated with the free wake models are presented together with the measurements in Appendix G, the figures G.7 to G.16. The results are shown as function of radial position along the blade at the four axial positions. The total horizontal velocity in the wake was requested, which is composed of the free stream wind vector and the wake induced velocity vector, since it has been attempted to filter out the velocities induced by the bound vortex. The free stream velocity is plotted in these results as a reference.

Analysis of results
In general the agreement between calculations and measurements is good, with the exception of the results near the tip. This may be caused by the tip vortex. Part of the discrepancy may be caused by the way how the influence of the tip vortex it weighted in the averaging procedure. Furthermore, the high peaks shown in the experiment can be due to the structure of the core of the vortex, a region where inviscid theory is not valid. NTUA performed a further analysis of the dependency of the results on time step, grid size and cut-off length, see [20], included as Appendix G in [21]. The best results were obtained with a uniformly spaced grid of 20x8 panels and a time step that divides each rotation into 48 steps. The best choice for the cut-off length, an essential numerical parameter in the vortex particle method, see Appendix Q and section 7.2.2 was 0.035R. However the cut-off length only influenced the results at the tip and the root of the blade.

In most of the calculations which are presented in this report, NTUA applied by approximation these parameters.

9.3.3 Wind gusts (case tun-gust)

Discussion of results
The results (calculated as well as measured) for the wind gust cases are given in Appendix G, the figures G.17 to G.26. As explained in section 9.2.3., the effects of structural flexibility and turbulence have been filtered out.

Analysis of results
In general the agreement between calculations and measurements is acceptable.

The overshoots in loads are, as expected from 8.3.2., very limited. Only a very small overshoot in loads is visible in the step down.

It must be noted however that the step in wind speed takes place in a period of 0.2 s, which is relatively large compared to the time scale in the dynamic inflow project which is expected to be less than 0.1 s.

Although strictly spoken outside the scope of the present investigation, which is focussed on dynamic effects, it is interesting to see that the three types of engineering models predict almost the same equilibrium level of axial induction. Also the induction from the free wake models is grouped very close together but there is a rather large difference with the level from the engineering models. In terms of loads, the differences are much less pronounced.
Conclusions and recommendations
As expected from theoretical considerations, developed during the project, the wind tunnel measurements revealed that a step on the wind speed does not give rise to clear dynamic inflow effects, and in principle these situations can be modelled with the classical stationary blade-element momentum theory. This implies also that the loads resulting from an incoming turbulent wind field are modelled properly with the classical theory, as is done until now. A definite conclusion cannot be drawn since the coherent wind gust which has been realised in the tunnel has a ramp time which is about twice the value of the dynamic inflow time scale.
10. IMPORTANCE OF DYNAMIC INFLOW UNDER AXISYMMETRIC CONDITIONS FOR DESIGN PURPOSES

In the previous sections it was shown both from measurements and calculations on the Tjæreborg turbine, that dynamic inflow can increase the fluctuations in the flatwise moments of the blades and the rotor shaft torque. This was especially true for steps on the pitch angle ($\Delta \theta \approx 2$ to 3 degrees). Dynamic inflow becomes more important when the pitching rate and the loading (the value of the axial induction factor) on the turbine increases. For above rated conditions the loading is low, so the dynamic inflow effect becomes less pronounced. It may be questioned whether dynamic inflow is important for lifetime calculations of pitch regulated turbines, since pitching actions normally take place at above rated wind speeds and probably the influence of dynamic inflow on the aerodynamic load calculations will be limited. For smaller turbines the dynamic inflow effects will even be very limited, since the time scale of the dynamic wake process increases proportionally with the diameter of the turbine. On the other hand it may be expected that the importance of dynamic inflow for constant $\lambda$ machines is large, since the loading for these turbines at $V_{\text{rated}}$ is high. (Note that this will only be true for fluctuations in the blade loads because the variable speed operation will diminish load fluctuations in the rotor shaft torque).

It is known from [22] that for the Tjæreborg turbine the fatigue damage of shaft and blade root components is mainly determined by the mass loads. This is a result of the relatively heavy blades of this machine. Consequently even small improvements in the prediction of the aerodynamic loads, as realized by the present engineering methods, is not expected to effect the total lifetime calculation. However at locations closer to the tip, the relative importance of aerodynamic loads on the total fatigue damage increases. Also for light weighted rotor blades the importance may be larger, since aerodynamic loads become more dominant and the pitching speed for these blades may be larger.

The dynamic inflow effect at a stop of the Tjæreborg is hardly visible. Since the contribution of starts and stops to the total fatigue damage of the Tjæreborg turbine is already very small [22], dynamic inflow at starts and stops will not effect the total lifetime of the Tjæreborg turbine.

The dynamic inflow effect for a coherent wind gust is negligible. This was found in calculations which were performed on the Tjæreborg geometry, but also on the coherent gust measurements in the DUT wind tunnel.

In summary, the design parameters which determine the importance of dynamic inflow are:
- The pitch rate $d\theta/dt$;
- The weight of the blade;
- The size of the turbine;
- Variable/fixed rotor speed;
11. SHORT DESCRIPTION OF MODELS FOR THE YAWED CASES

11.1 Modelling aspects

In this chapter a short overview is given of the different methods which have been used by the various participants to model the asymmetric cases. For a good understanding of the different aspects it is important to repeat the definitions of yaw angle and azimuth angle: The blade azimuth angle is defined as zero for the blade pointing down in vertical position, while the yaw angle is defined positive according to Fig. A.2 from Appendix A. Note that the axial induced velocity is defined to be perpendicular to the rotorplane (and not in wind speed direction).

The following aspects are of importance in yawed flow modelling, see also section 6, which describes the helicopter modelling:

1. Disk averaged induced velocity \( u_{i,0} \):

   For the aligned flow situation, the following expression is the basis for the axial momentum equation:

   \[
   F_{ax} = \rho S (V_\infty - u_{i,0}) 2 u_{i,0}
   \]  

   (11.1)

   This equates the total change of axial momentum of the mass flow through the rotor disk \( (V_\infty - u_i) \) is the axial velocity at the disk), to the thrust on the same. For this situation it can be shown that there is no net exchange of momentum (apart from viscous or turbulent mixing) between the mass flow across the rotor disk and the 'outer' flow.

   For the case of yaw misalignment Glauert proposed, see also equation 6.3:

   \[
   F_{ax} = \rho S \sqrt{V_\infty^2 + u_{i,0}^2} \cdot 2 u_{i,0}
   \]  

   (11.2)

   where \( V_\infty \) and \( u_{i,0} \) have to be added vectorially, and next normed.

   In fact (11.1) is a special case of (11.2). It is based on the fact that (11.2) is the correct expression for a yawangle of 90 degrees \( u_{i,0} \) normal to \( V_\infty \) looking at the rotordisk as a circular wing. Then (11.2) is valid for 90 degrees and 0 degrees yaw misalignment, and it is supposed to be true for inbetween values.

2. Advancing and retiring blade effect: For positive yaw, the blade will be retiring in the upper half plane and advancing in the lower half plane with respect to the inplane wind component. This would give a 1P variation of angle of attack and effective inflow velocity, see figure 11.1:

   \[
   \tan \phi = \frac{V_\infty \cos \phi_x - u_i}{\Omega r + V_\infty \sin \phi_x \cos \phi_{r,b}}
   \]  

   (11.3)

   \[
   W = \sqrt{(V_\infty \cos \phi_y - u_i)^2 + (\Omega r + V_\infty \sin \phi_x \cos \phi_{r,b})^2}
   \]  

   (11.4)

   The effect is less significant at the tip of the blade, where the rotational speed becomes dominant on the \( V_\infty \sin \phi_x \cos \phi_{r,b} \) term.

   The advancing and retiring blade effect is symmetric around zero azimuth and of \( \cos(\phi_{r,b}) \) type, i.e. giving rise to maximum thrust at vertical down position in a homogenous ambient flow with positive yaw, due to the higher effective velocity at that position.

   The standard methodology until today generally only has included this advancing and retiring blade effect. In this convential type of modelling there
will only be an aerodynamic tilt moment as a result of yaw, but, averaged over a rotor revolution, not a restoring yaw moment as is measured in reality. Rotor cone angles, or coning of the rotor due to flexibility will give a small restoring yaw moment in aerelastic calculations of this type, but very much smaller than those occurring in measurements. For a more detailed description of the advancing and retreating blade effect on the mechanical loads, reference is made to [23].

3. Skewed wake geometry with trailing vortices, see also figure 11.2: The proximity to the rotor plane of the vortices in the wake strongly influences the inflow. The trailing tip vorticity is on the average closer to the downwind side of the rotor plane and the root vortices pass closer to the downwind side.

The velocities induced by the trailing vortices are of the \( \sin \phi_{r,b} \) type, which corresponds to the Glauert type of modelling (see also equation 6.5):

\[
  u_t = u_{t,0}(1 - K_c \frac{r}{R} \sin \phi_{r,b})
\]

(11.5)

The sinusoidal induction distribution is more prominent at the tip, contrary to the cosinusoidal induction distribution from the advancing and retreating blade effect which becomes less at the tip.

This sinusoidal distribution of the axial induction velocity yields a higher value of the total axial velocity for the upwind half of the rotor plane and hence larger blade loads in this part (assuming linear aerodynamics), resulting in a restoring yaw moment.

The formula for \( K_c \) in equation 11.5 will depend on the shape of the wake. Several expressions for \( K_c \) have been developed in helicopter society, see section 6. They are all a function of the wake skew angle \( \chi \) (the angle between
SHORT DESCRIPTION OF MODELS FOR THE YAWED CASES

Figure 11.2  Skewed wake

the wake and the rotor axis), see figure A.3 from Appendix A:

\[ \tan \chi = \frac{V_\infty \sin \phi_y}{V_\infty \cos \phi_y - u_1} \]  \hspace{1cm} (11.6)

Equation 11.5 only refers to the axial induction component. In the course of the project it was argued that in addition to this, there is also an inplane velocity component induced by the skewed wake. This component will diminish the advancing and retreating blade effect and consequently the tilting moment, although the component is estimated to be of a smaller order of magnitude.

4. Skewed wake geometry, root vorticity: the skewed wake geometry is not only of importance for the trailing vorticity. Also the root vorticity will be released under a certain skew angle. The root vortex effects will be asymmetric around \( \phi_{R,b} = 270 \) degrees (in case of positive yaw) and they generate much higher harmonics, because they are felt by the passing blade almost impulsively.

It can be concluded that the hitherto common way of accounting for yaw misalignment effects gives a far too small restoring yawing moment due to the neglect of the skewed wake geometry influence on the axial induction. Furthermore the aerodynamic tilting moment due to yaw will be slightly overpredicted due the neglect of the inplane component from the skewed wake. Of course, often the more important contribution to the aerodynamic tilting moment comes from the vertical wind shear.

In the next chapter, the modelling of yawed flow in the engineering models, the DUT model and the ECN1w model is described. The free wake approaches from Unist. and NTUA are not repeated here since they automatically model asymmetric as well as axisymmetric conditions. It must be noted that the asymmetric cases were also calculated with the TA engineering model although TA is not an official participant in the project.
11.2 Models used in the project

**ECN, cylindrical wake model**

The induced velocities are calculated in a similar way as for the cylindrical vortex sheet model which is used for the axisymmetric conditions. However the shape of the wake is different. The wake skew angle is calculated from equation 11.6 with $V$ and $u_i$, the local values at the tip. This skew angle remains constant through the wake. A cyclic distribution for the vorticity $\gamma$ has been assumed. The advancing and retreating blade effect is taken into account according to equation 11.3 and 11.4. It must be noted that in this model a concentrated root vortex is present in the wake centreline, which contributes to the axial induced velocity. A detailed description of the model is given in Appendix J.

**ECN,d.e.**

The model calculates a disk averaged induction according to Glauert, equation 11.2, but the dynamic inflow term from section 7 is added, i.e. the equation is applied on annular ring level. Consequently $u_{i,a}$ is not the disc averaged value, but the value averaged over an annular ring. The distribution of induced velocity is given according to:

$$u_i = u_{i,d}[1 - f_{2,eca}(r/R)\tan\frac{\chi}{2}\sin\phi_p] \quad (11.7)$$

with $f_{2,eca}(r/R)$ according to the (static) Pitt and Peters model, considering only the thrust effect:

$$f_{2,eca}(r/R) = \frac{15\pi r}{64 R}$$

**DUT**

The DUT model for yawed flow is not essentially different from the model used in the axisymmetric cases, described in Appendix P. But some modifications were made for successful calculations of yawed flow cases. At first a numerical effort was performed to treat the different blades in a non-identical way, necessary for yawed flow calculations.

The application of PREDICCHAT and PREDICDYN in axisymmetric cases furthermore revealed the necessity to start the inflow calculations sufficiently far in front of the rotor. From tests a typical distance was found in the order of 10 rotor diameters. There is no reason for assuming that this would be different for yawed flow cases. But then a problem arose with respect to computer time needed to perform a yawed flow calculation. Yawed flow calculations are dynamic calculations all the way up from the beginning. Thus in numerical methods from a distance of 10 rotor diameters before the rotor the dynamic code has to be operational. Preprocessing with a steady PREDICCHAT calculation, as was always done in the axisymmetric situations to speed up the calculations, is thus strictly spoken, not possible. An engineering approach was adopted however, where it was assumed that the windturbine is originally operating in steady uniform perpendicular flow. This situation is calculated with PREDICCHAT. At some instant the rotor is instantaneously yawed into its desired position. Then the dynamic (PREDICDYN) calculations after this sudden yaw are continued until periodicity in the results is obtained. This again made it possible to combine steady preprocessing with dynamic calculations. From tests it turned out that in general periodicity was established after four consecutive revolutions.
Finally it must be realised that the acceleration potential theory is established upon a small perturbation approach. This means that the equations used in the codes assume small velocity perturbations with respect to the undisturbed axial velocity. In situations with a large yaw angle it cannot be expected that the method gives accurate results, if convergence is obtained at all.

TUDk

TUDk developed an engineering relation with a curve fitting procedure from an actuator disk vortex ring model. The local induced velocity was found to depend on the radial position, the azimuth angle and the wake skew angle according to:

\[ u_i = u_{i,0} \left[ 1 - f_{2,stick}(r/R) \tan^{2} \frac{\chi}{2} \sin \phi_{r,b} \right] \]  \hspace{1cm} (11.8)

This is a relation very similar to the relations from Glauert which are described above. The dependency on the radial position was found to be:

\[ f_{2,stick}(r/R) = \frac{r}{R} + 0.4 \left( \frac{r}{R} \right)^3 + 0.4 \left( \frac{r}{R} \right)^5 \]  \hspace{1cm} (11.9)

The rotor averaged induced velocity was found to be equal to the value from Glauert, equation 11.2. It turned out that the non-linear equation 11.9 exaggerated the amplitudes and better results were obtained without the higher order terms. The calculations which have been presented in this report were all done with the linear term only. The dynamic inflow term which is reported in section 7 is added to this, and the equations are applied on annular ring level. A detailed description of the model is given in Appendix N.

GH

GH have used the Pitt and Peters model, see Appendix M and also section 6, of dynamic inflow to calculate the induced velocity distribution for a yawed rotor. This model relates the induced velocity to the loading experienced by the rotor. It requires an iterative solution of the quasi-stationary induced flowfield. The Pitt and Peters model uses an average induction based on the overall loading of the disk, this has been modified at GH by using conventional combined blade element and momentum method to determine the mean inflow at any radial station but applies the same azimuthal and radial dependence of the variations about this mean obtained from the Pitt and Peters method. A detailed model description is given in Appendix M.

TA

The model is derived from a vortex filament method and cast into a curve fit algorithm. This yielded a relation for the axial induction factor according to:

\[ a = a_0 \cdot [1 - f_{r,ta}(r/R) \sin \phi_{r,b}] \]

The disk averaged induction factor was equal to the value from Glauert, but applied on annular ring level. The relation \( f_{r,ta}(r/R) \) was found to be a non-linear function, which increases with radius:

\[ c_1 = 2.629610^{-3} \]
\[ c_2 = 1.622210^{-3} \]
\[ c_3 = -1.111110^{-5} \]
\[ c_4 = -3.70371 \times 10^{-6} \]

\[ C_b = c_1 + c_2 \lambda + c_3 \lambda^2 + c_4 \lambda^3 \]

\[ \eta = 0.63 r/R [1 + 1.56 (r/R)^2] \]

\[ f_{2,ta} = \eta \cdot C_b \cdot \chi (\chi \text{ in degrees}) \]

TA performed the calculations, which are described in this report on a voluntary basis. However in the JOULE 2 project 'Dynamic Inflow, yawed flow conditions and partial span pitch control' TA is involved officially. A detailed model description will be given in the final report of that project.

11.2.1 Qualitative discussion of the differences between models

The free wake methods from NTUA and Unist, and the acceleration potential method from DUT automatically model all 4 effects which are mentioned in section 11.1 (advancing and retreating blade effect, skewed wake influence from trailing vorticities on the axial induction and inplane component and the skewed wake influence from the root vorticity). The ECN-sw method models all these effects with the exception of the inplane component induced by the skewed wake. The root vortex modelling in the ECN-sw method will be exaggerated, because it is concentrated in one vortex at the rotor centre line. A smear out of vorticity over the root section of the blade will be more realistic.

The engineering models, basically have added the Glaucert sinusoidal terms to the advancing and retreating blade effect. The in-plane component induced by the skewed wake and the root vorticity has generally not been implemented. The engineering models all apply the disc averaged velocity \( n_{1,0} \) from Glaucert on annular ring level.

The engineering models from TUDk and TA were both derived from more advanced wake models using a curve fit. The result which was found by TUDk confirmed the modelling proposed by Glaucert, with a sinusoidal induction distribution and a somewhat different non-linear radial distribution term: \( f_2 (r/R) \). However in a later stage of the project a linear radial distribution term has been applied again.

The procedure from TA also yielded a sinusoidal induction velocity distribution, as is the case in the Glaucert formula. The non-linear radial distribution term is less transparent: it does contain the shape of the wake in terms of the skew angle, but also the tip speed ratio \( \lambda \) is present.

The ECN-sw engineering model is very similar to the TUDk and consequently the Glaucert model. A linear, radial distribution term from the first harmonic in the Pitt and Peters model is used.

The GH engineering model is based on the Pitt and Peters model with 2 harmonics, but it is applied on annular ring level, while the original Pitt and Peters model is applied on disc level. It has the advantage as described in section 6 that the moment of momentum about the yawing axis which is present from the sinusoidal induction distribution is accounted for. The radial distribution term is linear for the first harmonic and quadratic for the second harmonics.

Finally, the tip correction factors in the engineering models, were left unchanged compared to the axisymmetric modelling, but in the TUDk and GH models the tip factor is only applied to the averaged induction \( n_{1,0} \) and not to the sine term.
In the axisymmetric case the tipfactor can be interpreted as due to the distance between trailing tip vortices, which cause a non uniformity in the induced velocity distribution that rotates with the blades. At least the trailing vortex geometry is unchanged following a blade. It must be noted however that in the case of yaw misalignment, this is not longer true.
12. YAWED CASES, TJÆREBORG

12.1 Available measurements

Four 10 minute time series at different yaw angles have been measured on the Tjæreborg turbine. A short description of this turbine is given in section 5.1.

These cases are denoted by case VII.1 to case VII.4.

12.1.1 Free stream conditions and load measurements of case VII

The free stream conditions (free stream wind speed, wind shear coefficient $\alpha$ (best fit from cup anemometers at 5 heights), and turbulence intensity $I$) are listed in table 12.1. The ambient wind conditions for the four measurement series are more or less comparable but the yaw angle varies between $-51^\circ$ and $+54^\circ$. A yaw angle of almost zero degrees has been added for reference.

<table>
<thead>
<tr>
<th>case</th>
<th>$\phi_y$ [°]</th>
<th>$V_\infty$ [m/s]</th>
<th>$\alpha$ [-]</th>
<th>$I$ [%]</th>
<th>$\theta$ [°]</th>
<th>$\Omega$ [rpm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>VII.1</td>
<td>32</td>
<td>8.5</td>
<td>0.31</td>
<td>8</td>
<td>0.5</td>
<td>22</td>
</tr>
<tr>
<td>VII.2</td>
<td>54</td>
<td>7.8</td>
<td>0.30</td>
<td>2</td>
<td>0.5</td>
<td>22</td>
</tr>
<tr>
<td>VII.3</td>
<td>-51</td>
<td>8.3</td>
<td>0.27</td>
<td>11</td>
<td>0.5</td>
<td>22</td>
</tr>
<tr>
<td>VII.4</td>
<td>-3</td>
<td>8.6</td>
<td>0.17</td>
<td>very low</td>
<td>0.5</td>
<td>22</td>
</tr>
</tbody>
</table>

The wind direction is measured with a fixed meteorological mast at 2D from the rotor.

Measurements are available of the blade root bending moments at all three blades. Furthermore the yawing moment at the front bearing is measured. However the large negative tilting moment introduces a large error into the yaw moment, due to the inaccuracy of the azimuth reading (30 kNm/(deg. azimuth)). Therefore the yawing and tilting moments which are used in the present project have been obtained from the flatwise moments at blade root from:

$$M_{yaw} = \sum_{ib=1}^{ib=B} M_{flat,ib}(\phi_{r,ib}) \cdot \sin(\phi_{v,ib}) \quad (12.1)$$

$$M_{tilt} = \sum_{ib=1}^{ib=B} M_{flat,ib}(\phi_{r,ib}) \cdot \cos(\phi_{v,ib}) \quad (12.2)$$

It must be noted that the yaw moment as defined above, is not the real yaw moment as experienced on the rotor hub, since the contribution of the blade root forces is not accounted for. However the effects which are of importance for the present project, are expected to be apparent in the yawing and tilting moments from equation 12.1.

An estimate for the real yawing moment can be obtained by multiplying the yawing moment from equation 12.1 with a calibration constant, which according to [24] = 1.16. The same holds for the tilting moment.
12.1.2 Discussion of binned averaged moments

The measurements of flatwise moments which are described in section 12.1.1 are binned averaged against the azimuth angle.

The results for all yaw angles are presented in Figure 12.1.

Figure 12.1 Measured flatwise moments at different yaw angles (Tjæreborg turbine)

The yawing moments which are derived from these flatwise moments with equation 12.1 are given in Figure 12.2. In Table 12.2 a Fourier decomposition is given.
of the binned averaged flatwise moment according to:

\[ M_{\text{flat}}(\phi_{r,b}) = \sum_{n=1}^{\infty} a_n \cdot \sin(n\phi_{r,b}) + b_n \cdot \cos(n\phi_{r,b}) \]  \hspace{1cm} (12.3)

\[ M_{\text{flat}}(\phi_{r,b}) = \sum_{n=1}^{\infty} A_n \cdot \cos(n\phi_{r,b} - \psi_n) \]  \hspace{1cm} (12.4)

\[ A_n = \sqrt{a_n^2 + b_n^2} \]  \hspace{1cm} (12.5)

\[ \psi_n = \arctan(a_n/b_n) \]  \hspace{1cm} (12.6)

Hence the phase \( \psi_n \) gives the position in the rotorplane where the maximum of the \( n^{th} \) harmonic is reached.

<table>
<thead>
<tr>
<th>( \phi_2 ) (case)</th>
<th>( A_0 )</th>
<th>( A_1 )</th>
<th>( \psi_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>32(VII.1)</td>
<td>575</td>
<td>57</td>
<td>127</td>
</tr>
<tr>
<td>54(VII.2)</td>
<td>323</td>
<td>52</td>
<td>110</td>
</tr>
<tr>
<td>-51(VII.3)</td>
<td>489</td>
<td>116</td>
<td>222</td>
</tr>
<tr>
<td>-3(VII.4)</td>
<td>643</td>
<td>37</td>
<td>198</td>
</tr>
</tbody>
</table>

Fig. 12.1 and table 12.2 show that, the phase of the binned averaged flatwise moment is dependent on the sign of the yaw angle. This is expected because the upstream part of the disk is exposed to higher winds, due to the sinusoidal induction distribution from equation 11.5: For zero yaw the phase \( \psi_1 \) (i.e. the maximum of the first harmonic) is reached at approximately the upper part of the rotor plane, due to the wind shear. For positive yaw angles the maximum of the first harmonic shifts to lower azimuth angles, since the upstream part is between \( \phi_{r,b} = 0 \) and \( \phi_{r,b} = 180 \). For negative yaw angles an opposite effect is found.

The 1P amplitude depends on the magnitude of the yaw angle, such that the amplitude increases with increasing (negative) yaw angle. However for a positive yaw angle the advancing and retreating effect counteracts the effects from the wind shear, by which the amplitudes are only slightly higher than for case VII.4 (zero yaw). Furthermore the case VII.4 is obscured by the fact that the wind shear is less than for the other cases. An equivalent wind shear would have given a higher 1P amplitude.

From this qualitative discussion of the binned averaged measured flatwise moment it can be concluded that at least the advancing and retreating blade effect and the sinusoidal axial induction distribution from the skewed wake have a notable influence, and therefore should be taken into account.

The influence of the \( 0^{th} \) and \( 1^{st} \) harmonics of the flatwise moment

\[ M_{\text{flat}} = A_0 + A_1 \cos(\phi_{r,b} - \psi_1) \]  \hspace{1cm} (12.7)

on the yawing moment can be assessed by substituting equation 12.7 in 12.1. This gives for a three blade turbine:

\[ M_{\text{yaw}} = -1.5A_1 \sin(\psi_1) \]  \hspace{1cm} (12.8)

constant over the rotorplane.

This equation then gives a measure for the rotor averaged yawing moment.
azimuthal variation in yawing moment which is visible in figure 12.2 is caused by
the higher harmonics in the flatwise moment.

With $\psi_1$ from table 12.2, equation 12.8 yields a stabilizing yawing moment for all
yawed cases (negative yawing moment for the positive yaw angles of case VII.1
and VII.2 and positive yawing moment for the negative yaw angles of case VII.3).
For the the non-yawed case (case VII.4) the binned averaged flatwise moment
tends to be cosinusoidal and in table 12.2 it can be seen that $\psi_1 \approx 180^\circ$. Then
equation 12.8 gives a mean zero yawing moment.
These observations are confirmed by the results from figure 12.2

12.2 Definition of calculation cases

12.2.1 General

Two cases on the Tjæreborg turbine at yawed conditions were defined. The
first one was denoted by case V. This preliminary case was defined before the
measurements from section 12.1 had become available and was used to get some
first experiences with the new yaw models. After the measurements had become
available, a new case (case VII) was defined in agreement with the measurement
conditions and since then, no updated results were supplied for case V, because
it appeared that the definition of case V was close to the measurement conditions
and would not deliver additional insights. Therefore, in this report the case VII
results are discussed only.

The description of the Tjæreborg turbine is given in Appendix T and Appendix U.

12.2.2 Definition of case VII.1 to VII.4 (azimuthal binned data under
yawed conditions)

The case consists of the simulation of the measured binned averaged results which
are described in section 12.1.1.

In all cases the rotor speed was 22 rpm. Structural flexibility, tower shadow and
tilt angle were taken into account by those participants who were able to. The
measurements were binned averaged over a 10 minute period. For the simulations,
only one revolution was reproduced. i.e. non-linear effects have been neglected.
For the low wind speed cases under consideration, the non-linearity will be very
limited. The wind shear has been estimated from measured 10 minute average
wind speeds at 5 different heights. The conditions for case VII.1 are:

- $V_{hub} = 8.5 \text{ m/s}$
- wind shear according to power law with exponent 0.31
- yaw angle = 32 deg
- pitch angle: 0.5 deg

The conditions for case VII.2 are:

- $V_{hub} = 7.8 \text{ m/s}$
- wind shear according to power law with exponent 0.30
- yaw angle = 54 deg
- pitch angle: 0.5 deg

The conditions for case VII.3 are:

- $V_{hub} = 8.3 \text{ m/s}$
- wind shear according to power law with exponent 0.27
- yaw angle = -51 deg
• pitch angle: 0.5 deg

The conditions for case VII.4 are:
• \( V_{hub} = 8.6 \, \text{m/s} \)
• wind shear according to power law with exponent 0.17
• yaw angle = -3 deg
• pitch angle: 0.5 deg

Required as function of azimuth:
• Axial induced velocities at \( r = 9 \, \text{m}, \, r = 15 \, \text{m}, (\approx 50\% \, \text{span}), \, r = 21 \, \text{m} (\approx 70\% \, \text{span}) \); \( r = 27 \, \text{m}(\approx 90\% \, \text{span}) \);
• Flatwise moments at \( r = 2.75 \, \text{m} \);
• Yawing and tilting moments which are derived from the flatwise moments on all three blades with equation 12.1 and 12.2. Note that the flatwise moment was measured on all three blades. The calculated flatwise moment which was given for 1 blade can be transformed easily to the flatwise moment on the other 2 blades.

12.2.3 Definition of summary data, case VII.5

In addition to the cases which examined the response of the turbine as function of azimuth angle for a particular yaw angle, a calculation round of so called 'summary data' was performed on a voluntary basis. In this case the rotor averaged loads and ranges (max-min) as function of yaw angle (0, 15, 30 and 45 deg) for several wind speeds (\( V = 7 \, \text{m/s}, \, V = 9 \, \text{m/s} \) and \( V = 11 \, \text{m/s} \)) were calculated. The rotor speed was 22 rpm.

The aerodynamic yawing moment, tilting moment, shaft power, and axial force were required. Wind shear was excluded, but NTUA and ECN took the effects of tilt angle, tower shadow and structural dynamics into account.

![Figure 12.3 Tjæreborg. Induced velocity at \( r = 9 \, \text{m} \) (30\%R); \( \phi_y = 32^\circ \); \( V = 8.5 \, \text{m/s} \) (CASE VII.1)](image)

The yawing and tilting moments were calculated at the hub location. As explained
in section 12.1.1, these moments differ from the values from equation 12.1 which are used for the cases VII.1 to VII.4.

A comparison is made with only a few measurements.

This case is denoted by case VII.5.

12.3 Results

12.3.1 Case VII.1 to VII.4

Discussion of results
In the figures H.1 to H.28 from Appendix H, the induced velocities, the yawing and tilting moments and the flatwise moments at blade root are presented as function of azimuth angle. The calculations of flatwise moments, yawing moments and tilting moments have been compared with measured results. As stated in the previous section, the calculations and measurements of the yawing and tilting moments have been derived from the flatwise moment data at blade root.

The DUT results for the large yaw angles, (54 deg (case VII.2) and -51 deg (case VII.3)) are not presented, because with the DUT model convergence is not established at these large yaw angles. The same is true for the ECN,iw results of induced velocities at r = 21 m and r = 27 m at \( \phi_r = 54 \) deg.

Structural flexibility has been taken into account by ECN,dc, GH, NTUA, TA and TUDk.

In the following subsection, the results are analysed. Since the yawing and tilting moments are driven by the flatwise moments, and these are largely determined by the induced velocities, the discussion starts at the induced velocities, followed by the flatwise moments and the yawing and tilting moments.

Analysis of results
In order to analyse the results of the induced velocities, six results have been copied from Appendix H which will be discussed in some detail. These results are given in the figures 12.3 to 12.8.

They show the results at a two positive and one negative yaw angle, at the inboard section and the tip section.

The following observations can be made:

- The influence of the root vortex is visible in the results from ECN,iw, DUT, NTUA and Unist. The influence is notably at \( \phi_{r,b} = 90^\circ \) for negative yaw and at \( \phi_{r,b} = 270^\circ \) for positive yaw. The trend in the Unist. and NTUA results is different from the trend in the ECN,iw (and DUT) results: This is explained by the fact that shedding of root vorticity at every blade is taken into account in the free wake approaches from Unist. and NTUA. In the ECN,iw method the root vorticity is concentrated in one single vortex. Obviously the free wake results will be more realistic.
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Figure 12.4 Tjæreborg, Induced velocity at $r = 27$ m (90%R); $\phi_y = 32^\circ$; $V = 8.5$ m/s (CASE VII.1)

Figure 12.5 Tjæreborg, Induced velocity at $r = 9$ m (30%R); $\phi_y = 54^\circ$; $V = 7.8$ m/s (CASE VII.2)
Figure 12.6 Tjæreborg, Induced velocity at \( r = 27 \text{ m (90\%R)} \); \( \phi_y = 54^\circ \); \( V = 7.8 \text{ m/s} \) (CASE VII.2)

Figure 12.7 Tjæreborg, Induced velocity at \( r = 9 \text{ m (30\%R)} \); \( \phi_y = -51^\circ \); \( V = 8.3 \text{ m/s} \) (CASE VII.3)
Joint investigation of dynamic inflow effects and implementation of an engineering method

Figure 12.8 Tjæreborg, induced velocity at \( r = 27 \, \text{m} \) (90% R); \( \phi_y = -51^\circ \); \( V = 8.3 \, \text{m/s} \)  
(CASE VII.3)

- Generally spoken the free wake results from NTUA and Unist are grouped closely together. Furthermore the engineering models from ECN,de and TUDk are grouped closely together.

- The results from TUDk, ECN, d.e., and TA are almost purely sinusoidal, no other harmonics are visible. In the GH model higher harmonics are also visible as expected from the Pitt and Peters model with 2 harmonics. However for case VII.2 GH applied the Pitt and Peters model with only the first harmonic terms, since the combination of large positive yaw and large shear led to convergence problems for the Pitt and Peters model with two harmonics.

- The induced velocity results cannot be compared with measurements. The comparison of engineering method results with free wake method results reveal that at the root considerable differences are found, which can be explained by the neglect of root vorticity in the engineering models. For practical purposes this may be of less importance, since the root section only gives a limited contribution to the loads. At the tip the free wake methods still show some influence of the root vortex at the large yaw angles (-51° and +54°), but hardly any at the more moderate yaw angle of +32°.

At the tip, the mean level of induction from the TUDk and ECN,de method compare reasonable well with the free wake results. The level in induction from the GH and TA methods is generally higher. Obviously the shape of the ECN,de TA and TUDk induction distribution differs at high yaw angles, since these engineering models assume pure sinusoidal induction distributions. It is interesting to note that the shape in GH induction distribution at negative yaw compares reasonable well with the free wake results. The amplitude from the TA model is in general somewhat higher than the amplitude from the free wake methods where the ECN,de gives a somewhat lower amplitude.
Figure 12.9  
*Tjæreborg, Flatwise moment at r = 2.75 m (blade root); φ_y = 32°; V = 8.5 m/s (CASE VII.1)*

Figure 12.10  
*Tjæreborg, Flatwise moment at r = 2.75 m (blade root); φ_y = 54°; V = 7.8 m/s (CASE VII.2)*
Figure 12.11  Tjæreborg, Flatwise moment at $r = 2.75 \text{ m (blade root)}$; $\phi_y = -51^\circ$; $V = 8.3 \text{ m/s (CASE VII.3)}$

Figure 12.12  Tjæreborg, Flatwise moment at $r = 2.75 \text{ m (blade root)}$; $\phi_y = -3^\circ$; $V = 8.6 \text{ m/s (CASE VII.4)}$
In the figures 12.9 to 12.12 the results of the binned averaged flatwise moments are copied from Appendix H for all 4 yaw angles. Calculated as well as measured results are shown.

The following observation can be made:

- The qualitative trend in the measured flatwise moments, which was discussed in section 12.1.2, is visible in most calculations: The 1P amplitude is largest for negative yaw, and the phase angle $\psi_1$ is between 90-180° for positive yaw and between 180-270° for negative yaw.
- In general, the calculational models which have included structural flexibility (ECN,de, GH, NTUA and TUDk) predict the measured shape better than the remaining models.
- The ECN,iw result for case VII2 show fluctuations, which are caused by numerical instabilities which occurred at this large yaw angle.
- The GH mean flatwise moments are consistently lower than the measured level which is explained by the higher induction from this model. Actually the same was found for the axisymmetric cases, but to a smaller extent. The ECN,de mean flatwise moments are consistently higher than the measured level and the other participants are sometimes higher and sometimes lower.
- It is interesting to note that the hump in measured flatwise moment around azimuth angle $\phi_{r,b} = 270°$ is also visible in the free wake methods, see figure 12.10. This may be explained by the root vorticity. However at a negative yaw angle the effect of root vorticity should be visible at azimuth angle $\phi_{r,b} = 90°$. For this situation, the free wake methods do not show a much better result. It must be noted that even if the observed effect is caused by the root vorticity, the hump is very small and therefore of less importance for practical purposes.

The yawing moments are copied from Appendix H in the figures 12.13 to 12.16. The following observation can be made:

- The calculated yawing moments are stabilizing as expected from the observations about the shift in phase $\psi_1$ of the flatwise moment. Averaged over a revolution, most calculated yawing moments agree reasonable with the measured values. The exception is in the ECN,iw model. The discrepancies obtained with this model are probably caused by the exaggeration of root vorticity effects. It is expected that the neglect of root vorticity in this model will yield much better results.
- The agreement in amplitude and phase of the 3P component in the yawing moment between the calculations mutually and between the calculations and measurements is moderate. Since a pure 1P variation of flatwise moment yields a constant yawing moment, see section 12.1.2, the 3P component of the yawing moment is caused by the higher harmonics of the flatwise moment. Therefore small changes in the flatwise moment yield large changes in the 3P component of the yawing moment. Although the calculations of the flatwise moments seem to agree reasonable well with the measurements, this is mainly true for the first harmonic, and less true for the higher harmonics. This could be expected since some participants neglect structural flexibility, tower shadow and root vorticity, which effect the higher harmonics in the flatwise moment.

The tilting moments are given in Appendix H, the figures H.7, H.14, H.21, and H.27. It is once again emphasized that these are only aerodynamic tilting moments, where the total tilting moment mainly comes from the weight of the rotor. The observations on the tilting moments are more or less the same as for the yawing moments: The $0^{th}$ and $1^{st}$ harmonic of the flatwise moment (equation
Figure 12.13  Tjæreborg, Yawing moment; \( \phi_y = 32^\circ; V = 8.5 \text{ m/s (CASE VII.1)} \)

Figure 12.14  Tjæreborg, Yawing moment; \( \phi_y = 54^\circ; V = 7.8 \text{ m/s (CASE VII.2)} \)

12.7) substituted in equation 12.2 gives a constant tilting moment of

\[
M_{\text{tilt}} = -1.5A_1 \cos \psi_1 
\]  

(12.9)

The 3P fluctuations which are notable in the tilting moment originate from the higher harmonics of the flatwise moment. Therefore these fluctuations are very sensitive to relatively small changes in the flatwise moment. Most participants predict (averaged over a revolution) a positive tilting moment. This would be expected from equation 12.9 since the phase \( \psi_1 \) in the flatwise moment is between
Figure 12.15  Tjæreborg, Yawing moment: $\phi_y = -51^\circ$; $V = 8.3$ m/s (CASE VII.3)

Figure 12.16  Tjæreborg, Yawing moment: $\phi_y = -3^\circ$; $V = 8.6$ m/s (CASE VII.4)

90° and 270° (table 12.2). The exception is in the TA results for the cases VII.1 and VII.2 (positive yaw). The explanation is that in these results the phase of the flatwise moment is slightly below 90°.

Conclusions and recommendations
This section has shown that the improved yaw models predict the shape of the binned averaged flatwise moment better than the conventional methods. The conventional methods only model the advancing and retreating blade effect, which
yields a sinusoidal flatwise moment and consequently a zero yawing moment. The newly developed engineering models predict the sign of the yawing moment correct, which is essential for yaw controlled turbines.

12.3.2 Case VII.5

Discussion of results

In Appendix H the summary data results from case VII.5 are given. The definition of this case is described in section 12.2.3. The mean value are plotted at the top of every page figure and the corresponding range is plotted below. Only a limited number of participants have supplied results, since this case was performed on a voluntary basis.

In the figures H.29 to H.34 the rotor averaged values and ranges (max-min) of the yawing moment are shown as function of (positive) yaw angle for three different wind speeds: $V_{\infty} = 7\text{m/s}$, $V_{\infty} = 9\text{m/s}$, $V_{\infty} = 12\text{m/s}$. For the yawing moment a limited comparison with measured data could be made. There the rotoraveraged values and ranges of the measured yawing moments which were used at the positive yaw angle cases VII.1 and VII.2 are determined. The averaged wind speeds at these cases was 7.8 m/s for case VII.2 and 8.5 m/s for case VII.1. The measurement point from case VII.2 is compared with the calculations at 7 m/s and the measurement point from case VII.1 is compared with the calculations at 9 m/s.

As explained in section 12.1.1 the 'real' yawing moment at the hub, which was requested for case VII.5 is $\approx 16\%$ higher than the values which are presented for the cases VII.1 and VII.2. The measured data which are used for the present case have been corrected in this way. It must be noted that the comparison with the measurements are obscured by the fact that in the measured data the effects of wind shear were present, which were ignored in the calculational results of all participants. In addition, TUDk and GH did not model the tower shadow and flexibility. Wind shear and tower shadow will in particular be relevant for the ranges. The wind shear will only have a limited influence on the mean level of the yawing moment, see section 12.1.2. Hence the mean level of the yawing moment is a more sensible quantity to compare, since it is mainly determined by yaw effects.

The calculated tilting moment and the axial force are presented in the figures H.35 to H.46. No comparison with measured data is made. The axial force is not measured, the tilting moment is mainly determined by the weight of the rotor.

Analysis of results

Generally spoken the qualitative behaviour of the results is in agreement with the expectations. For a good understanding of the trends which are visible in the figures it should be reminded that in the calculations the effect of wind shear were neglected. In these circumstances the flatwise moment (which determines tilting and yawing moment) is largely influenced by the advancing and retreating blade effect and by the skewed wake geometry effect on the axial induction distribution. For positive yaw, the advancing and retreating blade effect would give the highest loads at the blade position down ($\phi_{r,b} = 0^\circ$) and the skewed wake geometry gives the highest loads at $\phi_{r,b} = 90^\circ$.

The mean yawing moments are stabilizing and the magnitude (in an absolute sense) increases with the yaw angle. The mean yawing moments decrease from
\( V_\infty = 9 \text{m/s} \) to \( V_\infty = 12 \text{ m/s} \). This can be explained by the lower levels of axial induction factor at the higher wind speed. This leads to lower values of the wake skew angle (equation 11.6). This yields lower amplitudes in the sinusoidal induction distribution, since the factor \( K_r \) in equation 11.5 decreases. The ranges in yawing moment increase with yaw angle and wind speed. The ranges in yawing moment are determined by the higher harmonics in the flatwise moment, which will increase with yaw angle and wind speed.

When the calculated yawing moment results are compared with the (very limited number of) measured values the mean levels are in general somewhat overpredicted and the ranges are considerable underpredicted. The underprediction in ranges is as expected, due to the neglect of wind shear in the calculations. The mutual differences in mean yawing moment between the calculated results, become larger at larger yaw angle.

The mean tilting moments are negative, where positive tilting moments were found for the cases VII.1 to VII.4. These positive tilting moment were explained by the phase \( \psi_1 \) in flatwise moment, which appeared to be between 90° and 270°, section 12.3.1. However due to the neglect of wind shear in the present cases, \( \psi_1 \) is moved to a value between 0° and 90°. This will change the sign in the tilting moment. The magnitude of the mean tilting moment increases with wind speed and yaw angle, since the advancing and retreating blade effect increases with wind speed and yaw angle. Also the ranges in tilting moments increase with wind speed and yaw angle, similar to the ranges of the yawing moment.

The mutual agreement in calculated tilting moments is moderate.

The mean level of axial force decrease with yaw angle, due to the decrease in axial velocity. The ranges increase with wind speed and yaw angle. The mutual agreement in calculated mean axial force is reasonable.

**Conclusions and recommendations**

This section has shown that the mean yawing moment seems to be somewhat underpredicted, but the comparison with measured data is too limited to draw firm conclusions. The mutual agreement in calculated results of mean yawing moment, tilting moment and axial force is reasonable and sometimes moderate. For the tilting moment (where differences are largest) this has no practical meaning since in reality the mean tilting moment is determined by the weight of the blade.

The differences in ranges could partly be explained. Furthermore the ranges in yawing and tilting moment are very sensitive to the exact shape of the flatwise moment. Hence differences in this quantity can be expected to be large.
13. YAWED CASES, WIND TUNNEL

13.1 Available measurements

Four measurement series at different yaw angles have been measured on the wind tunnel model. A short description of the wind tunnel and the model is given in section 5.2.

These cases are denoted by case VI.1 to case VI.4.

13.1.1 Windtunnel conditions, load measurements and flow field measurements

The conditions of the four available measurement series are summarized in table 13.1.

Table 13.1 Conditions of the measurements

<table>
<thead>
<tr>
<th>case</th>
<th>( \phi_r ) [°]</th>
<th>( V_\infty ) [m/s]</th>
<th>( \theta ) [°]</th>
<th>( \Omega ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>VI.1</td>
<td>0</td>
<td>6.0</td>
<td>4.0</td>
<td>12</td>
</tr>
<tr>
<td>VI.2</td>
<td>10</td>
<td>6.0</td>
<td>4.0</td>
<td>12</td>
</tr>
<tr>
<td>VI.3</td>
<td>20</td>
<td>6.0</td>
<td>4.0</td>
<td>12</td>
</tr>
<tr>
<td>VI.4</td>
<td>30</td>
<td>6.0</td>
<td>4.0</td>
<td>12</td>
</tr>
</tbody>
</table>

The measurements are made at comparable conditions, the only difference is the yaw angle, which ranges from 0 to 30 degrees with an interval of 10 degrees. Measurements are available of the blade root bending moments at one of the two blades. From the blade root flatwise moment the yawing and tilting moments have been estimated in a similar way as it was done for the Tjæreborg turbine, see equation 12.1 and 12.2 (The flatwise moment from the second blade was taken equal to the value of the first blade but shifted 180 degrees in blade azimuth angle). Furthermore the horizontal flow velocities are measured in the near wake with a hot wire system, 3 cm (5% R) downstream of the rotor plane at three radial positions (50% R, 70% R and 90% R). The horizontal velocities are measured at several positions at the rotor plane. For one measurement point, the hotwire was fixed at a certain azimuthal and radial position and the blade was passing the hot wire during 10 revolutions. Therafter the hotwire was moved to another position.

Some additional information about the measurement accuracy and the instrumentation can be found in the section 9.1.1 and 9.1.3. Note that for the present measurement series the rotor shaft torque was not available, and the axial force measurements were filtered down to a very low frequency, such that they did not show variations induced by yaw.

13.1.2 Discussion of azimuthal binned averaged moments and wake velocities

In the figures 13.1 to 13.6 the following measured results are presented, for the 4 yaw angles:

- The flatwise moments at blade root;
- The yawing moments derived from the flatwise moments;
- The tilting moments derived from the flatwise moments;
The total horizontal velocity measured with the hotwire probe, just downstream of the rotor plane at three radial positions.

It is reminded that the measurement device of the horizontal velocities did not follow the blade, but was fixed at a certain azimuthal position while the blade passed.

Figure 13.1 Windunnel, Flatwise moment at different yaw angles: $\phi_y = 30^\circ$; $V = 6.0$ mis (CASE VI)

Figure 13.2 Windunnel, Yawing moment at different yaw angles: $\phi_y = 30^\circ$; $V = 6.0$ mis (CASE VI)
Joint investigation of dynamic inflow effects and implementation of an engineering method

Figure 13.3  Windtunnel, Tiltling moment at different yaw angles; $\phi_Y = 30^\circ$; $V = 6.0$ mts
(CASE VI)

First of all it can be observed that the flatwise moment for a yaw angle of zero degrees is not fully constant, as would be expected, from a uniform and stationary incoming flow field. Hence the fluctuations which can be observed are a consequence of non-uniformity and turbulence.

In the table below the Fourier decomposition of the flatwise moment is given for the four yaw angles (notations according to equation 12.4).

Table 13.2  Fourier decomposition of flatwise moment for yowed windtunnel cases

<table>
<thead>
<tr>
<th>$\phi_Y$ (case)</th>
<th>$\lambda_0$</th>
<th>$\lambda_1$</th>
<th>$\psi_1$</th>
</tr>
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<tbody>
<tr>
<td>°</td>
<td>Nm</td>
<td>Nm</td>
<td>°</td>
</tr>
<tr>
<td>0(VI.1)</td>
<td>1.48</td>
<td>0.07</td>
<td>141</td>
</tr>
<tr>
<td>10(VI.2)</td>
<td>1.47</td>
<td>0.24</td>
<td>131</td>
</tr>
<tr>
<td>20(VI.3)</td>
<td>1.30</td>
<td>0.26</td>
<td>114</td>
</tr>
<tr>
<td>30(VI.4)</td>
<td>0.95</td>
<td>0.34</td>
<td>120</td>
</tr>
</tbody>
</table>

Fig. 13.1 and table 13.2 show that, the phase of the binned averaged flatwise moment decreases from $141^\circ$ at zero yaw towards $90^\circ$, when the yaw angle is increased (although there is again a small increase when the yaw angle changes from $20^\circ$ to $30^\circ$). This is consistent with the sinusoidal induction distribution from equation 11.5 and with the advancing and retreating blade effect. The sinusoidal induction distribution would yield a phase angle of $90^\circ$ and the advancing and retreating blade angle would yield a phase angle of $0^\circ$.

The IP amplitude depends on the magnitude of the yaw angle, such that the amplitude increases with increasing yaw angle. Since there will be no wind shear in the wind tunnel, the relation between amplitude and yaw angle is more straightforward than for the Tjereborg cases, where the advancing and retreating blade effect could counteract the wind shear and decrease the amplitude.
The yawing and tilting moment which is derived from the 0th and 1st harmonics of the flatwise moment is for a 2-bladed turbine different than for a 3-bladed turbine (equation 12.8 and 12.9):

\[ M_{yaw}(\phi, b) = -A_1 \sin(2\phi, b) \cos \psi_1 - 2A_1 \sin^2(\phi, b) \sin \psi_1 \]  
\[ M_{tilt}(\phi, b) = -A_1 \sin(2\phi, b) \sin \psi_1 - 2A_1 \cos^2(\phi, b) \cos \psi_1 \]  

(13.1)  

(13.2)

Hence the first harmonic of the flatwise moment yields a variation of the yawing and tilting moment over the rotorplane, contrary to the situation at a three bladed turbine. The rotor averaged yawing moment and tilting moments are

\[ \bar{M}_{yaw} = -A_1 \sin(\psi_1) \]  
\[ \bar{M}_{tilt} = -A_1 \cos(\psi_1) \]  

(13.3)  

(13.4)

respectively.

With \( \psi_1 \) from table 13.2, equation 13.3 yields a stabilizing negative yawing moment for all cases, in agreement with the results from figure 13.2.

It might be doubted whether a negative yaw angle also would give a stabilizing yawing moment. For a stabilizing yawing moment, the value of \( \psi_1 \) should be between 180-360 degrees, where the value for a zero yaw angle is still far below 180 degrees (141 degrees). For a negative yaw angle the advancing and retreating blade effect and the sinusoidal axial induction distribution will shift the phase towards 180 degrees resp. 270 degrees, but this will be a continuous process and there will be no jump in phase when the yaw angle goes from positive to negative. It is reminded that for the Tjæreborg cases, the phase angle at zero yaw was much closer to 180 degrees, due to the wind shear.

Note that the discontinuity which can be observed in some of the yawing and tilting moment is caused by a relatively small discontinuity between the flatwise moment at a blade azimuth angle slightly below 360° and slightly above 0°.

The figures 13.4 to 13.6, show the measurements of the total horizontal velocities, just downstream of the rotorplane for a yaw angle of 30°. The trend which is seen in these measurements is at first sight peculiar: From the considerations given above, the highest velocities in the rotorplane would be expected in the upstream part of the disc, i.e. between 0 and 180 degrees. The measurements indicate an opposite trend. It must be noted however, that it is not necessarily the total horizontal velocity which is maximum in the upstream part of the disc, but the velocity axial to the rotorplane. In section 13.3.1, it will be shown that both free wake methods, which take into account all yaw effects can reproduce the trend in the measurements better than the simplified engineering models.
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Figure 13.4 Windtunnel, Measured horizontal velocities at 50% span; $V = 6.0$ m/s (CASE $V_I$)

Figure 13.5 Windtunnel, Measured horizontal velocities at 70% span; $V = 6.0$ m/s (CASE $V_I$)
13.2 Definition of calculation cases

13.2.1 General

The azimuthal binned averaged measurements which are described in section 13.1.2 have been reproduced by the calculational models which are reported in section 11.

The description of the wind tunnel model is given in Appendix 8.

13.2.2 Definition of case VI.1 to VI.4 (azimuthal binned data under yawed conditions)

In the simulation of the case VI.1 to VI.4 axisymmetric conditions have been assumed, with the exception of the yaw angle. Hence no wind shear and tower shadow are assumed. The yaw angle is 0 degrees for case VI.1, 10 degrees for case VI.2, 20 degrees for case VI.3 and 30 degrees for case VI.4. The rotor speed is 12 Hz, the incoming wind speed $V_\infty = 6$ m/s and the tip angle is 4 degrees.

Table 13.3 Required quantities for the wind tunnel case, case VI

<table>
<thead>
<tr>
<th>quantity</th>
<th>position</th>
<th>as function of</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{hor}, IP}$</td>
<td>50%R, 70%R, 90%R</td>
<td>$\phi_r$</td>
</tr>
<tr>
<td>$u_{1P}$</td>
<td>50%R, 70%R, 90%R</td>
<td>$\phi_r$</td>
</tr>
<tr>
<td>$u_i$</td>
<td>50%R, 70%R, 90%R</td>
<td>$\phi_{r,b}$</td>
</tr>
<tr>
<td>$M_{\text{rel}}$</td>
<td>$r = 0.128m$</td>
<td>$\phi_{r,b}$</td>
</tr>
</tbody>
</table>

- $V_{\text{hor}}$ is the total horizontal velocity at the position of the hot-wire. This position is fixed in the rotor plane and the blade passes the hot wire. 1P indicates that the velocity is averaged over 1 revolution. $V_{\text{hor}}$ can be compared with
measurements. The velocities are calculated at an axial distance of x/R=0.05 behind the rotor plane. \( V_{\text{hot}} \) can be calculated as

\[
V_{\text{hot}} = (V_{ax})^2 + (V_{tan} \cdot \cos(\phi_c))^2 + (V_{rad} \cdot \sin(\phi_c))^2
\]  

(13.5)

where \( \phi_c \) is the position of the hotwire.

- \( u_{11P} \) gives the axial induction in the rotorplane averaged over a revolution, i.e., the induction is calculated at a fixed position and the blade passes this position.
- \( u_{11} \), without extension 1P gives the 'standard' axial induced velocity following the blade.
- The flatwise moment \( M_{\text{fl}} \) has been compared with measured data. From the flatwise moment, the yawing and tilting moments have been derived.

13.3 Results

13.3.1 Case VI.1 to VI.4

Discussion of results

In Appendix I, the results of case VI are presented. Note that TA (which officially does not participate in the project) did not reproduce this case.

The yawing and tilting moments for the zero yaw case are not presented, since they are zero (calculated) or almost zero (measurements).

The total horizontal velocity which was measured just downstream of the rotor plane has only been reproduced with the free wake methods from NTUA and Unist. The engineering methods are less suited to predict this quantity since they are based on yaw models which consider the velocities axial to the rotorplane. For the zero yaw case (case VI.1), it is assumed that the inplane component can be neglected and the measured axial induced velocity which is added to the figures is taken equal to the free stream velocity - measured velocity.

In the following subsection, the results are analyzed. First the axial induced velocity following the blade (as function of \( \phi_{c,h} \)) is discussed. Then some comments are given on the 1P averaged axial induction in the rotorplane as function of \( \phi_c \). Not all models could predict this quantity, since some models only yield results at the blade. This is followed by the discussion of the total horizontal velocities in the rotorplane. Thereafter the flatwise moments and the yawing and tilting moments are commented.

Analysis of results

The results of the axial induced velocities at the blade are shown in Appendix I, the figures 1.4 to 1.6 for zero yaw (case VI.1), the figures 1.14 to 1.16 for 10 degrees yaw (case VI.2), the figures 1.26 to 1.28 for 20 degrees yaw (case VI.3), and the figures Appendix I to 1.40 for 30 degrees yaw (case VI.4). One more or less representative result for 70% span at 30 degrees yaw has been copied to figure 13.7.

The same comments can be given as for the Tjärneborg cases, see section 12.3.1:

- The engineering methods show an almost pure sinusoidal shape.
- The results from TUDk, ECN,de, and GH are close together at small yaw angles, but discrepancy with GH results increases with yaw angle, and the ECN,de result is having a slightly smaller amplitude than TUDk.
- The results from Unist and NTUA are close together.
The root vorticity effect, (the deviation from the sinusoidal shape around 270°) has a similar trend for the DUT and ECN,iw model, but it differs from the trend in the free wake approaches from NTUA and Unist. The free wake approaches will be more realistic.

The 1P averaged axial induction in the rotorplane at $\phi_{r}$, (not at blade position $\phi_{r,b}$) is given in Appendix I, in the figures I.1 to I.3 for zero yaw (caseVI.1), the figures I.11 to I.13 for 10 degrees yaw (caseVI.2), the figures I.23 to I.25 for 20 degrees yaw (caseVI.3), and the figures I.35 to I.37 for 30 degrees yaw (caseVI.4). One more or less representative result for 70% span at 30 degrees yaw has been copied to figure 13.8.

It must be noted that the differences between the blade induced velocity and the 1P averaged induced velocities as presently described can be attributed to the non-uniformity of the induction in the rotor plane. Hence it is a measure for the tip correction losses. It can be observed that the differences between the two types of results is small at the 50%R position, but at the tip position (90%R) most participants find appreciable differences.

Most of the observations which have been made for the blade induced velocities remain valid for this type of induced velocity.

The results of the total horizontal velocities are given in Appendix I, the figures I.7 to I.9 for zero yaw (caseVI.1), the figures I.17 to I.19 for 10 degrees yaw (caseVI.2), the figures I.29 to I.31 for 20 degrees yaw (caseVI.3), and the figures I.41 to I.43 for 30 degrees yaw (caseVI.4). The result at 30 degrees yaw and 70% span is copied to figure 13.9. In this figure the trend which would be expected from an engineering model, with a sinusoidal axial induction distribution is added also. In the engineering model result it is assumed that the total horizontal velocity is the difference between the free stream velocity and the axial induction. The discrepancies between the engineering model and the measured result is evident.
Despite some quantitative differences between the free wake model results and the measurements, it is remarkable to see that the trend in the free wake calculations is predicted much better than the expected result from the engineering models (this is in particular true for the NTUA result, but it is often found in the Unist. results as well): The minimum velocity is not reached at an azimuth angle of 270° as would yield from an engineering model, but somewhere between 90 and 180° for 50 and 70% span and at 180° at 90% span. The maximum is somewhere at 320° for 50 and 70% span and at 90° at 90% span, where an engineering model would yield the maximum around 90° at every radial position. The explanation was found to be the presence of the tangential horizontal velocity which is induced by the skewed wake. This component is automatically taken into account in the free wake methods, but not in the engineering methods.

The results of the flatwise moments are presented in the figures I.10, I.20, I.32 and I.44 from Appendix I.

In figure 13.10, the result at a yaw angle of 30° has been copied. The following observations can be made:

- All predicted flatwise moments are on a higher level than measured. This is not expected from the results of section 9.3.1, see also figure G.3, where the calculated axial force coefficients are compared with the measured values. Then the calculated axial force is underpredicted. The explanation is the presence of centrifugal stiffening effects, which were not taken into account in the calculations.

- The higher harmonics in the flatwise moment are underpredicted. This can be explained by the non-uniformity in the flow field and the tower shadow in combination with structural dynamic effects, which were ignored in the calculational results. This is confirmed by the fact that even in the aligned flow situation, there are considerable fluctuations in the measured flatwise moment, see figure 13.1. The axisymmetric flow conditions which are assumed in the
Calculations, do not give any fluctuation at all.

- Generally spoken the first harmonic of the flatwise moment is predicted reasonably well. The influence of increasing yaw angle on the amplitude and phase of the first harmonic in the flatwise moment, see section 13.1.2, is also found in the calculations. In yaw, the phase angle is close to 90° and the amplitude increases with yaw angle.

The results of the yawing moments are presented in the figures 1.21, 1.33 and 1.45 and the results of the tilting moments are presented in the figures 1.22, 1.34 and 1.46. In the figures 13.11, and 13.12 the results at a yaw angle of 30° have been copied.

The calculational yawing moments are of the $-\sin^2 \phi_{r,b}$ type and the tilting moments are of the $-\sin 2 \phi_{r,b}$ type. This is consistent with the equations 13.1 and 13.2, in case the phase angle of the flatwise moment, $\psi_v = 90^\circ$. The discrepancies with the measured yawing and tilting moments can be explained by the higher harmonics in the flatwise moment, due to tower and flow non-uniformity of the flow.

Conclusions and recommendations
This section has shown that the simplified engineering methods, which model yawed conditions by means of the Glauert sinusoidal axial induction distribution and the advancing and retreating blade effect, do not predict the details of the flowfield, i.e. the total horizontal velocity, just downstream of the rotorplane, well. The reason is that the tangential, horizontal velocity, which is induced by the skewed wake is not implemented. It is encouraging that the free wake methods which take into account all yaw effects predict this flow field much better. Despite of this, the results of blade and hub loads calculated by the engineering methods, show the same, satisfactorily, agreement with the measurements as the free wake methods.
Figure 13.10  Windtunnel, Flatwise moment; $\phi_y = 30^\circ$; $V = 6.0$ m/s (CASE VI.4)

Figure 13.11  Windtunnel, Yawing moment; $\phi_y = 30^\circ$; $V = 6.0$ m/s (CASE VI.4)
Figure 13.12 Wind tunnel, Tilting moment: $\phi_r = 30^\circ$; $V = 6.0 \text{ m/s}$ (CASE VI.A)
14. IMPORTANCE OF YAW MODELLING FOR PRACTICAL PURPOSES

The increases in amplitude of the flatwise moment on the Tjæreborg which are found due to the yaw misalignment appeared to be rather limited (in the order of 50 kNm). This is probably not very relevant for design purposes since a moderate turbulence level is expected to yield similar load fluctuations. Also the ranges in yawing and tilting moments, due to yaw are not very large. Hence the importance of the improved yaw modelling for fatigue consideration is limited.

However it is important to note that the conventional yaw models do not predict the correct phase of the flatwise moment and consequently the sign of the yawing moment. It is obvious that the sign of the yawing moment should be predicted well when the yaw stability of a turbine is considered. Therefore the improved yaw models are essential for the design of yawed controlled turbine.
15. CONCLUSIONS AND RECOMMENDATIONS

Conclusions
The most important conclusions which can be drawn from the project are:

- A clear dynamic inflow effect is present in the pitching step measurements which have been taken on the Tjæreborg 2MW turbine. The associated time scale which was found is in the order of 0.3 to 0.5 times the ratio of the rotor diameter and the free stream wind speed.

- The dynamic inflow effects can be relevant for fast pitching large machines, since the importance increases with pitching speed and size of the turbine.

- The measurements at coherent wind gusts in the Delft open jet tunnel do not give significant dynamic inflow effects. However, the time for the tunnel mechanism to build up the gust was large compared to the time scale in the dynamic inflow process. Calculations of coherent wind gusts, with a smaller time scale, also showed that the dynamic inflow effects were negligible.

- For pitching transients and wind gust steps, the engineering methods developed within the project, predict the most important dynamic inflow effects well, contrary to conventional methods. This was validated by comparing the calculations with measurements and with calculations from free wake methods.

- In the measurements under yawed conditions of blade root rowwise moments on the Tjæreborg turbine and on the wind tunnel model, the influence of the skewed wake on the axial induction distribution, and the advancing and retreating blade effect, was notable.

- The wind tunnel measurements under yawed conditions showed the presence of an inplane, horizontal, velocity component induced by the skewed wake. This component was taken into account by the free wake methods but not by the engineering methods. Therefore the details of the flow field are predicted better with the free wake methods.

- Despite of the discrepancies in flow field, the engineering methods predict the loads on the turbine in yaw with a reasonable degree of accuracy, and much better than the conventional methods. This is of particular importance for yaw controlled turbines. The advantage of the engineering methods is obviously the limited computational effort compared to the free wake methods.

- Although the project was focused on dynamic effects, the results indicated that in most cases the engineering models predict the same equilibrium levels of induction. Also the levels of induction from the free wake models are close together, however this level often differed from the engineering methods. The comparison with wind tunnel measurements under axisymmetric conditions revealed that the free wake methods predict the level of induction very reasonable. This indicates that the calculation of induction from the engineering methods should be improved.

Recommendations
Although considerable progress has been made in the development and the validation of the engineering models, it is inevitable that certain items could not be addressed. Some of them will be considered in the JOULE 2 project 'Dynamic Inflow, yawed conditions and partial span pitch control'. In particular partial span pitch control and the dependency of the dynamic inflow process on the turbine size will be treated in that project, together with a further validation of the yaw
models. Other topics which should be addressed in future investigations are:

- Improvement of tip loss factors in yawed conditions;
- Improvement of the prediction of mean induction in the engineering methods.
- Validation and eventually improvement of models in turbulent wake situations.
  In this situation, the flow in the wake may reverse and appreciable viscous effects may occur.
- Validation and eventually improvement of models under extreme yaw angles (> 60°).
- Validation and eventually improvement of models under dynamic yaw conditions.
REFERENCES


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APPENDIX A. CONVENTIONS, REFERENCE SYSTEMS AND NOTATIONS

A.1 (Blade) azimuth angle ($\phi_{r,b}$ and $\phi_r$)

See figure A.1.

![Blade azimuth angle and blade numbering](image)

Figure A.1 (Blade) azimuth angle and blade numbering

- **Blade azimuth angle**: $\phi_{r,b}$ = angle of blade number 1; **Blade numbering**, see section A.2.
- **Azimuth angle**: $\phi_r$ = angle in the rotorplane, not necessarily the position of the blade.

The zero azimuth is in vertical downward direction. The azimuth angle is positive in clockwise direction looking downstream to the rotor.

A.2 Blade numbering (1, 2 and 3)

See figure A.1. The order in which the blades pass the tower is for a three bladed turbine: 1,2,3

A.3 Turbine angle ($\phi_{\text{turb}}$)

The turbine angle gives the angle of the nacelle w.r.t. the true (not magnetic) North. When the turbine is oriented to the East (this means that the upstream direction is to the East) the turbine angle = 90°. See figure A.2.
A.4 Ambient wind conditions

The ambient wind conditions are described by means of the wind speeds, wind shears and wind directions.

- $V_{\infty}$: Ambient wind speed;
- $V_{hub}$: Ambient wind speed at hub height;
- $V_{up}$: Ambient wind speed at $h = \text{hub height} + R$;
- $V_{low}$: Ambient wind speed at $h = \text{hub height} - R$;
- $\delta V_{up}$: Relative vertical wind shear in upper rotor plane $= \frac{V_{up} - V_{hub}}{V_{hub}}$;
- $\delta V_{low}$: Relative vertical wind shear in upper rotor plane $= \frac{V_{low} - V_{hub}}{V_{hub}}$;
- $\alpha$: Wind shear coefficient: $V(h) = V_{hub}(h/h_{hub})^{\alpha}$;
- $\phi_{w,\text{hub}}$: Wind direction at hub height;
- $\phi_{w,up}$: Wind direction at $h = \text{hub height} + R$;
- $\phi_{w,low}$: Wind direction at $h = \text{hub height} - R$.

The wind direction is given w.r.t. true North. When the wind comes from the East, the wind direction = 90°, see figure A.2.

A.5 Yaw angle ($\phi_y$)

The yaw angle is given as the difference of the wind direction and the position of the nacelle, see figure A.2

$\phi_y = \phi_w - \phi_{\text{turb}}$

A.6 Wake skew angle ($\chi$)

The wake skew angle gives the angle between the wake centre line and the rotor axis direction, positive according to figure A.3.
A.7 Pitch angle ($\delta$) and twist ($\epsilon$)

See figure A.4, in which:
- $\beta = \text{local blade angle, angle between the chordline of the blade element and the rotorplane, positive when it points in the opposite wind direction.}$
  - full span pitch: $\beta = \theta + \epsilon$
  - partial span pitch:
    * inner part of the blade: $\beta = \epsilon$
    * tip: $\beta = \theta_t + \epsilon$

- $\theta = \text{pitch angle; } \theta_t = \text{tip angle.}$
  By definition: $\theta_t = 0^\circ$, if chordline of tip-station stands in the rotor plane, see figure A.5; Positive when it points in the opposite wind direction, and decreases the angle of attack.
- $\epsilon = \text{twist angle of the blade relative to tip station}$

A.8 Angle of attack and inflow conditions

The definition of the angle of attack and the inflow conditions, which are used in the present project is in agreement with the common definition according to blade element theory. The inflow angle ($\phi$) gives the angle between the effective (resultant) wind velocity at the blade element and the rotorplane. From velocity diagram, see figure A.6:

$$\tan \phi = \frac{V_{\infty} - u_i}{\omega \cdot r + V_r} = \frac{(1 - a) \cdot V_{\infty}}{(1 + a') V_r}$$  \hspace{1cm} (A.1)

(Under the assumption of zero yaw.)
in which:
- $V_r = \Omega \cdot r$
- $\Omega =$ rotational speed $= d\phi_r / dt$;
- $r =$ radial position;
- $u_i =$ induced velocity, axial to the rotor plane (positive when it is in the opposite direction of the undisturbed wind speed)
- $\omega =$ rotational induced velocity (positive when it is in the opposite direction of the rotational speed)
- $a =$ axial induction factor $= \frac{\Omega}{U}$
- $a' =$ tangential induction factor $= \frac{\omega}{\Omega}$

Then the effective incoming wind speed ($W$) and dynamic pressure ($q$) are defined as:

$$W = \sqrt{(1 - a)^2 \cdot V_{\infty}^2 + (1 + a')^2 V_s^2}$$
$$q = 0.5 \cdot \rho W^2$$

In addition, a dynamic pressure related to the rotational velocity ($V_r$) and the helical velocity ($V_h$) are introduced:

$$q_r = 0.5 \cdot \rho V_r^2$$
$$q_h = 0.5 \cdot \rho V_h^2$$

$$V_h = \sqrt{V_{\infty}^2 + V_r^2}$$

Note that the induced velocities $u_i$ is always (also in yawed flow conditions) defined perpendicular to the rotorplane.
Figure A.5  Pitch angle at tip station

The angle of attack ($\alpha$) is the angle between the effective wind speed and the chordline:

$$\alpha = \phi - \beta$$  \hspace{1cm} (A.2)

A.9 Aerodynamic forces and moments

The aerodynamic forces can be either actual forces or forces per unit length. Lowercase symbols ($n, t, l, d$ and $m$) are used to indicate forces and moments per unit length whereas the corresponding uppercase symbols ($N, T, L, D$ and $M$) represent actual forces/moments expressed in [N]/[Nm], see also figure A.6.
- The aerodynamic normal force per unit length $n$ [N/m] is perpendicular to the chord of the aerofoil section,
- The aerodynamic tangential force per unit length $t$ [N/m] is along the chord of the aerofoil section,
- The aerodynamic lift force per unit length $l$ [N/m] is perpendicular to the effective wind speed $W$; $l = n \cos \alpha - t \sin \alpha$,
- The aerodynamic drag force per unit length $d$ [N/m] is along the effective wind speed vector $W$ of the aerofoil section; $d = n \sin \alpha + t \cos \alpha$,
- The aerodynamic moment per unit length of the aerofoil section is $m$ [N]. The moment axis is located at the quarter chord line. Positive is nose-up.

A.10 Aerodynamic coefficients

In this chapter, the aerodynamic forces are made non-dimensional with the dynamic pressure $q$ and the chord length $c$. With the exception of the pressure coefficient, all section coefficients use the lowercase symbol $c$.
- $C_p = \text{the pressure coefficient} = (p - p_{\infty}) / q$
- $c_n = \text{the section normal force coefficient} = n / q;$
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Figure A.6  Velocity diagram, local aerodynamic forces and rotating coordinate system

- \( c_t \) = the section tangential force coefficient = \( t/q \);
- \( c_l \) = the section lift coefficient = \( l/q \);
- \( c_d \) = the section drag coefficient = \( d/q \);
- \( c_m \) = the section moment coefficient = \( m/(q \cdot c) \)

A.11 Unsteady conditions

For the determination of the time scale in the dynamic inflow project, an exponential behaviour for the function \( F \) has been assumed according to:

\[
F(t) = F_1 + \Delta F \cdot \left(1 - \exp^{-\alpha(t-t_1)/\gamma^0}\right)
\]

(A.3)

(see figure A.7). Then the time scale can be derived from:

\[
f(t) = \frac{t - t_1}{\ln((F_2 - F)/\Delta F)}
\]

(A.4)

Reduced frequencies and non-dimensional times can be defined. In the literature it is found that both the chord and the half chord is used for non-dimensionalisation. It is recommended to use the half chord:

- The reduced frequency \( (k) \) is given by: \( k = \omega \cdot c/(2 \cdot W) \)
  with:
  \( \omega \) = circular frequency of oscillation and \( W \) explained in section A.8.
- The non-dimensional effective pitch rate \( (\dot{\theta}^+) \) is given by: \( \dot{\theta}^+ = \dot{\theta} \cdot c/(2 \cdot W) \)
  with:
  \( \dot{\theta} \) = the pitch rate = \( d\theta/dt \).
- The non-dimensional angle of attack rate is given by: \( \alpha^+ = \dot{\alpha} \cdot c/(2 \cdot W) \)
  with:
  \( \dot{\alpha} \) = the angle of attack rate = \( d\alpha/dt \).
Figure A.7 Time scale determinations

- The non-dimensional time delay is given by: $\tau = \Delta t \cdot W/(2 \cdot c)$
- The non-dimensional time is having the same symbol:
  $\tau = t \cdot W/(2 \cdot c)$
- If some other quantity is introduced it was again recommended to use the half chord for the non-dimensionalisation. A non-dimensional length would thus be: $2 \cdot t \cdot W/c$

A.12 Reference systems

Three reference systems are introduced, see figure A.6 and figure A.8 to A.10.
- A fixed, non-rotating axis system $(xyz)_N$, related to the nacelle and with origin at the top tower centre;
- A fixed, non-rotating axis system $(xyz)_H$ with origin at the rotor hub centre;
- A rotating axis system $(xyz)_{rot}$ with origin at the leading edge of blade section $i$.

The tilt angle $(\alpha_t)$ is defined as the angle between the rotor shaft and the horizontal.
The cone angle $(\alpha_c)$ is defined as the angle between the blades and the rotor plane.

A.12.1 Fixed $(xyz)_H$ coordinate system

- $x_N, y_N, z_N$: right handed coordinate system, origin in top tower centre;
- $y_N$-axis: along the tower centre line, positive in vertical upward direction,
- $x_N$-axis: along the rotor shaft (assuming zero tilt angle);
- $z_N$-axis: to the right, looking downstream to the rotor.

Nacelle loads:
- $x_N$: $F_{ax,N}$ = axial force on nacelle, positive in downstream direction.
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- \(y_N\): \(F_{\text{vert}}\) = vertical force on nacelle, positive in upward direction
- \(z_N\): \(F_{\text{lat}}\) = lateral force on nacelle, positive when the force acts to the right, standing in front of the rotor.
- \(x_N\): \(M_{\text{torq},N}\) = Moment around \(x_N\)-axis
- \(y_N\): \(M_{\text{yaw}}\) = yaw moment, positive when it is a destabilizing yaw moment i.e. the yaw misalignment is increased by a positive yaw moment.
- \(-z_N\): \(M_{\text{tilt}}\) = tilt moment, positive when it increases the tilt angle

A.12.2 Fixed \((x y z)_H\) coordinate system

- \(x_H\), \(y_H\), \(z_H\): right handed coordinate system, origin in rotor centre;
- \(x_H\)-axis: along the rotor shaft, positive in downstream direction;
- \(y_H\)-axis: along the vertical upward direction (assuming zero tilt angle);
- \(z_H\)-axis: to the right, looking downstream to the rotor.

Hub loads:
- \(x_H\): \(F_{\text{ax}}\) = axial force along rotor shaft, positive in downstream direction
- \(y_H\): \(F_{\text{vert},H}\) = 'vertical' (assuming zero tilt angle) force on hub, positive in upward direction
- \(z_H\): \(F_{\text{lat},H}\) = lateral force on hub, positive when the force acts to the right, standing in front of the rotor;
- \(x_H\): \(M_{\text{torq},H}\) = Rotor shaft torque;
- \(y_H\): \(M_{\text{yaw},H}\) = 'yaw' moment (assuming zero tilt angle);
- \(-z_H\): \(M_{\text{tilt},H}\) = 'tilt' moment (assuming zero tilt angle), positive when it increases the tilt angle.

A.12.3 Rotating \((x y z)_{\text{rot}}\) coordinate system

- \(x_{\text{rot}}, y_{\text{rot}}, z_{\text{rot}}\): right handed coordinate system, origin at the leading edge of blade section \(i\).
- \(x_{\text{rot}}\)-axis: along chord line, positive from leading edge \((x=0)\) to trailing edge \((x=c)\);
- \(z_{\text{rot}}\)-axis: perpendicular to chord line; positive in upwind direction;
- \(y_{\text{rot}}\)-axis: along rotor blade axis;

Blade loads:
- \(z_{\text{rot}}\): \(F_{\text{flat}}\) = flat shear force, positive in downwind direction
- \(x_{\text{rot}}\): \(F_{\text{edge}}\) = edge shear force, positive in rotational direction
- \(z_{\text{rot}}\): \(M_{\text{edge}}\) = edge moment, positive in rotational direction
- \(x_{\text{rot}}\): \(M_{\text{flat}}\) = flat moment, positive in downwind direction.
Figure A.8  *Coordinate systems (φ_{r,b} = 270°)*

Figure A.9  *Coordinate systems in side view (φ_{r,b} = 180°)*
Figure A.10  Coordinate systems in front view
A.13 Abbreviations, symbols and units

Abbreviations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>HAWT</td>
<td>Horizontal Axis Wind Turbine</td>
</tr>
<tr>
<td>2D</td>
<td>Two-dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>Three-dimensional</td>
</tr>
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</table>

Symbols and units

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>[m²] blade cross sectional area</td>
</tr>
<tr>
<td>AR</td>
<td>[-] aspect ratio; for a HAWT blade taken as $S_o/(ar{c}^2)$</td>
</tr>
<tr>
<td>a</td>
<td>[-] axial induction factor, see section A.8</td>
</tr>
<tr>
<td>a'</td>
<td>[-] tangential induction factor, see section A.8</td>
</tr>
<tr>
<td>B</td>
<td>[-] number of blades</td>
</tr>
<tr>
<td>$C_{D,ax}$</td>
<td>[-] axial force coefficient: $C_{D,ax} = \frac{F_{ax}}{0.5 \rho V^2 R^2}$</td>
</tr>
<tr>
<td>$C_{l, \text{max}}$</td>
<td>[-] maximum lift coefficient</td>
</tr>
<tr>
<td>$C_p$</td>
<td>[-] pressure coefficient</td>
</tr>
<tr>
<td>$C_p$</td>
<td>[-] power coefficient, $C_p = \frac{P}{0.5 \rho V^2 R^5}$</td>
</tr>
<tr>
<td>c</td>
<td>[m] chord length</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>[m] mean aerofoil chord of the blade</td>
</tr>
<tr>
<td>$c_d$</td>
<td>[-] section drag coefficient, see section A.10</td>
</tr>
<tr>
<td>$c_{d,ax}$</td>
<td>[-] local axial force coefficient, $c_{d,ax} = \frac{F_{ax}}{0.5 \rho V^2 R^2}$</td>
</tr>
<tr>
<td>$c_c$</td>
<td>[-] section lift coefficient of profile, see section A.10</td>
</tr>
<tr>
<td>$c_l$</td>
<td>[-] section lift coefficient, see section A.10</td>
</tr>
<tr>
<td>$c_{l, \text{max}}$</td>
<td>[-] section maximum lift coefficient</td>
</tr>
<tr>
<td>$c_{\alpha}$</td>
<td>[rad⁻¹] aerodynamic section moment coefficient, with reference to 25% chord, positive when nose up, see section A.10</td>
</tr>
<tr>
<td>$c_n$</td>
<td>[-] section normal force coefficient, see section A.10</td>
</tr>
<tr>
<td>D</td>
<td>[m] or [kN] rotor diameter or drag, see section A.9</td>
</tr>
<tr>
<td>d</td>
<td>[N/m] section drag, see section A.9</td>
</tr>
<tr>
<td>$d_{ax}$</td>
<td>[m] rotor overhang</td>
</tr>
<tr>
<td>E</td>
<td>[N/m²] Young’s modulus</td>
</tr>
<tr>
<td>$F_{\text{flat}}$</td>
<td>[kN] flat shear force, see section A.12.3</td>
</tr>
<tr>
<td>$F_{\text{edge}}$</td>
<td>[kN] edge shear force, see section A.12.3</td>
</tr>
<tr>
<td>$F_{ax}$</td>
<td>[kN] axial force, see section A.12.1</td>
</tr>
<tr>
<td>$F_{\text{lat}}$</td>
<td>[kN] lateral force, see section A.12.1</td>
</tr>
<tr>
<td>$f_{ax}$</td>
<td>[N/m] local axial force</td>
</tr>
<tr>
<td>$f$</td>
<td>[s] time scale, see section A.11</td>
</tr>
<tr>
<td>$f_2$</td>
<td>[-] radial distribution term of induced velocity</td>
</tr>
<tr>
<td>g</td>
<td>[m/s²] gravitational acceleration</td>
</tr>
<tr>
<td>h</td>
<td>[m] height</td>
</tr>
<tr>
<td>I</td>
<td>[-] or [m⁴] turbulence intensity or moment of inertia</td>
</tr>
<tr>
<td>$K_c$</td>
<td>[-] coefficient in Glauert’s distribution of induced velocity</td>
</tr>
<tr>
<td>k</td>
<td>[-] reduced frequency, see section A.11</td>
</tr>
<tr>
<td>L</td>
<td>[N] lift, see section A.9</td>
</tr>
<tr>
<td>$l_{\text{lift}}$</td>
<td>[N/m] section lift force perpendicular to the effective wind speed, see section A.9</td>
</tr>
<tr>
<td>M</td>
<td>[-] Mach number</td>
</tr>
<tr>
<td>$M_{\text{edge}}$</td>
<td>[kNm] edgewise moment, see section A.12.3</td>
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<tr>
<td>$M_{\text{flat}}$</td>
<td>[kNm] flapping moment, see section A.12.3</td>
</tr>
<tr>
<td>$M_{\text{tilt}}$</td>
<td>[kNm] tilting moment, see section A.12.1</td>
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<td>Symbol</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>$M_{\text{torq}}$</td>
<td>[kNm]</td>
</tr>
<tr>
<td>$M_{\text{yaw}}$</td>
<td>[kNm]</td>
</tr>
<tr>
<td>$m$</td>
<td>[N]</td>
</tr>
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<td>$n$</td>
<td>[N/m]</td>
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<td>[Hz] or [W]</td>
</tr>
<tr>
<td>$p$</td>
<td>[Pa]</td>
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<tr>
<td>$q$</td>
<td>[Pa]</td>
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<tr>
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<td>[m]</td>
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<tr>
<td>$s$</td>
<td>[m]</td>
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<tr>
<td>$S$</td>
<td>[m²]</td>
</tr>
<tr>
<td>$S_b$</td>
<td>[m²]</td>
</tr>
<tr>
<td>$r$</td>
<td>[m]</td>
</tr>
<tr>
<td>$T$</td>
<td>[N]</td>
</tr>
<tr>
<td>$t$</td>
<td>[s] or [m] or [N/m]</td>
</tr>
<tr>
<td>$u$</td>
<td>[-]</td>
</tr>
<tr>
<td>$u_i$</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$V$</td>
<td>[m/s]</td>
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<td>$v_i$</td>
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<tr>
<td>$W$</td>
<td>[m/s]</td>
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<td>$\alpha$</td>
<td>[$^\circ$] or [-]</td>
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<tr>
<td>$\alpha_x$</td>
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<tr>
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<tr>
<td>$\dot{\alpha}$</td>
<td>[rad/s or $^\circ$/s]</td>
</tr>
<tr>
<td>$\alpha^*$</td>
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<tr>
<td>$\beta$</td>
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<tr>
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<tr>
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<td>$\dot{\theta}$</td>
<td>[rad/s or $^\circ$/s]</td>
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</tr>
<tr>
<td>$\lambda$</td>
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</tr>
<tr>
<td>$\lambda_r$</td>
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</tr>
<tr>
<td>$\mu$</td>
<td>[-]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>[kg/m³]</td>
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<td>Symbol</td>
<td>Unit</td>
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<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>θ</td>
<td></td>
</tr>
<tr>
<td>σ</td>
<td>[-]</td>
</tr>
<tr>
<td>φ</td>
<td>[°]</td>
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<tr>
<td>φ_{r,b}</td>
<td>[°]</td>
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<tr>
<td>φ_{t}</td>
<td>[°]</td>
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</tr>
<tr>
<td>φ_{urb}</td>
<td>[°]</td>
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<td>φ_{w}</td>
<td>[°]</td>
</tr>
<tr>
<td>Ω</td>
<td>[rad/s;rpm]</td>
</tr>
<tr>
<td>ω</td>
<td>[rad/s]</td>
</tr>
</tbody>
</table>

**subscripts**
- a: amplitude
- aer: aerodynamic
- bcm: according to blade-element-momentum notations
- dev: from measurement device
- el: electrical
- hub: hub height
- h: helical
- i: at spanwise location i
- in: cut-in
- j: at chordwise position j
- low: lower part of rotor plane
- m: derived from measurements, or mean
- min: minimum
- max: maximum
- out: cut-out
- pres: derived from pressure distribution
- r: rotational
- rat: rated
- s: shedding or stalling point
- stag: stagnation point
- tip: at the tip
- up: upper part of rotor plane
- 1P: 1P averaged
- ∞: free stream
APPENDIX B. RESULTS OF CASE I

Table B.1 Non-dimensional difference *) between equilibrium values for the step in wind speed (case I.1)

<table>
<thead>
<tr>
<th></th>
<th>$u_t(r = 15.0\text{m})$</th>
<th>$m_{\text{ref}}(r = 2.75\text{m})$</th>
<th>$m_{\text{eq}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECN, i.w.</td>
<td>-0.080</td>
<td>-0.261</td>
<td>-0.509</td>
</tr>
<tr>
<td>ECN, d.e.</td>
<td>0.015</td>
<td>-0.262</td>
<td>-0.496</td>
</tr>
<tr>
<td>TUD</td>
<td>0.034</td>
<td>-0.244</td>
<td>-0.488</td>
</tr>
<tr>
<td>GH</td>
<td>-0.020</td>
<td>-0.272</td>
<td>-0.498</td>
</tr>
<tr>
<td>TUDk</td>
<td>-0.029</td>
<td>-0.264</td>
<td>-0.495</td>
</tr>
<tr>
<td>Unist.</td>
<td>-0.106</td>
<td>-0.246</td>
<td>-0.475</td>
</tr>
</tbody>
</table>

*) Defined by $\Delta q/q_{\text{eq}}$

Table B.2 Non-dimensional difference *) between equilibrium values for the step in pitch angle (case I.2)

<table>
<thead>
<tr>
<th></th>
<th>$u_t(r = 15.0\text{m})$</th>
<th>$m_{\text{ref}}(r = 2.75\text{m})$</th>
<th>$m_{\text{eq}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECN, i.w.</td>
<td>-0.219</td>
<td>-0.148</td>
<td>-0.026</td>
</tr>
<tr>
<td>ECN, d.e.</td>
<td>-0.171</td>
<td>-0.134</td>
<td>-0.034</td>
</tr>
<tr>
<td>TUD</td>
<td>-0.170</td>
<td>-0.140</td>
<td>-0.039</td>
</tr>
<tr>
<td>GH</td>
<td>-0.188</td>
<td>-0.132</td>
<td>-0.027</td>
</tr>
<tr>
<td>TUDk</td>
<td>-0.175</td>
<td>-0.132</td>
<td>-0.025</td>
</tr>
<tr>
<td>Unist.</td>
<td>-0.114</td>
<td>-0.160</td>
<td>-0.102</td>
</tr>
</tbody>
</table>

*) Defined by $\Delta q/q_{\text{eq}}$

Figure B.1 Tjørelbørg: Non-dimensional flat moment at blade root: $V = 13\text{ m/s to 10 m/s}$
(CASE I.1(2))
Figure B.2 Tjæreborg: Non-dimensional flat moment at \( r = 15 \text{ m} \); \( V = 13 \text{ m/s to 10 m/s} \); (CASE 1.1)

Figure B.3 Tjæreborg: Non-dimensional flat moment at \( r = 21 \text{ m} \); \( V = 13 \text{ m/s to 10 m/s} \); (CASE 1.1)
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Figure B.4 Tjæreborg: Non-dimensional rotor shaft torque; $V = 13 \, \text{m/s to 10 m/s}$; (CASE 1.1)

Figure B.5 Tjæreborg: Non-dimensional axial force; $V = 13 \, \text{m/s to 10 m/s}$; (CASE 1.1)
Figure B.6  
**Tjæreborg:** Non-dimensional induced velocity at blade root; $V = 13$ m/s to 10 m/s; (CASE I.1)

Figure B.7  
**Tjæreborg:** Non-dimensional induced velocity at $r = 15$ m; $V = 13$ m/s to 10 m/s; (CASE I.1)
Figure B.8  Tjæreborg: Non-dimensional induced velocity at $r = 21$ m; $V = 13$ mts to 10 mts; (CASE I.1)

Figure B.9  Tjæreborg: Non-dimensional flat moment at blade root; $V = 10$ mts; $\theta = 0^\circ$ to $2^\circ$; (CASE I.2)
Figure B.10  Tjøeberg; Non-dimensional flat moment at \( r = 15 \text{ m} \); \( V = 10 \text{ m/s} \); \( \theta = 0^\circ \) to \( 2^\circ \); (CASE 1.2)

Figure B.11  Tjøeberg; Non-dimensional flat moment at \( r = 21 \text{ m} \); \( V = 10 \text{ m/s} \); \( \theta = 0^\circ \) to \( 2^\circ \); (CASE 1.2)
Figure B.12  Tjæreborg: Non-dimensional rotor shaft torque; \( V = 10 \text{ mls} \); \( \theta = 0^\circ \text{ to } 2^\circ \); (CASE 1.2)

Figure B.13  Tjæreborg: Non-dimensional axial force; \( V = 10 \text{ mls} \); \( \theta = 0^\circ \text{ to } 2^\circ \); (CASE 1.2)
Figure B.14  
*Tjørebørg: Non-dimensional induced velocity at blade root; \( V = 10 \) m/s; \( \theta \) = \( 0^\circ \) to \( 2^\circ \); (CASE 1.2)

Figure B.15  
*Tjørebørg: Non-dimensional induced velocity at \( r = 15 \) m; \( V = 10 \) m/s; \( \theta \) = \( 0^\circ \) to \( 2^\circ \); (CASE 1.2)
Figure B.16  Tjæreborg: Non-dimensional induced velocity at $r = 21$ m; $V = 10$ m/s; $\theta = 0^\circ$ to $2^\circ$; (CASE I.2)
Figure C.1  Tjæreborg; Flat moment at blade root; \( V = 7.4 \text{ m/s}; \theta = 1^\circ \text{ to } 3^\circ \); (CASE II.1)

Figure C.2  Tjæreborg; Flat moment at \( r = 15 \text{ m}; V = 7.4 \text{ m/s}; \theta = 1^\circ \text{ to } 3^\circ \); (CASE II.1)
Figure C.3  Tjæreborg: Flat moment at \( r = 21 \text{ m} \); \( V = 7.4 \text{ m/s} \); \( \theta = 1^\circ \) to \( 3^\circ \); (CASE II.1)

Figure C.4  Tjæreborg: Rotor shaft torque; \( V = 7.4 \text{ m/s} \); \( \theta = 1^\circ \) to \( 3^\circ \); (CASE II.1)
Figure C.5  Tjæreborg; Axial force; $V = 7.4 \text{ m/s}; \theta = 1^\circ \text{ to } 3^\circ$; (CASE II.I)

Figure C.6  Tjæreborg; Induced velocity at blade root; $V = 7.4 \text{ m/s}; \theta = 1^\circ \text{ to } 3^\circ$; (CASE II.I)
Figure C.7  Tjæreborg: Induced velocity at $r = 15$ m; $V = 7.4$ m/s; $\theta = 1^\circ$ to $3^\circ$; (CASE II.1)

Figure C.8  Tjæreborg: Induced velocity at $r = 21$ m; $V = 7.4$ m/s; $\theta = 1^\circ$ to $3^\circ$; (CASE II.1)
RESULTS OF CASE II

Figure C.9  Tjæreborg; Flat moment at blade root; \( V = 12.5 \text{ m/s}; \theta = 1.164^\circ \text{ to } 3.190^\circ;\) (CASE II.2)

Figure C.10  Tjæreborg; Flat moment at \( r = 15 \text{ m}; V = 12.5 \text{ m/s}; \theta = 1.164^\circ \text{ to } 3.190^\circ;\) (CASE II.2)
Figure C.11  Tjæreborg; Flat moment at $r = 21$ m; $V = 12.5$ m/s; $\theta = 1.164^\circ$ to $3.190^\circ$; (CASE II.2)

Figure C.12  Tjæreborg; Rotorshaft torque; $V = 12.5$ m/s; $\theta = 1.164^\circ$ to $3.190^\circ$; (CASE II.2)
Figure C.13  Tjæreborg: Axial force; $V = 12.5 \text{ m/s}$; $\theta = 1.164^\circ$ to $3.190^\circ$; (CASE II.2)

Figure C.14  Tjæreborg: Induced velocity at blade root; $V = 12.5 \text{ m/s}$; $\theta = 1.164^\circ$ to $3.190^\circ$; (CASE II.2)
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Figure C.15 Tjæreborg: Induced velocity at $r = 15 \text{ m}$; $V = 12.5 \text{ m/s}$; $\theta = 1.164^\circ$ to $3.190^\circ$; (CASE II.2)

Figure C.16 Tjæreborg: Induced velocity at $r = 21 \text{ m}$; $V = 12.5 \text{ m/s}$; $\theta = 1.164^\circ$ to $3.190^\circ$; (CASE II.2)
Figure C.17 Tjæreborg; Flat moment at blade root; \( V = 9.0 \) m/s; \( \theta = 0.21^\circ \) to \( 3.38^\circ \); (CASE II.3)

Figure C.18 Tjæreborg; Flat moment at \( r = 15 \) m; \( V = 9.0 \) m/s; \( \theta = 0.21^\circ \) to \( 3.38^\circ \); (CASE II.3)
Figure C.19  Tjæreborg; Flat moment at \( r = 21 \text{ m} \); \( V = 9.0 \text{ m/s} \); \( \theta = 0.21^\circ \text{ to } 3.38^\circ \); (CASE II.3)

Figure C.20  Tjæreborg; Rotor shaft torque; \( V = 9.0 \text{ m/s} \); \( \theta = 0.21^\circ \text{ to } 3.38^\circ \); (CASE II.3)
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Figure C.22  *Tjæreborg: Induced velocity at blade root; V = 9.0 m/s; θ = 0.21° to 3.38°; (CASE II.3)*
Figure C.23  Tjæreborg: Induced velocity at $r = 15 \, m$; $V = 9.0 \, m/s$; $\theta = 0.21^\circ \, to \, 3.38^\circ$;  
(CASE II.3)

Figure C.24  Tjæreborg: Induced velocity at $r = 21 \, m$; $V = 9.0 \, m/s$; $\theta = 0.21^\circ \, to \, 3.38^\circ$;  
(CASE II.3)
Figure C.25  Tjæreborg; Flat moment at blade root; $V = 8.7$ m/s; $0 = 0.070^\circ$ to $3.716^\circ$; (CASE II.4)

Figure C.26  Tjæreborg; Flat moment at $r = 15$ m; $V = 8.7$ m/s; $0 = 0.070^\circ$ to $3.716^\circ$; (CASE II.4)
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Figure C.27  Tjæreborg: Flat moment at $r = 21$ m; $V = 8.7$ m/s; $\theta = 0.070^\circ$ to $3.716^\circ$; (CASE II.4)

Figure C.28  Tjæreborg: Rotor shaft torque; $V = 8.7$ m/s; $\theta = 0.070^\circ$ to $3.716^\circ$; (CASE II.4)
Figure C.29  Tjæreborg; Axial force; \( V = 8.7 \text{ m/s} \); \( \theta = 0.070^\circ \) to \( 3.716^\circ \); (CASE II.4)

Figure C.30  Tjæreborg; Induced velocity at blade root; \( V = 8.7 \text{ m/s} \); \( \theta = 0.070^\circ \) to \( 3.716^\circ \); (CASE II.4)
Figure C.31  Tjæreborg: Induced velocity at $r = 15 \text{ m}$; $V = 8.7 \text{ m/s}$; $\theta = 0.070^\circ$ to $3.716^\circ$;  
(CASE II.4)

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(CASE II.4)
APPENDIX D. RESULTS OF TIME SCALE ANALYSIS
Table D.1  Comparison between calculated and measured time scales

<table>
<thead>
<tr>
<th>Ident.</th>
<th>1st step</th>
<th>2nd step</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>f</td>
<td>df/dt</td>
</tr>
<tr>
<td></td>
<td>calc.</td>
<td>meas.</td>
</tr>
<tr>
<td></td>
<td>calc.</td>
<td>meas.</td>
</tr>
<tr>
<td>II.1 flap</td>
<td>3.0(0.37)</td>
<td>3.75(0.46)</td>
</tr>
<tr>
<td>II.1 torq</td>
<td>3.0(0.37)</td>
<td>3.50(0.43)</td>
</tr>
<tr>
<td>II.2 flap</td>
<td>2.0(0.42)</td>
<td>1.3(0.26)</td>
</tr>
<tr>
<td>II.2 torq</td>
<td>2.0(0.42)</td>
<td>2.0(0.42)</td>
</tr>
<tr>
<td>II.3 flap</td>
<td>exponential</td>
<td>pitching</td>
</tr>
<tr>
<td>II.3 torq</td>
<td>exponential</td>
<td>pitching</td>
</tr>
<tr>
<td>II.4 flap</td>
<td>2.0(0.29)</td>
<td>2.7(0.39)</td>
</tr>
<tr>
<td>II.4 torq</td>
<td>2.5(0.36)</td>
<td>2.5(0.36)</td>
</tr>
<tr>
<td>II.5 flap</td>
<td>-</td>
<td>3.6(0.40)</td>
</tr>
<tr>
<td>II.5 torq</td>
<td>-</td>
<td>3.1(0.34)</td>
</tr>
<tr>
<td>II.6 flap</td>
<td>-</td>
<td>3.8(0.36)</td>
</tr>
<tr>
<td>II.6 torq</td>
<td>-</td>
<td>3.6(0.34)</td>
</tr>
</tbody>
</table>

*) Visually averaged values
( ) dimensionless value: f*U/D
Spread in calculations: ≈ ±0.15D/V∞
RESULTS OF TIME SCALE ANALYSIS

Figure D.1 Tjæreborg; Time scale of induced velocity at blade root; $V_{\infty} = 7.4$ m/s; $\theta = 1.0^\circ$ to $3.0^\circ$; (CASE II.1)

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Figure D.4 *Tjæreborg: Time scale of rotor shaft torque; \( V_\infty = 7.4 \text{ m/s}; \theta = 1.0^\circ \text{ to } 3.0^\circ \); (CASE II.1)"
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Tjøvelborg: Effect of different equilibrium values on time scale of flatwise moment; \( V_\infty = 7.4 \text{ m/s; } \theta = 1.0^\circ \text{ to } 3.0^\circ; \) (CASE II.1)

Figure D.8  
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Figure E.5  
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Figure E.6  
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(CASE IV.1)

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(CASE IV.1)
Figure F.5  

tjørehørg: Edge moment of blade 2 at blade root; safety stop; \( V = 7.09 \text{ m/s} \); (CASE IV.1)

Figure F.6  

tjørehørg: Edge moment of blade 3 at blade root; safety stop; \( V = 7.09 \text{ m/s} \); (CASE IV.1)
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TUN-CHAR, TUNGUST AND
TUNWAKE

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Figure G.2  DUT wind tunnel, $C_{D, ax}(\lambda)$ curve; $\theta = 2^\circ$; (CASE tun-char)
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Figure G.4  DUT wind tunnel, $C_p(\lambda)$ curve; $\theta = 0^\circ$; (CASE tun-char)
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Figure H.12  Tjæreborg, Flapwise moment at $r = 2.75$ m (blade root); $\phi_y = 54^\circ$; $V = 7.8$ m/s (CASE VII.2)
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Figure H.14  *Tjæreborg, Tilting moment; $\phi_y = 54^\circ$, $V = 7.8$ m/s (CASE VII.2)*
Figure H.15  Tjæreborg, Induced velocity at $r = 9$ m (30%R); $\phi_y = -51^\circ$; $V = 8.3$ m/s (CASE VII.3)

Figure H.16  Tjæreborg, Induced velocity at $r = 15$ m (50%R); $\phi_y = -51^\circ$; $V = 8.3$ m/s (CASE VII.3)
Figure H.17 Tjæreborg, Induced velocity at \( r = 21 \text{ m} \) (70%R); \( \phi_y = -51^\circ; \) \( V = 8.3 \text{ m/s} \) (CASE VII.3)

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Figure H.26 Tjæreborg, Yawing moment; $\phi_y = -3^\circ$; $V = 8.6$ m/s (CASE VII.4)
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**Figure H.28**  Tjæreborg, Flawwise at $r = 2.75$ m (blade root); $\phi_y = -3^\circ$; $V = 8.6$ m/s (CASE VII.4)
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Figure H.32 *Summary data Tjæreborg, Range in yawing moment at V_w = 9 m/s; (CASE VII.5)*
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Figure H.34 Summary data Tjæreborg, Range in yawing moment at $V_\infty = 12$ m/s; (CASE VII.5)
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Figure 1.27  Wind tunnel, Blade induced velocity at 70% span; $\phi_y = 20^\circ$; $V = 6.0$ m/s  
(CASE VI.3)

Figure 1.28  Wind tunnel, Blade induced velocity at 90% span; $\phi_y = 20^\circ$; $V = 6.0$ m/s  
(CASE VI.3)
Figure 1.29 Wind tunnel, Horizontal velocity at 50% span; $\phi_y = 20^\circ$; $V = 6.0 \text{ m/s}$ (CASE VI.3)

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Figure 1.36  Windtunnel, (1P averaged) axial induced velocity at 70% span; $\phi_s = 30^\circ$; $V = 6.0 \text{ m/s} (\text{CASE VI.4})$
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Figure I.38 **Wind tunnel, Blade induced velocity at 50% span; φr = 30°; V = 6.0 m/s (CASE VI.4)**
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Figure 1.40  Windtunnel, Blade induced velocity at 90% span: \( \phi_y = 30^\circ; V = 6.0 \text{ m/s} \) (CASE VI.4)
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Figure 1.44 Wind tunnel, Flapwise moment; \( \phi_y = 30^\circ; V = 6.0 \) m/s (CASE VI.4)
Figure IA5  Windtunnel, Yawing moment; $\phi_y = 30^\circ$; $V = 6.0$ m/s (CASE VI.4)

Figure IA6  Windtunnel, Tilting moment; $\phi_y = 30^\circ$; $V = 6.0$ m/s (CASE VI.4)
APPENDIX J. ECN, CYLINDRICAL WAKE MODEL

J.1 Introduction

This Appendix describes the ECN cylindrical wake model. First the description of the basic model is given for axisymmetric condition. This is followed by the modifications to this basic model which have been applied for yawed conditions.

J.2 Basic features and equations, axisymmetric conditions

In the ECN cylindrical wake model, the induced velocities are calculated by modelling the wake as a semi-infinite cylindrical vortex sheet of radius \( R \) equal to the rotor radius and a root vortex concentrated in the centre of the cylinder, see figure J.1.

![Cylindrical wake model](image)

Figure J.1 Cylindrical wake model

If \( \gamma \) is the tangential component of the vorticity density on the sheet, application of the Biot-Savart law gives for the induced velocity in axial direction:

\[
u_i(0, r) = \frac{1}{4\pi} \int_0^\infty \int_0^{2\pi} \frac{\gamma(x)(R - r\cos \phi)}{(x^2 + R^2 + r^2 - 2rr\cos \phi)^{1.5}} R d\phi dx \quad (J.1)
\]

This model has the reassuring feature that for \( \gamma = \text{constant} \) the induced velocity in the rotor plane equals:

\[
u_i(x = 0, r) = \frac{\gamma_i}{2} \quad \text{for} \quad \gamma_i = \text{constant} \quad (J.2)
\]

while at infinity (where the \( x \) integral extends from \(-\infty \) to \( \infty \)):

\[
u_i(x = \infty, r) = \gamma_i = 2\nu_i(x = 0, r) \quad (J.3)
\]

as was noted by de Vries (see [25]); This is in accordance with the momentum theory. See also Appendix K.
If $\gamma_{ax}$ is the axial component of the vorticity density on the sheet, the tangential velocities induced by the tip vortices are found from:

$$u'_{t,\text{sheet}}(0, r) = -\frac{1}{4\pi} \int_0^{2\pi} \int_0^\infty \frac{\gamma_{ax}(x) (R \cos \phi_r - r) + x \gamma_i(x) \sin \phi_r}{(x^2 + R^2 + r^2 - 2rR \cos \phi_r)^{1.5}} Rd\phi_r dx$$  \hspace{1cm} (J.4)

Additionally, there is a tangential velocity component induced by the root vortex:

$$u'_{r,\text{root}}(0, r) = -\frac{1}{4\pi r^2} \int_0^\infty \frac{\Gamma_{\text{root}}(x)}{(1 + x^2/r^2)^{1.5}} dx$$  \hspace{1cm} (J.5)

with $\Gamma_{\text{root}}$ the strength of the root vortex.

If the transport velocity of the wake vorticity is $V_{tr}$, then the vorticity present at position $x$ at time $t$, was released at an earlier time $\tau$:

$$\tau = t - \frac{x}{V_{tr}}$$  \hspace{1cm} (J.6)

With (J.6) the $x$ integration in (J.5) can be transformed to a $\tau$ integration.

$$u_t(0, r, t) = \frac{1}{4\pi} \int_0^{2\pi} (R - r \cos \phi_r) Rd\phi_r \int_0^\infty \gamma_i(-V_{tr} - [t - \tau] dV_{tr}/d\tau) d\tau$$

$$= \frac{1}{4\pi} \int_0^{2\pi} \int_t^\infty \frac{\gamma_i([t - \tau] dV_{tr}/d\tau) (R \cos \phi_r - r)}{(t - \tau)^2 V_{tr}^2 + P^2)^{1.5}} Rd\phi_r d\tau$$  \hspace{1cm} (J.7)

with $P^2 = R^2 + r^2 + 2rR \cos \phi_r$.

Equation (J.5) is transformed into:

$$u'_{t,\text{sheet}}(0, r, t) = \frac{1}{4\pi} \int_0^{2\pi} \int_t^\infty \frac{\gamma_{ax}(\tau) (R \cos \phi_r - r) + (t - \tau) \gamma_i \sin \phi_r}{(t - \tau)^2 V_{tr}^2 + P^2)^{1.5}} \frac{(V_{tr} + [t - \tau] dV_{tr}/d\tau) Rd\phi_r d\tau}{d\tau}$$  \hspace{1cm} (J.8)

Equation (J.5) gives (with $V_{tr}$ the axial convection velocity of the root vortex):

$$u'_{r,\text{root}}(0, r, t) = -\frac{1}{4\pi r^2} \int_t^\infty \frac{\Gamma_{\text{root}}(\tau)}{[t - \tau]^2 V_{tr}^2 + P^2)^{1.5}} (V_{tr} + [t - \tau] dV_{tr}/d\tau) d\tau$$  \hspace{1cm} (J.9)

Now if $\gamma_{ax}(\tau), \gamma_i(\tau)$ and $\Gamma_{\text{root}}(\tau)$ were known, the equations (J.7), (J.8) and (J.9) give in principal a way to evaluate $u_t$ and $u'_t$. A relation between $\gamma_{ax}(\tau), \gamma_i(\tau), \Gamma_{\text{root}}(\tau)$ and $C_{D,ax}$ can be obtained from the following considerations.

Let the bound vorticity of a blade be $\Gamma$. According to the Youkowsky formula, the axial force on this vortex will be (for an elemental distance $dr$ along the radius):

$$\rho \Omega \Gamma (1 + x') d\Gamma = \rho \Omega \Gamma d\tau$$  \hspace{1cm} (J.10)

If $\Gamma$ does not depend on $r$, the blade load distribution will be triangular and the induced velocity constant over the rotor plane. Also this is consistent with the shedding of vorticity only at the blade tip and hub and hence with the vortex sheet assumed.
Integration of (J.10) over the entire radius and summation over the blades (B) gives:

\[ \Gamma_n = C_{D,n} 0.5 \rho V_w^2 \pi R^2 = 0.5 \rho \Omega R^2 \Gamma B \] (J.11)

or

\[ \Gamma = \frac{\pi C_{D,a} V_w^2}{\Omega B} \] (J.12)

For the vortex sheet model, the root vortex strength \( \Gamma_{root} \) equals the bound vortex strength of all blades:

\[ \Gamma_{root} = B \Gamma \] (J.13)

For determining the tangential component of vorticity density, the shed vorticity \( \Gamma \) has to be smeared out over a distance \( dx \), equal to the distance over which the vortex is transported till the next blade passes:

\[ dx = \frac{2\pi}{\Omega B} V_{tr} \] (J.14)

The vorticity vector on the sheet will have an axial and tangential component (figure J.2). The angle \( \alpha_s \) between the tangential direction and the vector equals:

\[ \alpha_s = \arctan \left( \frac{V_{tr}}{\Omega R} \right) \] (J.15)

The tangential component of vorticity density can then be determined from:

\[ \gamma_t dx = \Gamma \cos \alpha_s \text{ with } dx \text{ defined as above \ } (J.16) \]

or, using (J.12) and (J.16)

\[ \gamma_t V_{tr} = \frac{C_{D,a} U^2}{2} \left( \cos \alpha_s \approx 1 \right) \] (J.17)

In a similar way the axial component of vorticity density can be determined, by smearing out the axial component of the vorticity vector over an arc length \( R d \phi \), between two blades:

\[ Rd \phi = R 2\pi / B \] (J.18)

---

Figure J.2. Shedding angle in cylindrical wake model
The axial component of vorticity density then follows from:

\[ \gamma_{ax} = \frac{C_{D,ax} U^2}{2\Omega R} \sin \alpha_s \]  \hspace{1cm} (J.19)

with

\[ \sin \alpha_s = \frac{V_{tr}}{\sqrt{\Omega^2 R^2 + V_{tr}^2}} \]  \hspace{1cm} (J.20)

Finally \( V_{tr} \) will be related to the wake velocity at the rotor disc. In fact we shall take:

\[ V_{tr} = V - u_i \]  \hspace{1cm} (J.21)

Since this gives the correct steady state solution:

\[ \gamma = 2u_i : u_i (V - u_i) = \frac{C_{D,ax} V^2}{4} \]  \hspace{1cm} (J.22)

Then equation (J.7) with (J.17) and (J.21) gives an integral relation for the axial induced velocity. From the equations (J.9) and (J.8) with (J.19), (J.20) and (J.21) an integral relation is obtained for the tangential induced velocity. These integral relations are solved numerically (see the next section).

### J.3 Integration procedure

For the evaluation of equation (J.7) it appeared to be sufficient to extend the backward time integral to a time corresponding to a transport distance of two rotor diameters (although in some cases three rotor diameters have been applied):

\[ \tau = -\frac{4R}{V_{tr}} \]  \hspace{1cm} (J.23)

However for:

\[ \frac{4R}{V_{tr}} < \tau < -\frac{2R}{V_{tr}} \text{(or) } D < x < 2D \]  \hspace{1cm} (J.24)

it holds that:

\[ 2rR \cos \theta \ll (t - \tau)^2 V_{tr}^2 + R^2 + r^2 \text{(or) } 2rR \cos \theta \ll x^2 + R^2 + r^2 \]  \hspace{1cm} (J.25)

Then the computational time can be reduced, by solving the integral in eqn. (J.7) analytically.

Let:

\[ g(\tau) = \frac{2rR}{b(\tau)} \]  \hspace{1cm} (J.26)

with:

\[ b(\tau) = (t - \tau)^2 V_{tr}^2 + R^2 + r^2 \]  \hspace{1cm} (J.27)

Then eqn (J.7) gives with (J.17):

\[ u_i(r, t) = \frac{R}{8\pi} \int \frac{C_{D,ax}(\tau) U^2(\tau)}{b(\tau)^{1.5}} \int_0^{2\pi} \frac{(R - r \cos \phi_s)}{(1 - g(\tau) \cos \phi_s)^{1.5}} \sin \phi_s \sin \phi d\phi d\tau \]  \hspace{1cm} (J.28)

with

\[ g(\tau) \cos (\phi_s) \ll 1 \]  \hspace{1cm} (J.29)

This yields:

\[ u_i(r, t) = \frac{R}{4} \int \frac{C_{D,ax}(\tau) U^2(\tau)}{b(\tau)^{1.5}} \int\left[r(1 + \frac{15}{16} g(\tau)^2) - 0.75rg(\tau)(1 + 1.0733g(\tau)^3)\right]d\tau \]  \hspace{1cm} (J.30)
(\(\frac{dY_{dr}}{dr}\) has been neglected).

The tangential velocities induced by the sheet vorticity appeared to be much smaller than those induced by the root vortex. Therefore it is assumed that for the far wake:

\[
D < x < 2D
\]  

(J.31)

the tangential velocities are induced by the root vortex only.

The integration of the equations (J.7), (J.30), (J.8) and (J.9) has been performed with Simpson’s.

For stationary axial symmetrical conditions, it is possible to solve eqn. (J.7) analytically (Appendix K):

\[
C_{D_{ac}} = 4a(1 - a)
\]  

(J.32)

with a constant over the rotor plane. The accuracy of the integration procedure for several values of the time step (\(\Delta t\)), azimuth step (\(\Delta \phi_r\)) and integration length, has been assessed by comparing the numerical results with the results from eqn J.32.

It appeared that acceptable results (i.e differences between the numerical result and (J.32) < 5%) could be obtained for \(\Delta t = 0.04\) s, \(\Delta \phi_r = 10^\circ\) and an integration length of 4R. The ‘analytical far wake integration’ (equation J.30) was started at \(x = 1R\) instead of \(x = 2R\) as stated before.

When \(\Delta \phi_r\) was increased to 20° the results for the inner blade elements were still acceptable. At the tip however, the value of \(x^2 + R^2 + r^2 - 2Rr\cos \theta\) in the denominator of (J.1) gets very small for \(x \approx 0\) and \(\phi_r \approx 0^\circ\). For \(\Delta \phi_r = 20^\circ\), this has led to an unacceptable difference of more than 20% at the tip. Therefore the following option has been implemented in which integration around the tip is performed analytically.

From Appendix K it can be found that the axial velocity \(du_i\) for a blade element near the tip, which is induced by the wake from:

\[
x_1 < x < x_2 \text{ and } - \phi_{r,1} < \phi_i < + \phi_{r,1}
\]  

(J.33)

can be approximated by:

\[
du_i = \frac{1}{2\pi} [\arcsin\left(\sqrt{\frac{x_2}{x_2^2 + a_0^2}} \sin \psi_1\right) - \arcsin\left(\sqrt{\frac{x_1}{x_1^2 + a_0^2}} \sin \psi_1\right)]
\]  

(J.34)

with:

\[
\sin \psi_1 = \frac{R \sin \phi_{r,1}}{R^2 + R^2 - 2Rr \cos \phi_{r,1}}
\]  

(J.35)

and

\[
a_0 = R - r
\]  

(J.36)

In the derivation of this equation, it has been assumed that: \(a = R^2 + r^2 - 2Rr \cos \phi_r\) is constant and equal to the value at \(\phi_r = 0\) \((a_0)\). This assumption is valid for small values of \(\phi_r\).

In the present calculations, equation J.34 has been applied up to \(x = 0.24/V_t\) with \(\phi_{r,1}\) taken as 8°. This yielded an induced velocity which was in good agreement with eqn (J.32) even at the tip with \(\Delta \phi_r = 20^\circ\).
J.4 Comparison with blade element momentum results

In the figures J.3 and J.4 the results for the axial and tangential induction factor, obtained with the cylindrical wake model have been compared with blade element - momentum model (PHATAS) results. They are given for stationary

Figure J.3 Comparison between cylindrical wake and PHATAS result

Figure J.4 Comparison between cylindrical wake and PHATAS result

axsymmetric conditions. As stated before, the axial induction factor calculated with the cylindrical wake model is given by eqn. J.32, which corresponds to the momentum theory solution, applied on disc level. Therefore the good agreement
in axial induction factor is not surprising.

It is interesting to see that also the agreement in tangential induction factor is good, with the exception of the root region, where the tangential induction factor calculated with the wake model is larger than the one calculated with Phatas. This deviation could be expected since the tangential velocity induced by the root vortex goes to infinity at \( r = 0 \).

### J.5 Effect of different transport velocities

In order to study the effect of different transport velocities of vorticity in the wake, calculations with the ECN i.w. model are performed. Results are shown in the figures J.5, J.6, J.7.

![Graph showing the effect of different transport velocities](image)

**Figure J.5** Thyeborg: Effect of different transport velocities on induced velocity at \( r = 21 \text{ m} \); ECN, i.w. model; \( V_\infty = 8.7 \text{ m/s}, \theta = 0.07^\circ \) to \( 3.716^\circ \) (case II.4)
Figure 1.6 Tjæreborg: Effect of different transport velocities on rotorshaft torque; ECN, iw. model; \( V_0 = 8.7 \text{ m/s}; \theta = 0.07^\circ \text{ to } 3.716^\circ \) (case II.4)

Figure 1.7 Tjæreborg: Effect of different transport velocities on flapwise moment at blade root; ECN, iw. model; \( V_0 = 8.7 \text{ m/s}; \theta = 0.07^\circ \text{ to } 3.716^\circ \) (case II.4)
In the ECNIw model, the value for the transport velocity is usually been taken equal to the value of the wind speed in the rotor plane:

\[ V_{tr} = V_{\infty} - u_{i} \quad (J.37) \]

With (J.37), the resulting equation for the induced velocity at stationary conditions can be solved analytically and the correct steady state value is obtained:

\[ a = 0.5 - 0.5 \sqrt{1 - C_{D,ax}} \quad (J.38) \]

See Appendix I. However, it can be argued that the transport velocity of the vorticity on the vortex sheet should be the average of the wake velocity (eqn. J.38) and the velocity of the surrounding free stream:

\[ V_{tr} = V - u_{i}/2 \quad (J.39) \]

In the figures J.5 to J.7 the effect of different transport velocities on the induced velocities and loads are shown. Three different values of the transport velocity have been applied:

\[ V_{tr} = V_{\infty} - u_{i} \]
\[ V_{tr} = V_{\infty} - u_{i}/2 \]
\[ V_{tr} = V_{\infty} - 1.33u_{i} \]

The figures show that changing the transport velocity leads to different time scales but also to different levels of loads and induced velocities. A comparison with measured results yields that eqn. (J.37) gives the right absolute level of the loads while eqn. (J.39) gives the right time scale. A possible explanation may be that the time scale is determined by the very near wake alone, while the absolute level may also be determined by vorticity in the far wake. In the very near wake, eqn. (J.37) will be more realistic. However, further downstream in the wake the induced velocity is increased and this will lead to lower wake velocities and consequently lower transport velocities.

### J.6 Yawed Conditions

The basic model which is used for the calculation of yawed conditions is in principle only slightly different from the axisymmetric conditions. The main differences are the implementation of the advancing and retreating blade effect, the distribution of vorticity over the rotor plane, and the skewed wake geometry.

#### J.6.1 Advancing and retreating blade effect

The advancing and retreating blade effect is modelled in the blade element equations in a similar manner as it is done for the engineering methods. The free stream velocity is decomposed into an axial and tangential component. Then the tangential component is added to the rotational speed, see also section 11.1.

#### J.6.2 Vorticity distribution

The bound vorticity on the blade is determined in a similar way as for the axisymmetric conditions. Then this bound vorticity is decomposed into an axial and tangential component at the tip, with the shedding angle \( \alpha_{s} \):

\[ \alpha_{s} = \arctan \frac{V_{\infty} \cos(\phi_{c}) - u_{i}(\phi_{d})}{\Omega R - V_{\infty} \sin(\phi_{c}) \cos(\phi_{d},b)} \]
The tangential component is again smeared out over the distance over which the vorticity is transported till the next blade passes. The axial component is smeared out over the tip arc length between the blades. Due to the differences in bound vorticity on the separate blades, this leads to different values of the trailed vorticity on every blade tip. The distribution of the vorticity between the blade tips is assumed to be harmonic, see figure J.8.

\[ \gamma(\phi_t) = \gamma_0 + \gamma_{c1} \cos \phi_t + \gamma_{s1} \sin \phi_t \]

\[ \gamma_0 = \sum_{ib=1}^{ib=B} \gamma(\phi_{c,ib})/B \]

\[ \gamma_{c1} = \sum_{ib=1}^{ib=B} 2\gamma(\phi_{c,ib}) \cdot \cos(\phi_{c,ib})/B \]

\[ \gamma_{s1} = \sum_{ib=1}^{ib=B} 2\gamma(\phi_{c,ib}) \cdot \sin(\phi_{c,ib})/B \]

![Diagram of azimuthal vorticity distribution at yawed conditions](image)

Figure J.8  azimuthal vorticity distribution at yawed conditions

The concentrated root vortex is found as the sum of all bound vortex strengths on the blade.

J.6.3 Skewed wake geometry

The geometry of the wake has been assumed to be skewed under the skew angle \( \chi \), see figure J.9. It must be noted however that the wake in the ECN/IW model is less smooth than presented in figure J.9. This is due to the fact that tip vorticity is assumed to be shedded under the local skew angle and transport velocity, which are a function of the azimuth \( \phi_t \), due to the variation of induction.

The vorticity is assumed to travel throughout the wake under the same skew angle and transport velocity as the shedding values at the rotor.
From this skewed wake the induction is calculated with the Biot Savart law. The resulting equations have much in common with the relations for the axisymmetric conditions, equation J.7, J.8 and J.9. However the vorticity and the transport velocity are now a function of azimuth and the geometric terms have also been changed, by the change of wake geometry.

J.6.4 Root vorticity

Due to the skew angle under which the concentrated root vortex in the ECNiw model is shedded, this root vortex induces a velocity in axial direction. The qualitative influence of it is sketched in figure J.10. For positive yaw the root vortex is mainly felt at an azimuth angle of 270°. For azimuth angle slightly below 270° a negative contribution is added to the axial induced velocity and for azimuth angle slightly above 270° a positive contribution is added. The influence of the root vortex on the Tjæreborg calculations is given in the figures J.11 and J.12. In particular at the root section the influence is large. Due to the simplification that all the vorticity is concentrated in one root vortex, it is expected that its influence is exaggerated.

J.7 Versions of computational model

For the axisymmetric cases, the basic model has been the same for all cases, the numerical details may have been different: In all Tjæreborg cases, a time step of 0.04 s and an azimuth step of 20° has been used. For the case 1 calculations, there appeared to be a bug in the integration procedure. This bug however did not effect the results in a severe way.

For all following cases the axial induced velocity has been calculated from a combination of the eqn's (J.7), (J.30) and (J.34): The wake has been integrated
up to $x=6R$, where eqn. (J.7) has been applied up to $x=1R$, and the integration around a blade tip from (J.34) until $x=0.24/V_tr$.

From $x=1R$ to $x=6R$, eqn. (J.30) has been applied. The tangential induced velocity is found from the equations (J.7), (J.8) but for $x>1R$ the velocities induced by the sheet (eqn. (J.8)) have been neglected.

For the asymmetric cases, the calculations, which are presented in this report have been performed with the same model. In all Tjæreborg cases, a time step of 0.04 s and an azimuth step of 6° has been used. This small azimuthal step was necessary, in order to avoid numerical instabilities at the tip. The wake length was extended until 5R downstream of the rotor.

The calculations on the wind tunnel model were performed with a time step of 0.002 sec and an azimuth step of 4°. The wake was extended until 6R downstream.

J.8 Computational time

The calculation of case II.1 ($\Delta t = 0.04$ s, $\Delta \phi_r = 20^\circ$, integration length up to 3 rotor diameters) has been taken 1 hour on a SUN (IPC) work station.

The calculations of the yawed cases require a much longer calculational time. This is a consequence of the smaller azimuthal step which was applied. No attempts have been made to reduce the computational effort, by implementing an analytical solution model near the tip section, and or by simplifying the far wake model, as was done for the axisymmetric cases, see section J.3. Although, in principle this could have been done, it was not considered worthwhile. It would cost much effort to implement these algorithms, while the ECN.iw model is only used to perform some test cases to give additional insights into yaw and dynamic inflow phenomena. With the ECN.iw test results, the development of the ECN.de
Joint investigation of dynamic inflow effects and implementation of an engineering method

![Graph](image)

Figure J.11  *Tjæreborg: Effect of root vorticity on induced velocity, near blade root at 4-30 degrees yaw at blade root*

engineering model is supported.

### J.9 Assumptions and limitations

The cylindrical wake model gives a very simplified picture of the wake. The main assumption is that the wake shape is prescribed (no wake expansion) and that vorticity is equally smeared out over a wake cylinder. The transport velocity of the vorticity is assumed constant. For sudden changes in transport velocity (i.e., for a step on the wind speed), this may lead to a 'hole' in the wake or to vorticity passing each other. Also the fact that vorticity is equally smeared out over a wake distance until the next blade passes, is only true for stationary conditions.

For yawed conditions, the results appeared to be largely influenced by the simplification of root vorticity, concentrated in one single vortex line. This can be improved by smearing out the root vorticity over an inner cylinder with radius equal to the blade root radius.

Furthermore, the wake rotation is neglected. Figure J.13 shows that in case of wind shear the higher wind at the upper part of the rotorplane gives more vorticity. Then the near wake induces higher axial velocities in the upper part, but the far wake induces lower velocities. Although the results can probably be improved by taking wake rotation into account, the improvement is expected to be marginal and it was not considered worthwhile to do this.
Figure J.12  Tjæreborg: Effect of root vorticity on induced velocity, near blade tip at +30 degrees yaw at blade root

Figure J.13  Wind shear in cylindrical wake model
APPENDIX K. ECN CYLINDRICAL WAKE MODEL FOR STATIONARY CONDITIONS

The axial velocities induced by the wake extending from \( x = x_1 \) to \( x = x_2 \) are in the ECNiw model given by:

\[
\mathbf{u}_i(0,r) = \frac{\gamma_i}{4\pi} \int_{x_1}^{x_2} \int_0^{2\pi} \frac{(R - \cos \phi)}{(x^2 + a^2)^{1.5}} R d\phi \, dx \tag{K.1}
\]

with \( a^2 = R^2 + r^2 - 2rR \cos \phi \) and \( \gamma_i \) is assumed to be constant.

Define:

\[
\sin \chi = \frac{a}{(x^2 + a^2)^{0.5}} \rightarrow \cos \chi \, d\chi = -\frac{ax \, dx}{(x^2 + a^2)^{1.5}} \tag{K.2}
\]

(see figure K.1). Then

![Diagram of cylindrical wake model](image)

**Figure K.1 Integration in cylindrical wake model**

\[
\frac{dx}{(x^2 + a^2)^{1.5}} = -\frac{1}{a^2} \sin \chi \, d\chi \tag{K.3}
\]

since

\[
\cos \chi = \frac{x}{(x^2 + a^2)^{0.5}} \tag{K.4}
\]

Substituting K.3 in K.1 gives:

\[
\mathbf{u}_i = \frac{\gamma_i}{4\pi} \int_0^{2\pi} \frac{-1}{a^2}(R - R \cos \phi_i) \left( \frac{x_2}{(x_2^2 + a^2)^{0.5}} - \frac{x_1}{(x_1^2 + a^2)^{0.5}} \right) d\phi_i \tag{K.5}
\]
Then a transformation from \( \phi_r \) to \( \psi \) (see figure K.1) is applied:

\[
\sin \psi = \frac{R \sin \phi_r}{a} = \frac{R \sin \phi_r}{(R^2 + r^2 - 2rR \cos \phi_r)^{0.5}}
\]

\( \text{K.6} \)

Consequently:

\[
\cos \psi \, d\psi = \frac{R \cos \phi_r \, a - rR^2 \sin^2 \phi_r}{a^3} \, d\phi_r
\]

\( \text{K.7} \)

This gives:

\[
d\psi = \frac{R(R - r \cos \phi_r)}{a^2} \, d\phi_r
\]

\( \text{K.8} \)

since:

\[
\cos \psi = \frac{(R \cos \phi_r - r)}{a}
\]

\( \text{K.9} \)

Substituting (K.8) in (K.5) gives:

\[
u_i = \frac{7\gamma}{4\pi} \int_0^{2\pi} \left( \frac{x_2}{(x_2^2 + a^2)^{3/2}} - \frac{x_1}{(x_1^2 + a^2)^{3/2}} \right) \, d\psi
\]

\( \text{K.10} \)

Then \( (u_i = 0) \) is (integration from \( x_1 = 0; x_2 \rightarrow \infty \)):

\[
u_i = \frac{7\gamma}{2}
\]

\( \text{K.11} \)

while \( (u_i = \infty) \) is: \( (x_1 = -\infty; x_2 \rightarrow \infty) \)

\[
u_i = \gamma_t
\]

\( \text{K.12} \)

Furthermore with \( \gamma_t \) taken as \( \frac{C_{D,\text{ax}}}{2(\nu - u_t)} \), the expression for \( C_{D,\text{ax}} \) becomes:

\[
C_{D,\text{ax}} = 4a(1 - a)
\]

\( \text{K.13} \)
APPENDIX L. ECN, DIFFERENTIAL EQUATION MODEL

L.1 Basic features and equations, axisymmetric model

The basis from the ECN differential equation model is the equation for the axial induced velocity which is obtained from the ECN cylindrical wake model I.7. For the sake of simplicity this equation is written in the following form:

\[ u_i(r, t) = \frac{1}{4\pi} \int_0^{2\pi} (R - r \cos \phi_e) R \, d\phi_e \int_0^{\infty} C_{D, ax}^e(\tau) \frac{V^2(\tau) d\tau}{[[t - \tau]^2 (V\varphi(\tau))^2 + P^2]^{1.5}} \]  \hspace{1cm} (L.1)

with \( P^2 = R^2 + r^2 + 2rR \cos \phi_e \).

Then the time derivative of equation of L.1 is taken:

\[ \frac{\delta u_i}{\delta t} = \frac{1}{4\pi} \int_0^{2\pi} \frac{(R - r \cos \phi_e) R \, d\phi_e}{P^3} \left[ \frac{C_{D, ax} \cdot V^2}{2} \right] \int t^{\infty} \frac{C_{D, ax}^e(\tau) V^2(\tau)(t - \tau) V \varphi(\tau) d\tau}{[[|t - \tau|^2 V\varphi(\tau)^2 + P^2]^{2.5}}} \]  \hspace{1cm} (L.2)

The remaining time integral can be manipulated somewhat more, but not to analytical form. Only for the steady case it can be shown to reduce to:

\[ \frac{C_{D, ax} \cdot V^2}{3P^3} \]

so that indeed:

\[ \frac{\delta u_i}{\delta t} \rightarrow 0 \]

For unsteady conditions it must be related to \( u_i \) in the rotor plane. In fact reasoning that the steady solution of momentum theory must correctly be preserved, it is proposed to write:

\[ \frac{\delta u_i}{\delta t} = \frac{1}{4\pi} \int_0^{2\pi} \frac{(R - r \cos \phi_e) R \, d\phi_e}{R^2 + r^2 + 2rR \cos \phi_e} \left[ \frac{C_{D, ax} \cdot V^2}{2} - 2u_i \cdot (V\infty - u_i) \right] \]  \hspace{1cm} (L.3)

For \( r=0 \), the \( \phi_e \) integral reduces to \( 1/2R \) so that

\[ 4R \frac{\delta u_i}{\delta t} = C_{D, ax} V^2 \infty - 4u_i (V\infty - u_i) \]  \hspace{1cm} (L.4)

Finally it is possible to retain \( r/R \) dependancy, present in L.3:

\[ 4Rf_a(r/R) \frac{\delta u_i}{\delta t} = C_{D, ax} V^2 \infty - 4u_i (V\infty - u_i) \]  \hspace{1cm} (L.5)

with

\[ f_a(r/R) = 2\pi \left[ \frac{1 - (r/R) \cos \phi_e}{[r^2 + (r/R)^2 - 2(r/R) \cos \phi_e]^{1.5}} \right] \]  \hspace{1cm} (L.6)

If moreover the 'local' value of \( C_{D, ax} \) is used

\[ C_{D, ax} = \frac{W^2}{V^2 \infty} \cos \phi \]  \hspace{1cm} (L.7)
then the model can be applied on element level. Physically, the approximation made on the element level is that the local vortex strength is equal to the 'constant' vortex strength which is shed at the blade tip. The model shows that the time delay closer to the tip is shorter than in the rotor centre. In fact at the tip the delay is reduced to zero as the tip only 'sees' the vortex just being shed (as far as the induced axial velocity is concerned). Finally the equation for the tangential in-plane momentum is modelled in the following way:

$$4R f_i(r/R) \frac{d(\omega \cdot r)}{dt} + 4a'(1-a)V_{\infty} \Omega r = \sigma \frac{W^2}{V_{\infty}^2} c_i \sin \phi$$  \hspace{1cm} (L.8)

where the $f_i(r/R)$ has again been derived from the cylindrical wake model as caused by the axial component of the tip vortex lines and the blade root vortices.

L.2 Yawed conditions

The method which is used to model yawed conditions is very similar to the Glaucert model. First the disk averaged induced velocity $(u_i, 0)$ is calculated from the Glaucert equation, applied on annular level, in combination with the dynamic term from L.5:

$$V_{\infty} f_i(r/R) \frac{\delta u_i, 0}{\delta t} + S = \Gamma_{\infty} \rho S_{\text{annulus}} \bar{V}_{\infty} + \bar{u}_{i, 0} \cdot 2 \bar{u}_{i, 0}$$  \hspace{1cm} (L.9)

Then the local induced velocity is calculated with the following equation from:

$$u_i(r/R, \phi) = u_{i, 0}[1 - f_2(r/R) \sin \phi \tan \chi/2]$$  \hspace{1cm} (L.10)

with $f_2(r/R)$ according to Pitt and Peters:

$$f_2(r/R) = 15\pi/32 \cdot r / R.$$

L.3 Versions of computational model

In principle no changes have been added to the model, although in the course of the project some bugs appeared which have been eliminated.

L.4 Computational time

The calculational time for the present model is in fact even less than it is for the stationary equilibrium wake model which has been implemented in the ECN dynamic load code PHATAS.
APPENDIX M. MODEL DEVELOPED AT
GARRAD AND HASSAN

Combined blade element and momentum theory, when used for time domain dynamic load calculations, conventionally assumes that the wake reacts instantaneously to changes in blade loading. This treatment is known as an equilibrium wake model. In fact changes in blade loading change the vorticity that is trailed into the rotor wake and the full effect of these changes take a finite time to change to a new equilibrium value.

The aerodynamics of yawed wind turbine rotors are complex and subject to considerable uncertainty. This situation arises because of several features of the flow experience by the yawed rotor:

- A skewed component of wind acting towards the tip of the yawed blade for one half of the rotational cycle and towards the root for the other half
- A cyclic change in angle of attack experienced by the blade as it alternatively advances into and retreats from the incident flow
- A complex wake structure giving rise to a major modification of the distribution of inflow velocities at the rotor disc

As discussed above it is well known that both the radial flow and unsteady effects referred to above are capable of modifying aerofoil characteristics significantly from two dimensional data. In addition, the yawed rotor may have an important influence on the form of the wake structure causing considerable variation of the inflow velocities as the blade rotates. For these reasons the standard approaches taken for aerodynamic modelling of wind turbines operating in axial flow are generally inadequate when the turbine is yawed out of the wind.

Work undertaken at Garrad Hassan has been aimed at providing a simple, validated, engineering model of dynamic inflow which accounts for both of the above deficiencies in the standard method of wind turbine aerodynamic analysis and is suitable for inclusion in an aeroelastic code.

The study of dynamic inflow was initiated nearly 40 years ago by an American helicopter aerodynamicist named Sissingh. Sissingh provided a mathematical model [GH.1] to explain the discrepancies observed by Coleman et al in 1945 [GH.2] and Amer in 1950 [GH.3] when they compared measured pitch-roll damping of a helicopter in forward flight with predictions based on standard momentum theory.

With this model Sissingh showed that the thrust perturbation created by a roll rate could create a variation in the induced flow field that substantially affected the roll damping. Several aerodynamicists have pursued the approach taken by Sissingh and dynamic inflow theory as applied to helicopters in forward flight is now a well developed method.

In brief, the theory provides a means of describing the dynamic dependence of the induced flow field at the rotor upon the loading that it experiences. At any point on the rotor disc \((\tau, \phi)\), the induced velocity \(\mathbf{u}\) is assumed to be represented by a truncated Fourier series with prescribed radial distributions:
\[ u = u_0 + u_s (\tau/R) \sin \psi + u_c (\tau/R) \cos \psi + u_{2s} (\tau/R)^2 \sin 2\psi + u_{2c} (\tau/R)^2 \cos 2\psi \] (M.1)

The coefficients of the Fourier series can be expressed as a vector, \( u \), so that:

\[ u^T = [u_0, u_s, u_c, u_{2s}, u_{2c}] \] (M.2)

According to dynamic inflow theory, the elements of \( u \) are assumed to be linearly related to the harmonics of the disc loading of the rotor. The relationship takes the form of a first order differential equation:

\[ [M]u + [L]^{-1}u = C \] (M.3)

The vector \( C \) contains the coefficients of the rotor disc load harmonics.

The apparent mass matrix \([M]\) and the static coupling matrix \([L]\) in equation M.3 may be combined to give a matrix of time constants \([\tau]\)

\[ [\tau] = [L][M] \] (M.4)

In the absence of transient loading of the rotor, this matrix reduces to zero and a 'quasi-static' inflow model results:

\[ u = [L]C \] (M.5)

The elements of the static coupling matrix \([L]\) were first derived by Curtiss and Shupe [GH.4] using classical momentum theory. This approach provided a model which correlated well with experimental data from a helicopter in hover. However, in forward flight, a situation similar to that of a wind turbine rotor operating in yaw, the momentum theory model was found to give less than adequate correlation with test data. Various models were proposed as replacements to that based on momentum theory but none proved to be entirely satisfactory.

Finally, in 1980, Pitt and Peters derived both apparent mass \([M]\) and a static coupling \([L]\) matrices which provided an inflow model which was self-consistent in both hover and forward flight. Their approach, detailed in [GH.5], was based on potential flow theory with the pressure discontinuity across an actuator disc representation of the rotor being modelled by a general family of potential pressure distributions developed by Kinner [GH.3].

These distributions were used to derive the total pressure on the rotor in terms of Legendre functions in an ellipsoidal co-ordinate system. The ellipsoidal co-ordinate system becomes necessary because of the skewed profile of the wake behind the rotor.

The resulting model provides a means by which the inflow distribution can be calculated from the rotor loads for any magnitude of 'wake skew angle'; from \( \alpha = 0^\circ \) correspondence to the helicopter in forward flight to \( \alpha = 90^\circ \) corresponding to hover. Furthermore, in hover, the model reduces to that given by use of
classical momentum theory as first proposed by Curtiss and Shupe [GH.4]. The nature of the model is such that its adaptation from a description of the inflow experienced by a helicopter rotor in skewed flow to that experienced by a wind turbine rotor in yaw is relatively straightforward. A wind turbine rotor operating in steady yawed flow is adequately represented by the quasi-static coupling matrix inflow model of equation M.5 and the elements of the quasi-static coupling matrix [L] are defined below. Only in the case of transient rotor loading caused by gross changes in wind speed and direction across the disc is the full dynamic inflow model required.

The Pitt and Peters dynamic inflow model has received substantial validation in the helicopter field. Such validation is documented in papers by Gaonkar et al [GH.6] [GH.7] and in the conclusion of [GH.7] the authors state: 'The Pitt model, developed from first principles, provides excellent data correlation in forward flight and is identical to momentum theory in hover. Thus it appears to be the best currently available model for rotor analysis'.

M.1 Uniform Axial Flow

The model used for uniform axial flow was based on the part of the Pitt and Peters model developed for calculating transient response to blade pitching.

The model was originally developed for an actuator disc with assumptions made concerning the distribution of inflow across the disc. Here the model is applied at blade element or actuator annuli level since this avoids any assumptions about the distribution of inflow across the disc.

For a blade element, bounding radii $R_1, R_2$, in uniform axial flow, the elemental thrust, $dT$, can be expressed as:

$$dT = 2U_\infty a + U_\infty m_A \dot{\alpha}$$  \hspace{1cm} (M.6)

where $m$ is the mass flow through the annulus and $m_A$ is the apparent mass acted upon by the annulus and $\dot{\alpha}$ is the axial induction factor. Blade element analysis gives:

$$T = 0.5\rho V^2 \sigma dAC_T$$  \hspace{1cm} (M.7)

where

$$dA = \pi \cdot (R_2^2 - R_1^2)$$  \hspace{1cm} (M.8)

and

$C_T$ is the axial force coefficient
$\rho$ is the density of air
$\sigma$ is the local solidity of the blade element
$V$ is the effective velocity at the blade element

For a disc of radius $R$ the apparent mass upon which it acts is given approximately by potential theory, Tuckerman, [GH.8], as $(8/3)\rho \cdot R^3$.

The mass flow through the annular element is $\rho U_\infty (1 - a)dA$ so equation M.6 can be expressed as:

$$1/\rho V^2 C_T \sigma \pi (R_2^2 - R_1^2) = 2 \rho U_\infty a (1 - a) \pi (R_2^2 - R_1^2) + (8/3)\rho (R_2^3 - R_1^3) U_\infty \dot{\alpha}$$  \hspace{1cm} (M.9)
\[
\frac{\sigma V^2}{U_{\infty}^2} \cdot C_T = 4a(1 - a) + \frac{16(R^3 \hat{\omega}^2 - R^3)}{3\pi U_{\infty}(R^2 \hat{\omega}^2 - R^2)}
\] (M.10)

This last equation can be used to replace the blade element and momentum theory equilibrium equation for the calculation of axial inflow. It can be directly integrated to give time dependent values of inflow for each blade element. The tangential inflow is obtained in the usual manner and so depends on the time dependent axial value. The equation introduces a time lag into the calculation of inflow which is dependent on radial station, through local tip speed ratio and effective apparent mass.

The values of time lag for each blade element calculated in this manner are expected to underestimate the effects of dynamic inflow, as each element is treated independently with no consideration of the three dimensional nature of the wake or the possibly dominant effect of the tip vortex. The application is, however, consistent with blade element theory and provides a simple and computationally inexpensive method of calculating the time dependent behaviour of the rotor under conditions of dynamic inflow.

The equations above have been formulated directly from the Pitt and Peters model as it appears in the literature and so are couched in terms of the axial induction factor. The model was changed during the course of the project so that the induced velocity is calculated directly. This doesn't change the equations used but does affect the calculational results for wind gust cases. The preliminary calculations for the Tjaereborg machine (Chapter 8) used the induction factor method but those for the wind tunnel model used the induced velocity method.

### M.2 Yawed Flow

The loads experienced by a wind turbine rotor operating in yaw are notoriously difficult to predict.

It has already been stated that the aerodynamics of the flow experienced by a yawed wind turbine rotor are similar to those of a helicopter in forward flight. In the case of the helicopter, a skewed wake will cause the induced velocities to be reduced at the upwind edge and increased at the downwind edge of the rotor with a nearly linear variation along the axis aligned with the flight direction. A similar situation arises with a yawed wind turbine rotor so that a blade advancing into the wind will experience lower induced velocity that a blade retreating from the wind, see Figure 6.1.

Given this similarity, it seems clear that the model developed by Pitt and Peters [GH.5] for the helicopter problem is likely to be appropriate for the yawed wind turbine rotor. The application of the approach to wind turbines was first carried out by Swift beginning in 1980 as a PhD project at Washington University, US, [GH.9]. It has also been implemented by Hales and Garside [4.16] and to varying degrees been adapted by Hansen et al in YAWDYN [GH.10], [GH.11], [GH.12].

The Pitt and Peters model as applied in its full form to a yawed wind turbine
rotor is described below.

The quasi-static inflow model described by equation M.5 is modified slightly to give the following:

\[ \mathbf{a} = [L] \mathbf{C}' \]  

(M.11)

Here the vector \( \mathbf{a} \) is obtained by non-dimensionalising the inflow velocity vector \( \mathbf{u} \) by the mean axial wind speed \( U_{ax} \):

\[
\mathbf{a}^* = \frac{1}{U_{ax}} \mathbf{u}
\]

\[
= \frac{1}{U_{ax}} [u_o, u_z, u_c, u_2o, u_2c]
\]

\[
= [a_o, a_z, a_c, a_2o, a_2c]
\]  

(M.12)

Consequently it is the axial induction factor \( a(r, \psi) \) which is represented by a Fourier series:

\[
a = a_o + a_r (r/R) \cos \psi + a_c (r/R) \sin \psi + a_{2o} (r/R)^2 \sin 2\psi + a_{2c} (r/R)^2 \cos 2\psi
\]  

(M.13)

The vector \( \mathbf{C}' \) contains the coefficients of the harmonics of the rotor disc loads:

\[
\mathbf{C}' = [C_T, C_L, C_M, C_{2L}, C_M]
\]  

(M.14)

It should be noted here that these coefficients refer to disc loading. The potential flow theory involved in the derivation of the Pitt and Peters model assumes an actuator disc across which there is a pressure drop that varies continuously with radius and azimuth. In this way the induced velocities can be related to the distribution of vorticity in the entire wake and not, as in blade element theory, to the loading at a particular radial station and rotor azimuth. The consequence of the use of disc load coefficients is that the quasi-static inflow model given by equation M.11 can only be solved by first integrating the rotor loads over a complete azimuthal revolution. This procedure is repeated on an iterative basis until the vector of inflow coefficients has converged to a final solution. The static coupling matrix [L] is defined for the full inflow model as appropriate to a helicopter rotor in skewed flow in [GH.6].

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APPENDIX N. TUDK MODEL

The model developed by TUDk is an 'engineering' model that uses a modified Blade Element Momentum method to calculate the induced velocities at the rotor. The modifications are introduced in order to describe the dynamics of the induced velocities in response to changing rotor load and to calculate the average level and azimuthal variation of the induced velocities in yawed operation. The need for these modifications has been described in the general sections of this report.

The model is developed for and has been integrated into an aero-elastic time domain model for wind turbine load prediction. The model has been developed in two stages, the axisymmetric case with load variations and the yawed case. They will be described separately below.

N.1 Axisymmetric model

In general, the standard Blade Element Momentum model performs quite well under stationary axial flow conditions. For dynamic load situations the first choice would therefore be to calculate the induced velocities in a quasi-static way assuming instantaneous equilibrium between load and induction. However, early measurements performed by TUDk (TUDk.1) and the measurements performed within this project has clearly shown that the quasi-static approach is inadequate for load predictions for cases with pitching transients. It was also demonstrated in [1] that the introduction of a simple first order differential equation describing a time lag between the load and the induced velocity would improve the results considerably. This has been the starting point for the present model.

The time constant(s) in the suggested model should be expected to be functions of wind speed, rotor load and possibly also the radial position. In order to investigate these relations, a vortex model for the wake behind a rotor with a uniform disk load was used [TUDk.2]. The uniform disk load results in the production of vorticity at the rim of the disk at a rate proportional to the load. This vorticity is convected downstream resulting in a continuous vortex sheet at the surface of the wake with a vorticity density proportional to the load and inversely proportional to the convection velocity, which is assumed to be the mean value of the calculated velocities just outside and just inside the wake and therefore gradually decreasing downstream. The wake expansion is determined from continuity of mass flow inside the wake. To obtain numerical results, the vortex sheet is lumped into a large number of discreet vortex rings and the expansion and convection velocity is determined by iteration.

For the present purpose the vortex model is applied for several values of static disk loading (thrust coefficients). When the iteration is complete, an additional infinitesimal increase in loading is assumed at t=0. This results in a proportional stepwise increase in the strength of the vortex rings with the step propagating downstream with the local value of the convection velocity. It is therefore easy to calculate the development in time of the induced velocities at the disk at different radial stations. The observed change of the induced velocity is finally normalized with the total change when the propagation is complete. The result is a number graphs with step response functions for the induced velocities at different radii for different thrust coefficients. In the following, these functions will be analyzed as the response of linear filters with the quasi-static induced velocity as input and
the actual induced velocity as output. At first sight the response functions look like 1. order time lag functions starting from 0 with a finite rate of change and approaching 1 gradually. Also, the initial slope increases with increasing r/R. However, with increasing time the response is less dependent of r/R which is consistent with the fact that this part of the response is produced by the change in the far part of the wake. Consequently, a 1. order response with only one time constant is a rather poor fit to the calculated response.

Based on the author's experience in the design of analogue electronic filters it was decided to model the response as two filters connected in series. This requires the introduction of an intermediate variable as the output of the first filter and input to the second. The step response of the first filter is an initial step of k times the input step (k < 1) followed by a 1. order time lag response with a time constant of \( \tau_1 \) for the remaining part. The second filter is an ordinary 1. order time lag response with a shorter time constant \( \tau_2 \) which 'rounds the corner' of the initial step from the first filter. The combined response of the filters can be described as two coupled 1. order differential equations in the variables \( x = \text{input}, (\text{quasi static induced velocity}), y = \text{intermediate variable}, z = \text{output}, (\text{actual induced velocity}) \):

\[
\begin{align*}
y + \tau_1 \frac{dy}{dt} &= x + k \cdot x \cdot \tau_1 \frac{dx}{dt} \\
z + \tau_2 \frac{dz}{dt} &= y
\end{align*}
\]

with \( k = 0.6 \) and:

\[
\begin{align*}
\tau_1 &= \frac{1.1}{(1 - 1.3a)} \frac{R}{U} \\
\tau_2 &= [0.39 - 0.26(\frac{r}{R})^2] \cdot \tau_1
\end{align*}
\]

\( \tau_1 \) is common to all radial stations but a function of the load by the induction \( a \), while \( \tau_2 \) is a fraction of \( \tau_1 \) depending on \( r/R \). In the expression for \( \tau_1 \), the factor of 1.3 times \( a \) is unexpected large but is needed to produce a good fit to the data from the vortex wake model. If only the effect of the convection velocity was included, a factor less than 1 would be expected. The result is probably an effect of the wake expansion. However, a should not be allowed to exceed 0.5 in this expression.

Regarding the size of the time constants it should also be noted that the resulting time scales for the induced velocities after a pitch step will be smaller due to the feedback from the induction to the loads.

When implementing this method into a typical time marching scheme where the time steps are small compared to the dynamic inflow time constants, the time derivatives in the above differential equations can be calculated as backward differences. Also, the quasi-static induced velocity can be calculated from the load based on the induced velocity from the previous time step. This makes the calculation fast with no need for iterations in the induced velocities.

The present dynamic inflow model is based on results for an actuator disk with uniform load. In the general case the load will not be uniform, but as the Blade Element Momentum method assumes complete independence of radial stations,
one could for each radial station assume that the loads at the other stations are the same as for the station in question, thereby creating a uniform disk load equal to the local disk load.

Finally, the actuator disk model only considers the dynamics of the axial induced velocity. In the implementation of the model the tangential induced velocity will be assumed to behave similarly and with the same time scales.

N.2 Model for yawed flow

The primary objectives of the TUDk model for induced velocities in yawed flow are the following:

- Reformulation of the momentum equation for the calculation of the azimuthally averaged axial induced velocity.
- Inclusion of an azimuthal variation of the axial induced velocity.

It was decided from the start that the usual blade element assumption of independence between radial stations should be retained and that any additional effects from the rotational terms of the wake velocities should be neglected. During the project some checks of these assumptions were made, which indicated that the largest discrepancies could be expected from the breakdown of the radial independence. However, both effects would primarily reduce the accuracy for the inner parts of the rotor where the importance of the induced velocities is small regarding load predictions.

In order to analyze the two main objectives, the actuator disk vortex wake model from [TUDk.2] is used again, however this time in a simplified version without wake expansion and with the convection velocity assumed constant along the length of the wake. It is shown in [TUDk.2] that this cylindrical wake model in axial flow produces exactly the same equation for the induced velocity at the disk as the momentum method, when the convection velocity for the vorticity of the vortex model is equal to the velocity term used for the mass flow in the momentum equation. This corresponds to a convection velocity equal to the average value of the total velocity outside and inside of the far wake (the roller bearing analogy). This basic similarity between the two methods is assumed to hold also for the yawed case, for which the axial induced velocities \( u_i \) at the disk can be calculated easily with a skewed version of the vortex wake model. The vortex rings used to model the continuous vorticity density on the wake surface is simply kept parallel to the skewed rotor disk. The method is described in more detail in [TUDk.3], but the result is simple:

- The distribution of \( u_i \) is referenced to the mean value - antisymmetric around the vertical diameter (assuming the skew angle is in the horizontal plane) with the higher values at the downwind side of the disk. Along the vertical diameter \( u_i \) is constant and therefore equal to the average axial induced velocity over the disc.
- This average \( u_i,0 \) is identical to the value found in the axial case assuming identical load and convection velocity \( V_c \). This implies that the value of \( u_i,0 \) can be calculated from the momentum equation using the full disk area and \( V_c = |\vec{V} + \vec{u}_{i,0}| \) (vector sum) for the velocity in the mass flow calculation.
- The azimuthal variation of \( u_i \) at constant radius is approximately sinusoidal with an amplitude proportional to \( u_{i,0} \) and \( \tan(\chi/2) \). The proportionality constant is a function \( f(\chi/R) \) with a slope of \( 1/R \) at the center but increasing progressively towards the outer radial positions. A simplified polynomial fit to the calculated
values is suggested below.

These results are in complete agreement with the model for yawed flow proposed by Glauert. While this may not be a proof of Glauert’s model, it does add to its credibility.

The implementation of the method in a standard time marching scheme for an aeroelastic model will proceed through the following steps:

1. Calculate the loads on the blades at each radial station in the normal way combining the instantaneous wind and the induced velocities from the previous step.

2. Calculate new quasi-static values for the averaged induced velocities (tangential and axial) from the blade element momentum equation with the modification for the mass flow as specified above and including the normal tip corrections.

3. Calculate new dynamic values of the induced velocities from the quasi-static ones using the dynamic equations specified in the previous paragraph for the axial, unsteady case. It is further suggested to average the velocities over the number of blades so only one value for each radial position is used.

4. Modify the axial induced velocity to account for the azimuthal variation.

In the original version of the model as reported in [TUDk.3] the following expression was used for the azimuthal variation:

\[ u_i = u_{i,0} \cdot (1 + f(r/R) \cdot \tan\left(\frac{x}{\ell}\right) \cdot \cos(\phi_{r,b} - \phi_{r,w})) \]  \hspace{1cm} (N.1)

where R is the rotor radius, \( \phi_{r,b} \) the azimuth angle, \( \phi_{r,w} \) the azimuth angle where a blade points 'downwind' and finally

\[ f(r/R) = r/R + 0.4 \cdot (r/R)^3 + 0.4 \cdot (r/R)^5 \]  \hspace{1cm} (N.2)

Some typical results from this version of the model has been reported in [TUDk.3]. The resulting yaw moments appear to be too large by a factor of 1.5 to 2 when compared with measurements and the more advanced free wake models of this project. This indicates that the azimuthal amplitude of the induced velocities is too large at the outer part of the blades. One explanation could be that the higher order terms of eq. N.2 resulting from the actuator disk model are unrealistic for a real rotor with limited number of blades. It has therefore been decided only to use the first linear term in the current version. A second reason is an error in the implementation of the tip correction. It should be noted, that the values for \( u_i,0 \) as calculated above include the tip correction, which means that they are larger by a factor of \( 1/F_{tip} \) than the true disk average, which should be used for the amplitudes.

These modifications produce the following revised expression for the azimuthal variations (final version):

\[ u_i = u_{i,0} \cdot (1 + F_{tip} \cdot r/R \cdot \tan\left(\frac{x}{\ell}\right) \cdot \cos(\phi_{r,b} - \phi_{r,w})) \]  \hspace{1cm} (N.3)

The TUDk-calculations for yaw shown in this report are all using this last expression.

The use of \( \cos(\phi_{r,b} - \phi_{r,w}) \) is more general than the \( -\sin(\phi_{r,b}) \) term normally used, as it allows for other directions of the skew angle than the pure horizontal direction. Also, the calculation of wake direction should take the possible dynamics of the
wake direction into account (i.e. response to wind direction changes or turbine yawing motion). One way to do this is to calculate a quasi-static instantaneous wake velocity vector \( \vec{V}_{w,qs} = \vec{V} + 0.75 \cdot \vec{u}_r(r = 0.7R) \) (added as vectors) in an earth-fixed frame of reference and then calculate a dynamic value of \( \vec{V}_w \) using a simple 1. order time lag filter with a time constant of \( R/V \). The purpose of the factor 0.75 is to simulate the direction of the first part of the wake instead of the far wake. The dynamic value of the \( \vec{V}_w \) vector can now be projected on the rotor plane to find the value of \( \phi_{r,w} \) and the skew angle \( \chi \).

N.3 References


APPENDIX O. MODEL DEVELOPED BY UNIST.

O.1 Theoretical Background

For an incompressible, irrotational - except at the locations of the discontinuity surfaces - and inviscid fluid there exists a scalar potential

\[ \Phi = \phi_\infty + \phi \]  

(O.1)

so that the continuity equation can be rewritten in terms of a perturbation potential \( \phi \)

\[ \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \]  

(O.2)

called Laplace equation with \( \phi_\infty \) as the potential of the free stream velocity. Eq. (O.2) can be integrated over a lifting and/or nonlifting bodies with surface \( S \) using Green’s theorem

\[ \phi = \frac{1}{4\pi} \int_S \vec{\sigma} \cdot \left( \frac{1}{r} \right) \, ds + \frac{1}{4\pi} \int_s \mu \vec{n} \cdot \nabla \left( \frac{1}{r} \right) \, ds. \]  

(O.3)

where

\[ r = | \vec{z} - \vec{x} | \]  

(O.4)

represents the distance between the point \( \vec{x} \), for which \( \phi \) is to be evaluated, and the actual point of the integration area \( \vec{z} \) (fig. O.1). \( \sigma \) is the strength of a source and \( \mu \)

![Figure O.1 Integration area](image)

the strength of a doublet on the panel. Hence the flow field around solid and/or lift
producing surfaces can be modelled by superposition of the singularities $\sigma$ and $\mu$ with the free stream velocity.

It is possible to solve the problem either, on the level of the potential $\Phi$ or on the level of the velocities $\vec{V}$, which are related in the following way

$$\vec{V} = \nabla \Phi.$$  \hspace{1cm} (O.5)

In the latter case eq. (O.1) becomes

$$\vec{V} = \nabla \Phi = \vec{V}_\infty + \vec{V}_\text{rot} + \nabla \phi = ....$$  \hspace{1cm} (O.6)

In a practical approach the desired configuration will be covered with a set of quadrangular panels.

Within the current low-order method ROVLM the singularities are constant over one panel. The influence of blade thickness is neglected, and the desired blade configuration is covered with a set of quadrangular panels each containing a dou- blet of constant strength which is equivalent to a closed vortex ring.

The induced velocities of a closed vortex ring of length $C$ can be calculated by the Biot-Savart law (fig. O.2)

![Diagram of a closed vortex ring](image)

Figure O.2  \textit{Arbitrary closed vortex ring}

$$\vec{V} = -\frac{\Gamma}{4\pi} \int_C \frac{\vec{r} \times d\vec{s}}{|r|^3}.$$  \hspace{1cm} (O.7)

For a straight vortex line with starting point $\vec{x}_1$ and end point $\vec{x}_2$ equation (O.7) reduces to

$$\vec{V}_{i,j} = \frac{\Gamma_k}{4\pi r} \left( \vec{e}_{i2} \cdot (\vec{e}_1 - \vec{e}_2) \right) \cdot r_{i2} \cdot \vec{e}_v, \quad \vec{\epsilon}_v = \vec{e}_{i2} \times \vec{e}_1$$  \hspace{1cm} (O.8)
where \( r \) is the distance of point \( \vec{x}_p \) to the vortex line, \( \vec{e}_{12} \) is the unit vector directing along the vortex line from the starting point \( \vec{x}_1 \) to the end point \( \vec{x}_2 \), and \( \vec{e}_{1p} \) and \( \vec{e}_{2p} \) are the unit vectors pointing from the starting and end point to point \( \vec{x}_p \) (fig. O.3). Where \( r_{12} \) is the distance between \( \vec{x}_1 \) and \( \vec{x}_2 \).

![Image of doublet panel formed by four straight vortex filaments](image.png)

**Figure O.3** *Doublet panel formed by four straight vortex filaments*

The induction of a complete panel is obtained by summing the contributions of the four vortex lines forming the panel. (see eq. O.11)

The vortex strength of the doublet panels are calculated by building the difference of the doublet strengths, e.g. (fig. O.4)

\[
\Gamma_k = \mu_{i+1,j} - \mu_{i,j} \quad (O.9)
\]

of two neighbour panels considering the individual signs defined in the next section.

**O.1.1 Networks and Coordinate Systems**

A set of panels on a surface is called a network. We assume that a discrete representation of the true network surface is provided by a grid of corner points \( \vec{p}_{inet}(i,j) \) with

\[
\text{inet} = 1, \text{innet} \\
i = 1, nx \\
j = 1, ny
\]

where the elements of the position vector \( \vec{p}_{inet} \) of the network \( \text{inet} \) are resolved in a global \( (x,y,z) \) coordinate system as depicted in fig. O.5. The direction of \( i \) and \( j \) defines locally a third direction on a panel, building a right-hand-system. The axis of the doublet and the orientation of the panel are defined positively when their axis points in the opposite direction mentioned above (see fig. O.6). Furthermore, a vortex line pointing in positive \( i \) or \( j \) direction of a network is
considered positively. The reason for that is the following. It is convenient to have the sign of the vortex located on the 1/4 line of a wing or rotor blade positive in "normal operation", because the lift gained by the law of Kutta-Joukowski is positive too.

The simulation, which will be described in the next section, is performed in several coordinate systems (Fig. 0.7). The rotor blade is defined in the coordinate system $B$ with the origin on the axis of rotation. The x-axis points chordwise back from the leading edge to the trailing edge, the y-axis in radial direction from the blade root to the tip and the z-axis so that a right-hand-system is established. Afterwards the rotor blade is multiplied and rotated in the desired azimuthal position. The coordinate system $D$ rotates with its origin on the rotation axis together with the blades. The coordinate system $M$ is identical to the position of the coordinate system $D$ before the start of blade rotation and does not move. The inertial coordinate system $K'$ is located on the ground in the center of the tower.

Within version 2.4 of ROVLM all calculations are performed regarding to the coordinate system of the first rotor blade. In the Versions 3.x the reference is changed to an inertial coordinate system $K'$ fixed with the base of the tower.

O.1.2 Free Wake Simulation

A traditional panel method includes only panels on the examined configuration, and no effect of wakes emanating from trailing edges. In the current method the aerodynamic behaviour is approximated by the rotor blades and the corresponding wakes.
As mentioned above the rotor blades are discretized into \( N \) panels, each carrying a doublet element of constant strength \( \mu \). Thus, a linear system of \( N \) equations can be formulated:

\[
\begin{pmatrix}
    a_{1,1} & a_{1,2} & \cdots & a_{1,N} \\
    a_{2,1} & a_{2,2} & \cdots & a_{2,N} \\
    a_{3,1} & a_{3,2} & \cdots & a_{3,N} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{N,1} & a_{N,2} & \cdots & a_{N,N}
\end{pmatrix}
\begin{pmatrix}
    \mu_1 \\
    \mu_2 \\
    \mu_3 \\
    \vdots \\
    \mu_N
\end{pmatrix} =
\begin{pmatrix}
    R_1 \\
    R_2 \\
    R_3 \\
    \vdots \\
    R_N
\end{pmatrix}.
\]  

(O.10)

The elements of the Aerodynamic Influence Coefficient (AIC) matrix \( a_{ij} \) are calculated according to

\[
a_{ij} = \vec{n}_j \cdot \frac{1}{4\pi} \left( \vec{\epsilon}_v \cdot \frac{1}{r_{12}} (\vec{\epsilon}_{12} \cdot (\vec{\epsilon}_{1p} - \vec{\epsilon}_{2p})) \cdot r_{12} + \vec{\epsilon}_v \cdot \frac{1}{r_{23}} (\vec{\epsilon}_{23} \cdot (\vec{\epsilon}_{2p} - \vec{\epsilon}_{3p})) \cdot r_{23} + \vec{\epsilon}_v \cdot \frac{1}{r_{34}} (\vec{\epsilon}_{34} \cdot (\vec{\epsilon}_{3p} - \vec{\epsilon}_{4p})) \cdot r_{34} + \vec{\epsilon}_v \cdot \frac{1}{r_{41}} (\vec{\epsilon}_{41} \cdot (\vec{\epsilon}_{4p} - \vec{\epsilon}_{1p})) \cdot r_{41} \right),
\]  

(O.11)

representing the influence of a quadrilateral panel \( i \) with unit singularity strength at collocation point \( j \), see fig. O.3. Where \( \vec{n}_j \) is the panel normal vector at the control point \( P \) of panel \( j \) and \( r_k \) is the distance between the control point and the \( k = 1 \cdots 4 \) vortex filaments. The right-hand side vector \( \mathcal{R} \) for a panel \( j \) is determined by

\[
\mathcal{R}_j = -\vec{n}_j \cdot (\vec{V}_{\infty}(\vec{x}, t) + \vec{\Omega} \times \vec{x}_j)
\]  

(O.12)

and contains the local velocity due to the sum of rotational motion of the blade and the free-stream velocity perpendicular to the panel surface. That means the kinematic boundary condition — i.e., no flow normal to the panels — is satisfied at
the blade collocation points $\vec{x}_j$. A first solution for the unknown doublet strengths $\mu$ is performed and yields the perturbation potential $\phi_B$ of the rotor blade, compare Eq. (O.3).

Along the trailing edge of the blades wake abutting points are introduced, and located at the corners of the vortex-rings of the wing-bound lattice.

According to Eq. (O.6) the induced velocities are evaluated at these points by

$$\vec{V}_{i,j}(\vec{x}, t) = \vec{V}_{\phi \psi}(\vec{x}, t) + \vec{V}_{\phi \nu \gamma}(\vec{x}, t) + \vec{V}_{\infty}(\vec{x}, t). \quad (O.13)$$

Note that the second part of the right hand side of Eq. (O.13) equals zero — i.e. there is no shedded vorticity at this moment.

Extending the model into a time-dependant framework, a time increment $\Delta t$ has to be chosen:

$$\Delta t = \frac{\Delta \Psi}{\Omega_0} \quad (\Delta \Psi = 5^\circ \ldots 30^\circ), \quad (O.14)$$

where $\Delta \Psi$ is the azimuthal angle step of the blades within each time step. Since the convection velocities at the wake abutting points from Equation (O.13) with the time increment $\Delta t$ a dislocation vector is determined.

$$\vec{x}_{i,j}'(t + \Delta t) = \vec{x}_{i,j}(t) + \Delta t \cdot (\vec{V}_{i,j}(\vec{x}, t) + \vec{V}_{\infty}(\vec{x}, t)). \quad (O.15)$$

To obtain the final position of the free vortex sheet relatively to the blade, the rotational motion of the rotor has to be taken into account.

$$\vec{x}_{i,j}(t + \Delta t) = \vec{x}_{i,j} '(t + \Delta t) + \Delta t \cdot (\vec{\Omega} \times \vec{x}_{i,j} '(t + \Delta t)). \quad (O.16)$$
The procedure described above provides a first row of free-wake elements behind the trailing edge, see also Fig. O.8. According to Kelvin's Theorem

$$\frac{\partial \Gamma}{\partial t} = 0$$  \hspace{1cm} (O.17)

(dynamic boundary condition), the singularity strength of these wake elements is set equal to the corresponding doublet strength of the preceding solution at the blade.

$$\mu^{\text{wake}}_{I,j}(t + \Delta t) = \mu^{T,B}(t).$$  \hspace{1cm} (O.18)

If we consider the influence of the free vortex sheet(s) $\vec{\nu}_{\text{free}}$, the right-hand side vector is recalculated by extending Eq. (O.12) to

$$R_j = -\bar{n}_j \cdot \left( \vec{V}_{\text{free}}(\bar{x}_j, t) + \vec{\Omega} \times \bar{x}_j + \vec{\nu}_{\text{free}}(\bar{x}, t) \right).$$  \hspace{1cm} (O.19)

The solution of the linear system of equations (O.10) provides new values for the blade-bound singularity strengths $\mu$. Now the velocities at all corners of the wake elements are recalculated according to Eq. (O.13). Convection due to the local velocity and the rotational motion yields an additional rank of wake elements behind the blade.

A vortex-ring once shed at the trailing edge of the blade keeps its circulation throughout the whole computation, i.e. the singularity distribution over the complete wake surface is known at each time. If this procedure is repeated, the wake length increases step by step and the bound circulation at the rotary wing changes simultaneously as shown in Fig. O.9.
Assuming that there is no geometrical variation of the rotor configuration, the AIC-matrix of the rotor remains constant for all time steps.

The variation of length, shape and position of the free vortex sheets with time effects that the influences of the wake networks onto all examined points — collocation points and points defining the separated vortex sheets — have to be determined again and again. A flow chart of the ROVLM-programm is presented in Fig. O.10. Evidently the computational effort increases as the length of the wake grows.

Time dependant pitching of a rotor blade can be included into the simulation if we use the following approach. For small pitch angles (\( \alpha_c < 5 \cdots 10^\circ \)) the AIC matrix changes only marginally in the side diagonals, furthermore the wake leaves the trailing edge at nearly the same position. Therefore it is possible to simulate blade pitching just by rotating the normal vectors of the blade panels, because the main effects are included, i.e. change of the blade bound vorticity and shedding of opposite orientated vortices into the wake according to the change of the blade bound vorticity. This approach is only used within the versions 2.x of ROVLM. In the versions 3.x of ROVLM, pitching of the blade is simulated by a full rotation of the complete blade geometry. All unit vectors are determined new after pitching.

O.1.3 Load Calculations

Once the geometry and the singularities of the wake are known for one time step the potential loads on the blade bound vortices can be determined by the law of Kutta-Joukowski

\[
 f_{pot} = \rho \Gamma \int_C \vec{dl} \times \vec{V}. \tag{O.20}
\]
Therefore, the blade is covered with load points in the middle of the vortex filaments forming the blade doublets (fig. O.11) and fig. O.12. Together with the induced velocities the potential forces are determined at these points. In the next step all forces are reduced to the center of the blade panels to get one representative force per panel. Forces on spar – chordwise – filaments contribute to the forces of the neighbour panels and are split accordingly. The blade panels are now treated as flat plates with a \( c_{10} \) of \( 2\pi \). To consider the effect of profile camber, the zero angle of attack is included as additional angle of attack. If we now use the potential forces \( \vec{f}_{pot} \) per panel to determine an effective angle of attack

\[
\alpha_{eff} = \frac{\left| \vec{f}_{pot} \right|}{\rho \bar{V}^2 \pi dS}
\]  

(0.21)

and correct the aerodynamic coefficients with 2D profile tables

\[
c_l = f_{2D}(\alpha_{eff}) \]  

(0.22)

\[
c_d = f_{2D}(\alpha_{eff}) \]  

(0.23)

we get, after integration along the radius, rotor torque, thrust and flapping moments

\[
\tilde{M}_q = \int_R \vec{r} \times \vec{f}_{tq} \, dr,
\]

(0.24)

\[
\tilde{F}_{ax} = \int_R \vec{f}_{ax} \, dr,
\]

(0.25)

\[
\tilde{M}_{fl} = \int_R \vec{r} \times \vec{f}_{tq} \, dr.
\]

(0.26)
Joint investigation of dynamic inflow effects and implementation of an engineering method

Figure O.10  Flow chart of ROVLM

It should be mentioned that the load calculation as described above is quasi steady, since up to now no term containing the change of doublet strength with time is included.

O.2 Assumptions and limitations

In this section the assumptions and features of the models for the flow, the wind turbine and their interaction will be summarized.

Within this potential method the flow is inviscid, quasi-steady and with exception of the wake and the blade irrotational. Since the velocities are small compared to the speed of sound, it is assumed that it is also incompressible and isentropic. The vorticity is assumed to be concentrated in infinite small vortex cores forming infinite thin vortex layers. Convection of vorticity (vortex shedding) is included whereas diffusion (time varying core sizes) is neglected at the moment.

The model of the flow field around the turbine involves a uniform flow according to an arbitrary prescribed wind speed at the hub height with an infinite high gust propagation velocity throughout the field. The effect of the earth boundary layer is modelled using the power law

\[ u(h) = u_{ref} \left( \frac{h}{h_{ref}} \right)^\alpha \]  \hspace{1cm} (O.27)

where \( u(h) \) is the velocity at the height \( h \) and \( u_{ref} \) is the velocity at the reference height \( h_{ref} \) and \( \alpha \) the shear exponent. Turbulence of the incoming flow can be simulated using the SANDIA model. The wake points emanating from the prescribed separation line on the blades are covered with doublets of constant strength and
are convected in the flow field by an Euler update (see eq. (O.16)). Interaction of different wakes with blades and other wakes is included in the model. To avoid unrealistic high induced velocities, the core of the vortices can be modelled in two ways: (a) cut-off influence of the vortex at a certain small distance, (b) consider an exponential damping.

Since in the project mainly the aerodynamic behaviour is of interest, only the aerodynamic characteristics are modelled in detail. The rotor blades are covered with several radial panels and are assumed to be infinite thin, which means that displacements effects are neglected. Several radial stations correspond to the prescribed 2D profile data. The effect of camber is modelled by adding the zero angle of attack to the actual pitch angle. For regions of \( \alpha \) where the profile is in stall the stall formula for the flat plate according to Riegels is applied. The aerodynamic effects of the nacelle is neglected at the moment but can be included if the model will be extended. Several tower models are included based on potential flow around cylinders and measured data." Since no control system and no structural dynamics is included, the rotor speed must be prescribed.

O.3 Versions of the model

Within the project the following version of ROVLM are used:
- 2.4.1: for cases I.1(1) and I.2(1), within this version an error occurred during integration. Corrected results were send to ECN after the Munich Meeting,
- 2.4: for cases I.1(2), I.2(2), II.1-4,
- 2.6: for cases IV.1, IV.2,
- 3.3: for hybrid wake calculations,
- 3.5: for cases IV.1, IV.2.
Figure O.12  *Load Calculation*

- 3.6: all other cases

The different versions are described below.

<table>
<thead>
<tr>
<th>Versions of ROVLM</th>
</tr>
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<tbody>
<tr>
<td><strong>Version</strong></td>
</tr>
<tr>
<td>2.4.1</td>
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<tr>
<td>2.4</td>
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<td>2.6</td>
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<tr>
<td>3.0-3.3</td>
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<tr>
<td>3.5</td>
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<tr>
<td>3.6</td>
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</table>

**O.4 Computational effort**

A major problem for the simulation of the measurements is the computational effort. Fig. O.13 shows the required calculation time for different azimuthal angle steps to simulate four revolutions of Debra case LC122. Even for 30° or 40° the geometry of the wake is well represented with a calculation time reduced to a fraction of that for 5°. The according variation for rotor thrust and torque referring to the solution with 15° is below 2% for the mentioned case. To reduce the computational effort a hybrid wake model was employed for the calculations.
Figure O.13 Computational effort

In a free wake calculation the wake can completely influence itself. In a hybrid wake model the influence of the wake on itself is limited to a specified region, e.g. namely the near wake. First calculations (fig. O.14) show encouraging results with reduced computational effort and small errors (fig. O.15). The maximum error in the case depicted above is about 2.6%. As depicted in figure O.13 the computational effort for a free wake calculation increases with the number of wake elements. The effort for case I.3, using a hybrid wake model with a free wake region extending about one rotor radius downstream needed about 13.2 hours on a IBM RISC System/6000. Another case, including a tree bladed turbine and a near wake extending 0.5R downstream, calculated on a PC-80486 took about 7 hours for 4 rotor revolutions. A simple prescribed wake can already be calculated within some minutes on the IBM RISC System/6000.

O.5 Possible extensions

Due to the general formulation and the modular programming of the 3.x versions it is possible to employ several extensions to the aerodynamic part of the model:
- calculate rotor speed out of resulting aerodynamic torque,
- use distribution of the zero angle of attack along blade,
- implementation of a turbulence model (already included),
- nonuniform gusts,
- calculation of velocities and pressure at prescribed points in the flow field (already included),
- consider the displacement effects of the rotor blades (already included),
- modelling of the rotor / tower interaction (2D model included),
- extension to unsteady load calculation,
- coupling with "simple" aerelastic model,
Joint investigation of dynamic inflow effects and implementation of an engineering method

Figure O.14  *Comparison hybrid wake/free wake - wake geometry*

- noise prediction (already implemented)

Any coupling with aeroelastic codes is possible – for axissymmetric wake geometries already performed within the MILLx code – but will require the development of a complete new code.
Figure O.15  *Comparison hybrid wake* *free wake - doublet history*
APPENDIX P. MODEL DEVELOPED BY DUT

P.1 General description
Within the Institute for Wind Energy of T.U. Delft an aerodynamic model has been developed a number of years ago based upon the asymptotic acceleration potential theory [DUT.1, DUT.2]. Under the assumption of incompressible, inviscid and irrotational flow, it can be shown that the pressure perturbation in the complete flow field is given by a Laplace equation and acts as an acceleration potential function. In the model the rotor blades are represented as discrete surfaces on which a pressure discontinuity is present. The model implies the presence of spanwise and chordwise pressure distributions, which are composed of analytical asymptotic solutions for the Laplace equation. This makes the approach equivalent to a lifting surface model. A similar approach for the determination of loads on helicopter blades was described by Van Holten [DUT.3]. In the first order approximation used for the present purpose, the chordwise pressure distribution is restricted to a flat plate type analogon. Integration of the accelerations experienced by particles of air, which travel from far upstream to the rotor blade, determines the velocities in the rotor plane. With these velocities the aerodynamic loads can be calculated.

A further advantage is that the asymptotic acceleration method gives a kind of intrinsic possibility to simplify or elaborate the models in specific areas of interest. In its simplest appearance the model is equivalent to a lifting line model with an axially delinearized wake. With more elaborated codes it is e.g. possible to calculate the near wake and the dynamic loads caused by coherent variations in the windspeed, and pitching of the blades (dynamic inflow).

In order to understand the phenomena of importance for dynamic inflow, it is convenient to consider the wake structure in terms of a vorticity representation. The wake vorticity exists of shed and trailing vorticity, both time dependent. The vorticity is formed at the blade and convected downstream with the local total velocity, which is partly wake induced. Equally, the strength of the trailing and shed vorticity depends on the wake through its effect on the inflow angles. This mutual interaction is taken into account in the dynamic inflow modelling.

P.2 Basic features and equations
The two important equations for incompressible inviscid flow are:

\[ \nabla \cdot \mathbf{V} = 0 \]  \hspace{1cm} (P.1)

the continuity equation expressing the conservation of mass, and the Euler equation:

\[ \frac{D \mathbf{V}}{Dt} = -\nabla p \]  \hspace{1cm} (P.2)

for the conservation of momentum.

When the fluid is also assumed to be irrotational:

\[ \nabla \times \mathbf{V} = 0 \]  \hspace{1cm} (P.3)

the following equations can be derived:
\[
\frac{\partial u}{\partial t} + \frac{1}{2} \frac{\partial}{\partial x} \left( u^2 + v^2 + w^2 \right) = -\frac{1}{\rho} \frac{\partial p}{\partial x} \\
\frac{\partial v}{\partial t} + \frac{1}{2} \frac{\partial}{\partial y} \left( u^2 + v^2 + w^2 \right) = -\frac{1}{\rho} \frac{\partial p}{\partial y} \\
\frac{\partial w}{\partial t} + \frac{1}{2} \frac{\partial}{\partial z} \left( u^2 + v^2 + w^2 \right) = -\frac{1}{\rho} \frac{\partial p}{\partial z}
\] (P.4)

The undisturbed windspeed \( \mathbf{V} \) is assumed to be parallel to the \( x \)-axis. The rotorplane is situated in the \( yz \) plane. If it is assumed that the perturbations in the velocities in \( x \), \( y \) and \( z \) directions are small with respect to the undisturbed windspeed \( \mathbf{V} = (V, 0, 0) \) then the linearized equation (D.4) can be written as:

\[
\frac{\partial u'}{\partial t} + V \frac{\partial u'}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \\
\frac{\partial v'}{\partial t} + V \frac{\partial v'}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \\
\frac{\partial w'}{\partial t} + V \frac{\partial w'}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x}
\] (P.5)

Here the suffix ' indicates a velocity perturbation. The first of these equations is now differentiated with respect to \( x \); the second with respect to \( y \) and the third with respect to \( z \). Then they are added and finally the continuity equation, eq. P.1 is substituted. This results into:

\[
\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = 0
\] (P.6)

This Laplace equation and equation P.5 show that the pressure, or more accurately the pressure perturbation, is acting as an acceleration potential function.

**P.3 Asymptotic considerations**

Asymptotic expansion techniques are often used in finding solutions of Laplace equations. The lifting line approach is such an example: Often the span of a lifting surface is large with respect to the chord (e.g. an aircraft wing or a windturbine rotor blade). At some distance of such a surface (in the far field) the experienced accelerations will be equivalent to those felt by a line on which the load is concentrated. So in this area the wing can be modelled with a pressure dipole line. Close to the surface the experienced accelerations will be dominated by the chordwise load distribution. So in the near field the experienced accelerations will be almost two dimensional, and can be modelled with two dimensional pressure distributions. In the method used here both asymptotic approximations are combined with the use of a common field term. This third term is needed to cancel the far field term in the vicinity of the lifting surface, as well as the near field term far away from the aerodynamic active surface. In such a way both terms are matched to give an expression which is valid in the whole field.
P.4 The first order boundary value problem and its solution

The situation for a one bladed rotor will first be considered in order to keep the expressions as simple as possible. In a later stage it will be shown that the expressions can be easily adapted for a multibladed configurations.

In the application of asymptotics, a choice is always made for the order of magnitude that will be neglected. In order to keep the expressions relatively simple, only the first order problem and its solution are presented here. The computer codes developed are also based upon the first order solution.

The first (obvious) boundary condition is the fact that the pressure perturbations should vanish far away from the aerodynamical active surface:

\[ p \to 0 \quad \text{when} \quad x^2 + y^2 + z^2 \to \infty \quad (P.7) \]

The second boundary condition can be described as a linearized kinematic boundary condition; for a flat plate aerofoil located in the \( y_b-z_b \) plane this yields:

\[ \frac{\partial p}{\partial x_b} = 0 \quad \text{on the rotor blade} \quad (P.8) \]

The projection of the rotor blade is assumed to be situated in the \( y-z_b \) plane (the rotor blade plane). The blade surface is described as a surface function with function values along the \( x_b \) axis.

Finally the Kutta-Joukowski condition has to be satisfied:

\[ p = -\infty \quad (P.9) \]

at the leading edge of the rotor blade in such a way that

\[ \frac{u}{\Omega r + v \sin \phi_{1,b} + w \cos \phi_{1,b}} = \theta_{\phi}(y, t) \quad \text{on the rotor blade} \quad (P.10) \]

In the equations P.8 and P.10 the index \( b \) indicates a coordinate system related to the blade. The expression \( \theta_{\phi}(y, t) \) represents the (time dependent) setting angle of the blade, i.e., the local angle with respect to the plane of rotation.

For a fast running wind turbine rotor, under the assumptions that the velocities in the rotor plane are small with respect to the rotational velocity \( \Omega r \) and that the conditions on the rotor blade plane can be transformed to the rotor plane, the expressions P.8 and P.10 can be written as:

\[ \frac{\partial p}{\partial x} = 0 \quad \text{on the rotor blade} \quad (P.11) \]

\[ \frac{u}{\Omega r} = \theta_{\phi}(y, t) \quad \text{on the rotor blade} \quad (P.12) \]

The general first order solution for the problem P.6, P.7 and P.11 can then be derived:

\[ \frac{P}{0.5 \rho V^2} = \frac{1}{2} \frac{u(y, \phi)}{\sqrt[4]{x^2 + y^2 + z^2}} \left( \frac{\sin(\phi)}{\cos(y + \cos(\phi))} \right) + \]
\[
\frac{1}{2\pi} \frac{\tau_{b} V \chi(y)}{r_{b}} \phi(y) \sin(\chi) + \frac{1}{\pi} \sum_{n=1}^{\infty} A_{n}(t) P_{n}^{1}(\cos \theta) Q_{n}^{1}(\cosh \nu) \sin \chi \quad \text{(P.13)}
\]

In P.13 the expression \( \phi(y) \) yields the chord distribution.

The first expression on the right hand side of equation (D.13) is the near field term written in local elliptical coordinates \( \phi \) and \( \eta \). The third expression is the far field term, written in prolate spheroidal coordinates \( \theta, \nu \) and \( \chi \); and the middle expression in the right hand side is the common field expression written in circular cylinder coordinates \( y, r_{b} \) and \( \chi \). The \( P_{n}^{1} \) and \( Q_{n}^{1} \) functions represent associate Legendre functions of the first and second kind. Legendre functions are the natural solutions for problems written in prolate spheroidal coordinates. In equation P.13 it can be seen that close to the blade the common field expression exhibits a singular behaviour (caused by \( r_{b}^{-1} \)). This behaviour is also found in the far field term (when the \( Q_{n}^{1} \) term is evaluated), but with the opposite sign. Thus the total expression does not have this essential singularity.

In expression P.13 the function \( k(y, t) \) is used. This is the lift distribution over the blade. It can also be expressed in terms of associate Legendre functions of the first kind:

\[
L \left( \frac{y}{b/2}, t \right) = 1/2\rho V^{2} b \sqrt{1 - \frac{y}{b/2}^{2}} \sum_{n=1}^{\infty} A_{n}(t) P_{n}^{1}(\frac{y}{b/2}) \quad \text{(P.14)}
\]

The coefficients \( A_{n}(t) \) are not determined at this stage. This is done by application of the final boundary condition:

\[
- \frac{V}{\Omega R} \frac{(\frac{V}{\Omega R})}{R} \frac{1}{b} \int_{-\infty}^{0} \frac{\partial p}{\partial \left(\frac{y}{b/2}\right)} \Omega dt + \frac{V}{\Omega R} \frac{R}{r} \mu(y, t) = 0 \quad \text{(P.15)}
\]

for a particle of air arriving at the midchordline of the turbine blade.

The situation of a multibladed rotor can now be tackled with a straight forward approach. The solution P.13 for a one bladed rotor can simply be expanded with similar equations for the other blades, all in their own local blade coordinates. The determination of the coefficients \( A_{n}(t) \) implies of course more labour, but is in principle unchanged.

P.5 Determination of the coefficients \( A_{n}(t) \)

The kinematic boundary condition P.15 states that the velocity found at the rotorblade should be such that the particles of air move tangential to the blade surface. Suppose the pressure distribution is known. Then the accelerations in the field can be determined, and thus the path of the particles of air, and their velocities in the rotor plane. However nothing is known a priori with respect to the path, since it is determined by a pressure field which still contains the unknown coefficients \( A_{n}(t) \).

There are however possibilities to start up the process to satisfy the final boundary condition. In these processes the boundary conditions will be satisfied in a number of collocation points distributed over the blade. At first the stationary situation will be considered. This is the situation where the coefficients \( A_{n} \) are independent of time. The load distribution over the blade is thus a function of the position on
the blade (see eg. eq. P.14 for the spanwise lift distribution).

In its simplest implementation (code PREDICHA1) an iterative procedure is developed for the calculation of the stationary coefficients $A_n$. It starts from an assumed straight, unperturbed path, which is travelled by the particles of air with an imposed constant speed. For the speed the value $0.6667V$ is used, the optimum value in the rotorplane determined by axial momentum theory. The accelerations experienced during the travel are integrated in order to obtain the velocities at the collocation points. This yields a first guess of the stationary coefficients $A_n$.

With the now well determined pressure field the perturbed axial velocities of the particles travelling to the collocation points can be calculated. Within the code PREDICHA1 the velocity is kept constant along the straight unperturbed path, although its value is collocation point dependent. In repeating this iterative procedure the ultimate stationary coefficients are determined.

In a more elaborated version (PREDICHA2) the option of a perturbed path with time depended velocity can be chosen. Essentially it does not differ too much from the PREDICHA1 approach. The first iteration is in fact identical. But from the second iteration step onwards it takes more "bookkeeping". Furthermore the integration of the pressure field together with the determination of the perturbed path takes place "backwards", i.e. travelling back in time and away from the considered collocation point because the starting position of a particle of air at $t = -\infty$ is a priory not known.

The path and the velocities of the particles of air determined in the $i$'th iteration step are used to calculate the accelerations in the $i+1$'th iteration experienced from the $i$'th pressure distribution. This leads to modifications in the path and the velocities and thus to the next iteration step.

Iteration is continued until convergence is established based upon the velocities calculated at the collocation points (i.e. the calculated velocities must match the pressure distribution).

In the vortex representation there is an analogous procedure which is often referred to as wake relaxation.

The time dependent (dynamic inflow) solutions are obtained according to the following procedure:

First a stationary calculation is carried out using PREDICHA1. This calculation uses the initial values of the relevant parameters (such as pitch angle and wind speed at $t = 0$).

With the now well determined stationary pressure field the unsteady paths of the particles (and their time and position dependent velocities) are calculated using a step by step variation procedure. Every next step the whole process of determination of the accelerations (now time dependent), the velocities and the paths is repeated, thus calculating the dynamic inflow velocities in the rotorplane. The latter process is equivalent to a vortex wake calculation with a dynamically
varying wake (a process called dynamic wake adaptation). The code in which this calculation is implemented is designated PREDICDYN.

P.6 The calculation of loads and inflow angles

Once the coefficients \( A_n(t) \) are determined according to the above described procedure the spanwise load over the rotor blade is given by equation P.14.

In figure P.1 a typical example is shown of a spanwise load distribution. It shows the variation in the spanwise load caused by a stepwise variation in the windspeed at \( t = 6 \). Fig. P.2 shows the corresponding contour plot. In this figure a kind of periodic changes in the lift coefficient distribution can be seen which starts out after some seconds. The period is equal to about one second, which is the blade passage frequency. This is a typical example of a fluctuation in the wake (shed vorticity) generating variations in the instantaneous blade loads.

The chordwise load distribution is assumed to be equal to the theoretical result for the flat plate. Within the near field representation used in the present method the chordwise pressure distribution can be obtained from (see also the first term on the r.h.s. of eq. D.13):

\[
\frac{p}{0.5 \rho V^2} = -\frac{1}{\pi} \frac{l(y, t)}{0.5 \rho V^2 c(y) (\cosh \eta + \cos \phi)} \sin \phi \tag{P.16}
\]

A typical example of the chordwise load distributions used in the acceleration potential method is given in figure P.3.

With the definition of the local elliptical coordinate system:

\[
z_e = \frac{c(y)}{2} \cosh \eta \cos \phi \tag{P.17}
\]

\[
x_e = \frac{c(y)}{2} \cosh \eta \cos \phi \tag{P.18}
\]

And substituting \(|x_e| < 0.5 c(y); x_e = 0\), into P.16 yields:

\[
\frac{p}{0.5 \rho V^2} = -\frac{1}{\pi} \frac{l(y, t)}{0.5 \rho V^2 c(y) (1 + z_e)} \sqrt{1 - z_e^2} \tag{P.19}
\]

which is the corresponding pressure distribution over the plate.

The velocity potential \( \Phi \) corresponding to the near pressure field P.16 is given by:

\[
\frac{\Phi}{0.5 \rho V^2} = -\frac{1}{\pi} \frac{l(y, t)}{0.5 \rho V^2 c(y)} (e^{-\eta} \sin \phi - \phi) \tag{P.20}
\]

which is the well known (two dimensional) potential for the Birkmann velocity distribution over a flat plate under an angle of attack \( \alpha \). For the \( \alpha \) related to P.20 the following relation holds:

\[
\alpha(y, t) = -\frac{1}{2\pi} \frac{l(y, t)}{0.5 \rho V^2 c(y)} \tag{P.21}
\]

In the three dimensional situation the definition of an angle of attack is not straightforward. Of course eq. P.21 can be used as the definition of the "incidence angle", a kind of 3-D equivalent of the (two dimensional) angle of attack. Apart
from the incidence angle there is also the geometrical angle of attack \( \alpha_{geo} \), determined by the undisturbed inflow velocity and the rotational velocity component.

The difference between \( \alpha_{geo} \) and \( \alpha(Z_b, \theta) \), defined by P.21 is then designated the induced angle \( \alpha_{ind} \), equivalent to the induced angle in the Prandtl hypothesis for the flow over a wing.

Under this hypothesis it is assumed that the characteristics of a specific blade section are identical to the 2-d characteristics of the considered airfoil under an angle of attack equal to \( \alpha_{geo} + \alpha_{ind} \). The induced angle \( \alpha_{ind} \) is attributed to the effect of the outer flow (related to the 3-D geometry) on the local 2-d flow. But under the hypothesis this is the only effect of the 3-D flow that is taken into account.

With the present method it is however possible to calculate the induced angle directly by using expression P.13. The induced angle is now determined by integration of the common field and the far field terms (2nd and 3rd term of expression P.13). Integration of the near field term P.16 and addition of the geometric angle of attack \( \alpha_{geo} \) concludes the calculation of the flow angle. This flow angle should then satisfy the tangential flow boundary condition P.15.

Using the far and common field expressions for the determination of inflow angles and velocities is a method with a better physical basis than implementation of the Prandtl hypothesis, and is therefore used for the present calculations. Moreover recent research has shown [DUT.4] that in an equivalent situation, the calculation of incidence angles from the near field term integration, and the method using the Prandtl hypothesis with eq. P.21 give different results, especially in the blade root area. The difference can there be attributed to a kind of "swept flow" effect caused by the more or less circular inflow path w.r.t. to the blade, which is not taken into account when using eq. P.21. Figure P.4 shows this effect for the Tjaereborg geometry at various operational conditions. It shows the ratio of the angle determined by eq. P.21 and the angle calculated from the near field integration. The effect does not seem to be too sensitive for variation in inflow, but is definitely geometry dependent.

P.7 Assumptions and limitations

As mentioned in the general description the method is restricted to incompressible, inviscid and irrotational flow. The specific equations in the code have been derived under the assumptions of a large aspect ratio (slender rotor blades) and high values for the tipspeed ratio \( \lambda \) (typically \( \lambda \geq 4 \)). With this assumptions the model becomes a small perturbation problem, which is solved for the "linear perturbation" situation (first order solutions).

The blade sections are modelled as flat plate airfoils. In the inviscid body of the program the drag is equal to zero and the lift is linear with the calculated inflow angle. In a "viscous" subroutine the lift and drag of the various blade sections is determined using the calculated (inviscid) inflow angle and velocities together with tabulated 2-dimensional (measured) \( c_\alpha \cdot \alpha \) and \( c_\alpha \cdot \alpha \) values. When the 2-dimensional values are given for a limited set of \( \alpha \)'s the viscous subroutine CDALCX extends the range to the required values. This is carried out based upon the calculated inflow angle. No effort is made to set up an iteration scheme for
correcting the potential $c_l$ value with its viscous analogon. The reason is twofold: there is no evident way of performing such iteration (what is the inviscid potential flow equivalent of a viscous $c_l$?) and besides that it would increase the calculation time considerably.

For the calculations presented in the present report the tabulated 2- dimensional $c_l - \alpha$ and $c_d - \alpha$ values for the various sections have been synthesized into one $c_l - \alpha$ and one $c_d - \alpha$ curve.

The model is developed for uniform undisturbed flow. Therefore it is, at present, not possible to calculate windsheared conditions. The model does assume a zero core angle of the rotor and does not take into account blade flexibility.

Gravitational loads have been implemented in a more or less rudimental way, in order to make comparisons possible with strain gauge measurements on windturbine used in the experiments.

P.8 VIAX development

In a parallel fundamental research project of the Institute for Wind Energy of the Delft University of Technology a code was developed for determination of the velocity field around the rotor. The code is based upon the same pressure distribution model as described above, and it can e.g. be used to determine the complete wake structure behind a windturbine rotor operating in uniform perpendicular flow. At present there are two codes operational, named VIAX and VIAX 2. Within the codes a PREDICHAT or a PREDICHAT 2 approach is used respectively for the determination of the loads and the wake velocities. Results from VIAX using the PREDICHAT approach for the determination of wake velocities (particles are assumed to experience the same accelerations as found along straight paths) have been published in [DUT.5]). In figure P.5 the velocity distribution calculated with VIAX 2 (with the PREDICHAT 2 approach) is showed in a plane just before the rotorplane for the Tjaereborg geometry. It should be emphasised that these "pictures" are steady in a rotating frame. When one examines the figures along circle (segments) an idea can be obtained of the experienced velocity fluctuations close to the rotor blade. Reference [DUT.5] gives more details concerning the (calculated and measured) velocity fluctuations.

P.9 Improvements in the calculation procedure

At the start of the project the stationary code PREDICHAT 1 was available. Validation of the code with other predictions and with a large number of experimental results was already established. Furthermore there was a preliminary version available in which the radial perturbations of the paths of the particles of air were accounted for.

In the first round of calculations within the project it was necessary to calculate stepwise variations in either windspeed or pitch angle setting. The first approach to tackle such situation was to use PREDICHAT (currently designated as PREDICHAT 1) for the determination of the "initial" (just before the stepwise variation) and the "ultimate" (long after the stepwise variation) condition. Within PREDICHAT 1 the paths of the particles of air are unperturbed (straight), and
their imposed velocities are constant. This made it fairly easy to establish a procedure for the calculation of stepwise variations. By combining the accelerations calculated along the path experienced by the particle in the initial situation and in the ultimate situation dynamic inflow velocity signals could be calculated at the collocation points, leading to dynamic inflow blade loads.

This procedure however needed an extra interpolation routine for the moments at which the stepwise variations take place. Suppose for example that the aerodynamic loads have to be determined three seconds after a step in windspeed or pitch angle. The "ultimate" accelerations and the "ultimate" imposed transport velocity are used for the determination of the accelerations during the last three seconds. For the accelerations in the time prior to the stepwise variation the "initial" accelerations should be used according to the procedure. Since the "initial" and the "ultimate" imposed transport velocity of the considered particles is in general different, an interpolation must be performed between the position obtained while travelling under the "ultimate" transport velocity during the last three seconds and the position travelling under the "initial" transport velocity.

The cases I.1 and I.2 have been calculated using such procedure.

In the mean time, as part of the fundamental modelling research at the Institute for Wind Energy of the Delft University of Technology, the PREDICHAT 2 code became available. For the second round of calculations, using case I.2, this improved code was used. This means that at the starting point of the dynamic inflow calculations ("at t=0") the particles of air moving to the collocation points have travelled along a curved path with a varying velocity. This steady situation is then used as the input for PREDICDYN. In this dynamic code the complete flow is modelled according to a time marching scheme. For particles of air travelling along a (now time dependent) path towards the collocation points use is made of the accelerations experienced by the initial steady rotor calculation of the paths when t < 0.

In this way the prescribed unsteady environment for t ≥ 0 is extended towards t = - ∞ with a steady situation.

The cases II.1 to II.4; and the cases IV.1 and IV.2 were calculated using the combination PREDICHAT 2 and PREDICDYN.

In figure P.6 a comparison is shown between the calculated axial (induced) velocities of particles of air travelling to collocation points on the blade for "constant" and "variable" axial velocity. The "constant velocity" results are obtained for the situation where the accelerations are integrated along a straight unperturbed path, and the "variable velocity" is obtained for a real perturbed path. The general behaviour is quite similar. The influence of the blade passages can be distinguished clearly on the right hand side (close to the rotor plane). Note that the accelerations are sometimes positive along the trajectory!!

The influence of the improved modelling on the general result is rather limited. Figures P.7, P.8 and P.9 show comparisons between PREDICHAT 1 and PREDICHAT 2 results (for stationary situations) together with the measured result. In figure P.7 the calculations have been performed for the Tjaereborg geome-
try, whereas in P.8 the WEG MS-1 geometry is used. In fig. P.9 the comparison is shown with the results of other codes examined in the CEC project "Windturbine Benchmark Exercise", [DUT.6]. From this comparisons it can be seen that the predictability of the curves seems to be slightly better when using the original PREDICTHY code. The reason for it is not clear; The trajectory modelling used in PREDICTHY 2 is expected to be closer related to reality than the rather forced straight path constant velocity path method used in older PREDICAT version for the integration of the accelerations.

In general it can however be said that both PREDICAT and PREDICAT 2 predict overall blade loads with an accuracy equal to other methods currently used. Its advantage may be that much more detail can be obtained with regard to load and velocity distributions. The extension into the dynamic code PREDICY was, after all, straight forward and did not give much trouble. Lifting surface methods require much more computing power, computer force and computing time. Furthermore these highly numerical approaches are less equipped for interpretation of the results in terms of physical phenomena, necessary for further understanding of the fundamental origin of the calculated dynamic loads.

P.10 Computing time

The computing time necessary to run PREDICAT in its basic mode (PREDICAT 1) is very short (in the order of some seconds). PREDICAT 2 calculations take a few minutes on a MS-DOS PC type (286, 386 and 486 processor) computer.

The computing time is very dependent upon the accuracy required. On the MS-DOS 486 PC running at a clock frequency of 25 MHz, the computer on which the programs are currently operational, an accuracy of $10^{-6}$ in the non dimensional axial inflow velocity at the collocation points normally results in a computing time of 3 to 5 minutes for PREDICAT 2.

With an accuracy of $10^{-4}$ in the velocity the calculation of 40 seconds of dynamic inflow conditions, with a time pitch of 0.025 seconds using PREDICDYN takes up to 6 hours, where 90 minutes is sufficient for $10^{-5}$ accuracy. The transients calculated in case IV took some 12 hours of calculation time, after some 15 minutes of PREDICAT 2 calculations and some other preprocessing procedures.

P.11 Possible extensions

For the future there are plans to develop the model further for application in (linear) sheared and yawed flow. Furthermore the introduction of the second order terms will be considered. This gives the possibility of introducing a more realistic near field pressure distribution, and allows the introduction of unsteady airfoil models.

There are at present no plans for further improvement of the geometric representation (such as the introduction of a cone angle), nor is the extension with a structural dynamical model (e.g. introduction of blade flexibility) foreseen.

There is however no fundamental obstacle for introducing these kind of geometric or structural dynamic improvements. In the same way a (dynamic) gearbox,
generator, controller and tower representation can, in principle, be introduced.

The policy however within the Institute for Wind Energy has been to use the acceleration potential approach as quite a powerful tool for analysis of aerodynamic problems. It is adequate for rather fundamental research, and does not give the bother of all kind of peripheral problems in setting up structural dynamics and further models.

P.12 References


DUT.5 N.J. Vermeer, G.J.W. van Bussel, "Velocity measurements in the near wake of a model rotor and comparison with theoretical results", ECWEC Conference Proceedings, Madrid, Spain, 1990

Figure P.1  Spanwise load distribution for stepwise variation in wind speed

Figure P.2  Contour plot of load distribution shown in fig. P.1
Figure P.3  Chordwise pressure distribution

Figure P.4  Deviation of decay line for various radial stations. Tjæreborg geometry
**Figure P.5** Velocity distribution just in front of Tjæreborg rotor

**Figure P.6** Axial induction velocities experienced by particles of air with a constant and a variable axial speed
Figure P.7 Comparison of $C_p(\lambda)$ and $C_{D_{ax}}(\lambda)$ curves predicted with PREDICHAT and PREDICHAT 2, Tjæreborg geometry.

Figure P.8 Comparison of predicted $C_p(\lambda)$ and $C_{D_{ax}}(\lambda)$ from PREDICHAT and PREDICHAT 2, with measurements, WEG-MS1 geometry.
Figure P9  Comparison of predicted $C_P(\lambda)$ and $C_{D_{ax}}(\lambda)$ in benchmark exercise, WEG-MS1 geometry
APPENDIX Q. MODEL DEVELOPED BY NTUA AND GEMH

The present document constitutes an overview of the tools that were used by NTUA-FS (National Technical University of Athens, Fluids Section) and GEMH (Groupe d’Energetique et de Mecanique du Havre) within the framework of the JOUR-0083C project. NTUA-FS joined the research group of the project during its extension as an associate partner to ECN (the coordinator). GEMH participated as a subcontractor to NTUA-FS.

Within the present project NTUA-FS participated with GENUVP, the vortex particle free-wake aerodynamic model, that was developed in the past jointly with GEMH. Major developments on GENUVP were not made within JOUR-0083C. This does not mean that there were no improvements. In fact when the project started, NTUA-FS had UPROP which is an earlier version of the vortex model. UPROP performed unsteady aerodynamic calculations around rotors only. Gradually and within the framework of different projects UPROP evolved into a multi-body aerodynamic model that could treat complicated geometries such as the Rotor-Tower configuration, the Rotor-Nacelle-Wing configuration of an aircraft, etc. Also additional physical features were added such as: (a) the turbulent evolution of the wake of a wind turbine (JOUR-0087 project on wake effects), (b) the aerelastic modelling of the blades of a wind turbine by means of the bending theory of beams (JOU2CT92-0113 project aiming at the development of new design tools) and (c) the prediction of noise emissions from wind turbines (JOU2-CT92-0148 project). Thus GENUVP emerged as a computational environment encompassing all these activities.

Within JOUR-0083C, GENUVP was used basically as an aerodynamic tool. However structural as well as tower effects were included in some cases. The main objective of that contribution was to assist the development of better engineering tools. To this end predictions were provided for all the cases that were investigated within the project. As regards NTUA-FS and GEMH, their main interest was to validate the capability of GENUVP to predict the dominating physical mechanisms that define the behaviour of a wind turbine in steady as well as dynamic conditions. The present document corresponds to the first part of the reporting of the work. The second, concerning the refinement of vortex methods as well as in depth analysis of some unsteady situations, will be delivered as annex to the JOU2-CT92-0186 project.

Q.1 Description of the method

The response of an HAWT to dynamic inflow conditions is a special case of the aerodynamic performance problem of rotors. In terms of its basic physical features, the corresponding flow is an example of a three-dimensional and non-linear vortex or rotational flow [NTUA.1; NTUA.4]. Assuming the fluid incompressible and inviscid, the spatially distributed vorticity will correspond to the wakes generated by the flow around the blades. For a theoretical analysis of the dynamic inflow effects, Vortex methods are among the most cost effective numerical
models [NTUA.5+NTUA.19]. Within this context, a computational environment has been developed by the name GENUVP (GENeral Unsteady Vortex Particle method). GENUVP is an unsteady code based on the vortex particle approximation of the free vorticity. In brief the modelling is defined along the following guidelines:

According to Helmholtz's decomposition theorem [NTUA.20] the velocity field is made up of an irrotational part representing the disturbance due to the presence of the solid boundaries and a rotational part representing the disturbance due to the wakes. In order to determine the irrotational part a Neumann boundary value problem for the Laplace equation is solved. On the other hand the rotational part of the velocity field is determined directly as convolution of the vorticity contained in the wakes. As the fluid is assumed inviscid, there is no link between the irrotational and the rotational part of the velocity field. In order to bring these parts into contact, a link can be defined that models the vorticity emission process observed in real flows. Mathematically this link is based on the Kutta condition (more exactly on an appropriate formulation of Kelvin's theorem). The fulfilment of the Kutta condition permits to define quantitatively the conversion of the bound-vorticity into free-vorticity. This mechanism constitutes the inviscid analog of the vorticity production process already existing in all the conventional aerodynamic models.

Q.1.1 Formulation of the problem

We consider the unsteady flow of an inviscid and incompressible fluid around a combination of \( N_B \) three-dimensional bodies \( B_k \) with boundaries \( S_k, k = 1(1)N_B \) that form the configuration of a wind turbine. Each component of the configuration can be regarded as either a non-lifting body or as a lifting one according to its operational characteristics. Non-lifting bodies are the nacelle and the tower of the turbine whereas the blades are lifting bodies in the sense that they generate wakes. According to the geometrical assumptions made, the blades can be modeled either as lifting surfaces (thin wing assumption) or as thick wings. For the purposes of the present work only the blades of the rotor were included. Moreover in order to keep the level of the computational requirements reasonable, thickness effects were neglected. Thus the configuration considered were comprised \( N_B \) thin wings (more details can be found in [NTUA.18,H19]). Under the assumptions introduced for the fluid, the flow can be modeled as follows.

In order to describe the geometry of the flow, a fixed co-ordinate system with respect to the blades is introduced. All the flow quantities are defined with respect to this system. Let \( D \subset \mathbb{R}^3 \) denote the flow field, \( S \) its boundary and \( \vec{u} \) the outward unit normal to \( S \) (Figure Q.1). Moreover let \( \vec{u}(\vec{x}; t) \in D, t \geq 0 \) denote the velocity field. According to Helmholtz's decomposition theorem [NTUA.20], \( \vec{u}(\vec{x}; t) \) takes the form:

\[
\vec{u}(\vec{x}; t) = \vec{U}_r(\vec{x}; t) + \nabla \phi(\vec{x}; t), \ vec{x} \in D, t \geq 0
\]  

(Q.1)

where \( \vec{U}_r(\cdot; t) \) is a given div-free velocity field and \( \phi(\cdot; t) \) the disturbance velocity potential. This term is set to include the inflow velocity \( \vec{U}_{in}(\cdot; t) \) as well as the velocity induced by the free vorticity. Therefore \( \vec{U}_r(\cdot; t) \) represents the rotational part of the flow. As regards the scalar potential \( \phi(\cdot; t) \), it is defined so as to give the
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absolute perturbation velocity when differentiated with respect to the co-ordinate variables of the system introduced.

For an incompressible fluid, the velocity potential will be the solution of a boundary value problem for the Laplace equation. However the presence of lifting bodies imposes the introduction of the wakes they generate as active boundaries of the flow. Let \( S_{W_k}, \ k = 1(1)N_B \) denote the vortex sheets shed by the \( N_B \) lifting bodies, and \( \vec{\nu}_{W_k} \) their outward unit normals respectively (In the sequel the subscript "W" will be used to denote quantities corresponding to the wakes of the flow). Clearly

\[
\partial D \equiv S = \bigcup_{k=1}^{N_B} S_k + \bigcup_{k=1}^{N_B} S_{W_k}
\]

Within the framework of the potential flow theory the velocity potential can be represented by means of surface singularity distributions [NTUA.18]. For the case of thin wings, dipole distributions defined over their surface and their wakes are used. In this connection it is reminded that a dipole distribution \( \mu(\vec{x}); \vec{x} \in \Sigma \) defined on a surface \( \Sigma \), introduces a discontinuity of the scalar potential (Figure Q.2):

\[
\mu(\vec{x}) = -[\phi](\vec{x}), \ \vec{x} \in \Sigma
\]

whereas the potential itself is given by:

\[
\phi_{\mu}(\vec{x}_0) = -\int_{\Sigma} \frac{\vec{v}(\vec{x}) \cdot (\vec{x}_0 - \vec{x})}{4\pi |\vec{x}_0 - \vec{x}|^3} d\Sigma(\vec{x})
\]

>From equation Q.4 the corresponding velocity field \( \vec{u}_{\mu}(\cdot) \) is obtained by differentiation. Using Stokes theorem, \( \vec{u}_{\mu}(\cdot) \) takes the form [NTUA.18]:

\[
\vec{u}_{\mu}(\vec{x}_0) = \nabla_{\vec{x}_0} \phi_{\mu}(\vec{x}_0) = \int_{\Sigma} \frac{\nabla_{\vec{x}} \mu(\vec{x}) \wedge \vec{v}(\vec{x})}{4\pi |\vec{x}_0 - \vec{x}|^3} \wedge (\vec{x}_0 - \vec{x}) d\Sigma(\vec{x}) +
\]

\[
\int_{\partial \Sigma} \mu(\vec{x}) \frac{d\vec{u}(\vec{x}) \wedge (\vec{x}_0 - \vec{x})}{4\pi |\vec{x}_0 - \vec{x}|^3}
\]

where \( \nabla_{\vec{x}_0}(\cdot) \) denotes differentiation with respect to \( \vec{x}_0 \). It is well known that as \( \vec{x}_0 \) approaches \( \Sigma \) the velocity becomes discontinuous. In particular if \( [\vec{u}_{\mu}] \) denotes the velocity discontinuity defined on \( \Sigma \) we have:

\[
[u_{\mu}](\vec{x}) \cdot \vec{v}(\vec{x}) = 0, \ \vec{x} \in \Sigma
\]

\[
\vec{v}(\vec{x}) \wedge [\vec{u}_{\mu}](\vec{x}) = \nabla_{\vec{x}} \mu(\vec{x}) \wedge \vec{v}(\vec{x}) = \vec{\gamma}(\vec{x}), \ \vec{x} \in \Sigma
\]

where \( \vec{\gamma}(\vec{x}) \) denotes the intensity of the surface vorticity and \( \nabla_{\vec{x}}(\cdot) \) the superficial differential operator. From Q.5 and Q.6 we deduce that a surface on which a dipole distribution is defined, corresponds to a vortex sheet, i.e. a surface with tangential velocity discontinuity.

Following the notations defined above, by means of Green's theorem the following representation theorem for the velocity potential \( \phi(\vec{x}_0, t), \vec{x}_0 \in D, \ t \geq 0 \) is
obtained:

\[ \Phi_k(\bar{x}_0; t) = -\sum_{k=1}^{N_B} \int_{S_k} \mu_k(\bar{x}; t) \cdot \frac{\bar{\nu}(\bar{x}) \cdot (\bar{x}_0 - \bar{x})}{4\pi |\bar{x}_0 - \bar{x}|^3} dS(\bar{x}) \quad (I) \]

\[ -\sum_{k=1}^{N_B} \int_{S_{W_k}} \mu_{W_k}(\bar{x}; t) \cdot \frac{\bar{\nu}_{W_k}(\bar{x}; t) \cdot (\bar{x}_0 - \bar{x})}{4\pi |\bar{x}_0 - \bar{x}|^3} dS_{W_k}(\bar{x}) \quad (II) \]

where,

- \( \mu_k(\cdot ; t) \) denotes the dipole distribution of the \( k \)-th thin lifting surface (term I),
- \( \mu_{W_k}(\cdot ; t) \) the dipole distributions of the vortex sheet originating from the \( k \)-th thin lifting surface (term II).

Due to the unsteady character of the inflow conditions, the unknown distributions \( \mu_k(\cdot ; t) \) and \( \mu_{W_k}(\cdot ; t) \) are time dependent. Besides that, as the vortex sheets \( S_{W_k} \) are freely moving material surfaces, the geometry of the problem is also time varying. Consequently, the problem to be solved is a free-boundary evolution problem with unknown surface distributions \( \mu_k(\cdot ; t) \) and \( \mu_{W_k}(\cdot ; t) \) as well as the geometry of the vortex sheets \( S_{W_k} \). In order to determine the unknown fields of the problem we dispose two types of conditions:

(a) the kinematic ones and more specifically the non-entry conditions on all the solid surfaces, and the conditions of material motion of the vortex sheets.

(b) the dynamic conditions, i.e. the requirement of zero pressure jump throughout the vortex sheets.

Let \( \bar{U}_B(\cdot ; t) \) denote the rigid body velocity distribution defined on the solid boundaries of the configuration. Then the non-entry conditions on all the solid surfaces take the form:

\[ \frac{\partial \Phi_k}{\partial \nu}(\bar{x}; t) = \bar{\nu}(\bar{x}; t) \cdot [\bar{U}_B(\bar{x}; t) - \bar{U}_s(\bar{x}; t)], \quad \bar{x} \in S_k(t), \quad k = 1(1)N_B \quad (Q.8) \]

The application of the Neumann condition Q.8 for the dipole formulation of the potential \( \Phi(\cdot ; t) \) given by Q.7 leads to a Fredholm equation of the 1st type, requiring the evaluation of integrals in the Cauchy principle value sense. However, relation Q.5 permits a more flexible approach especially if a piecewise constant approximation of the dipole distributions is chosen. In this case the first term in the right hand member of Q.5 is equal to zero. The remaining term in Q.5 is the contribution of a line vorticity distribution (usually termed as vortex lattice) defined on the boundary \( \partial \Sigma \) of \( \Sigma \). Thus in the case of a piecewise constant approximation of the dipole distributions, the velocity induced by a thin lifting surface is obtained through summation of the contributions of the vortex lattices corresponding to the elements of the grid used for \( S_k \), and \( S_{W_k} \), \( k = 1(1)N_B \). Let \( S_{\xi}, \xi = 1(1)E_k \) and \( S_{W_{\xi}}, \xi = 1(1)E_{W_k} \), denote the elements of \( S_k \) and \( S_{W_k}, k = 1(1)N_B \) respectively \( (E_k \) and \( E_{W_k} \) are the number of elements that form \( S_k \) and \( S_{W_k} \) respectively). The boundaries of these elements will be denoted by \( \partial S_{\xi} \) and \( \partial S_{W_{\xi}} \) respectively (Figure Q.3). Since \( \mu_k(\cdot ; t) \) and \( \mu_{W_k}(\cdot ; t) \) are chosen to be piecewise constant, let \( \mu_k^i(\cdot ) \) and \( \mu_{W_k}^i(\cdot ) \) denote respectively their intensities for every element.

According to the above remarks the kinematic condition Q.8 takes the form:

\[ \bar{x} \in S_B : \bar{\nu}(\bar{x}_0; t) \sum_{k=1}^{N_B} \sum_{i=1}^{E_k} \mu_k^i(t) \int_{\partial S_{\xi}^i} d\bar{M}(\bar{x}) \wedge (\bar{x}_0 - \bar{x}) = \frac{4\pi |\bar{x}_0 - \bar{x}|^3}{4\pi |\bar{x}_0 - \bar{x}|^3} \]

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\[ -\nu(\varepsilon_0; t) \sum_{k=1}^{N_B} \sum_{e=1}^{E_k} \mu_{\nu k}(t) \int_{S_{\nu k}} \frac{dH(\varepsilon) \wedge (\varepsilon_0 - \varepsilon)}{4\pi |\varepsilon_0 - \varepsilon|^3} + \]

\[ + \vartheta(\varepsilon_0; t) \cdot [\tilde{U}_D(\varepsilon_0; t) - \tilde{U}_L(\varepsilon_0; t)] \]

Condition Q9 will be used to give the discrete equations for the unknown intensities \( \mu_{\nu k}(t) \), \( e = 1(1)E_k \), \( k = 1(1)N_B \). This is done by applying Q9 to the centers \( \varepsilon_0^k \) of the elements that form the solid boundaries \( S_k \), i.e. for \( \varepsilon_0^k = \varepsilon_0^k \). As regards the contribution of the wakes, note that in equation Q9 all the corresponding terms appear in the right hand side of the equation as if they were considered given. In fact these terms represent the dynamic excitation of the flow. Thus if we split the kinematic from the dynamic part of the problem, then the wake terms will become part of the forcing of the kinematic part. As the remaining unknowns \( \mu_{\nu k}(t) \) are closely related to the circulation distribution of the corresponding lifting bodies, it is clear that they can be defined only by means of the dynamic conditions of the problem.

Let,

\[ \varepsilon_0^k \in S_0 : \varepsilon_0^k = \varepsilon_0(\xi_1^k, \xi_2^k, t), \xi_1^k \in [-1, 1], \xi_2^k \geq 0, t \geq 0 \quad (Q.10) \]

denote a parametric representation of a vortex sheet \( S_0 \) shed from a lifting component of the configuration along its trailing and possibly its tip edges, i.e. the vorticity emission line (Figure Q.4). Clearly \( S_0 \) can be regarded as a surface generated by the sequence of material lines leaving the emission line. In order to keep track with the history of the vortex shedding, a point \( \varepsilon_0^k(\xi_1^k, \xi_2^k, t) \) is identified as the position at time \( t \) of a material element that was shed at time \( \xi_2^k \) and at the point along the emission line defined by \( \xi_1^k \). Consequently \( \varepsilon_0^k(\xi_1^k, t; t) \) represents the current position of the emission line. Moreover the lines \( \xi_1^k = ct \) are formed by the material elements shed by the same point of the emission line.

Having defined the history of the vortex sheets, the zero pressure jump condition can be recast in an explicit form, permitting the determination of the remaining unknowns \( \mu_{\nu k}(t) \). In this connection let us first consider the dynamics of a vortex sheet \( \Sigma(t) \), i.e. a moving surface carrying a dipole distribution \( \mu(\cdot; t) \). Let

\[ \xi \in \Sigma(t) : \varepsilon_\Sigma(\xi; t), \xi = (\xi_1, \xi_2) \in \mathbb{R}^2, t \geq 0 \quad (Q.11) \]

denote the Lagrangian representation of \( \Sigma(t) \). The evolution of \( \Sigma(t) \) is defined by its equation of motion Q.12a and the zero pressure jump condition Q.12b obtained by applying Bernoulli's equation to the two faces of \( \Sigma(t) \):

\[ \frac{d\varepsilon_\Sigma(\xi; t)}{dt} = \tilde{U}_m(\varepsilon_\Sigma; t) \]

\[ \frac{\partial}{\partial t}([\tilde{U}_m(x; t) - [\tilde{u}_m]([\varepsilon_\Sigma; t) = 0 \quad (Q.12) \]

In Q.12 \( \tilde{U}_m(\cdot; t) \) and \([\tilde{u}_m](\cdot; t) \) denote the mean and the jump of the velocity field, both defined as surface fields, i.e. for points of \( \Sigma(t) \) only. From Q.3 and Q.6b we have that, for \( \xi \in \mathbb{R}^2, t \geq 0, \mu(\xi; t) = -|\psi| \xi; t \) and \([\tilde{u}_m](\xi; t) = \nabla \Sigma \mu(\xi) \). Thus if,

\[ \frac{d\mu}{dt}(\cdot) = \frac{\partial}{\partial \varepsilon}[\tilde{U}_m \cdot \nabla \Sigma](\cdot) \quad (Q.13) \]
is the superficial material time derivative, then Q.12b yields the following condition:

$$\frac{d_{\mu} \mu}{dt} (\cdot) = 0 \quad (Q.14)$$

It follows from condition Q.14 that the dipole distribution defining a vortex sheet is conserved materially and thus Q.14 is equivalent to Kelvin's theorem. As regards the case of a vortex sheet S\textsubscript{W} shed from a lifting body condition Q.14 can be used in two ways.

At first in accordance to the time history defined by Q.10, we obtain:

$$\mu_W (\vec{x}_W (\xi^1, \xi^2; t); t) = \mu_W (\vec{x}_W (\xi^1, \xi^2; \xi^3); \xi^3) \quad (Q.15)$$

Equation Q.15 simply states that the intensity of the dipole distribution carried by the material element \( \xi \) is equal to the value this element had when it first shed from the emission line of the body. Provided that a piecewise constant approximation of the dipole distribution is used, the application of Q.15 to all the surface elements \( S_{W_k} \) of the wakes, will reduce the remaining unknown degrees of freedom to the values of \( \mu_W (\cdot; t) \) along the corresponding emission lines only. Moreover condition Q.14 can be used concerning the emission itself. In this case and for piecewise constant approximation of the dipole distributions, condition Q.14 leads to equal values of \( \mu_W (\cdot; t) \) and \( \mu_W (\cdot; t) \) for the elements adjacent to the emission line. Consequently the system of equations for the unknown fields \( \mu_s (\cdot; t) \) and \( \mu_W (\cdot; t) \) is completed.

In order to conclude the formulation of the problem, we have to add the equations of motion for the vortex sheets defining the wakes of the lifting components of the configuration:

$$\frac{d\vec{x}_W}{dt} = \vec{U}_s (\vec{x}_W; t) + \nabla \phi (\vec{x}_W; t) - \vec{U}_p (\vec{x}_W; t) \quad (Q.16)$$

Theoretical results as well as experimental and numerical evidence suggest that in time, a free vortex sheet \( S_W \) looses its smoothness because of the singular character of the integrals involved in the calculation of \( \nabla \phi (\vec{x}_W; t) \). In order to overcome this difficulty a generalization of the vorticity field is introduced. Based on Q.6 the generalized vorticity field associated with a vortex sheet can be defined:

$$\omega (\vec{x}; t) = \nabla \wedge \vec{u}_s (\vec{x}; t) = \frac{\partial_s (\vec{z} - \vec{z}_c)}{	ext{surface vorticity}} \cdot \left[ \nabla \mu (\vec{z}_\Sigma; t) \wedge \vec{u} (\vec{z}_\Sigma; t) \right] + \frac{\delta_{\Sigma} (\vec{z} - \vec{z}_\Sigma; t) \vec{F} (\vec{z}_\Sigma; t)}{	ext{line vorticity}} \quad (Q.17)$$

where \( \delta_{\Sigma} (\cdot) \) and \( \delta_{\Sigma S}(\cdot) \) denote the surface and line Dirac functions defined on the interior and the boundary of \( \Sigma (t) \) respectively and \( \vec{F}(\cdot, t) \) the unit tangential to \( \partial \Sigma \) vector (Figure Q.2). It is noted that if \( \mu (\cdot; t) \) is constant then there is no surface term. The above generalization permits the application of the vortex particle approximation. More specifically the surface and line vorticities carried by the wake surfaces \( S_{W_k} \) are considered as generalized spatially distributed vorticity.
Q.1.2 The Computational Model

Since the problem is formulated in time, a time marching scheme was defined. Let $\Delta t$ denote the time step of the scheme. According to the analysis given in the previous section, all information concerning the vortex sheets of the flow, is known from the previous steps, except of the near part, i.e. the part generated during the current time step. Consequently different approximations can be used for the near ("new" part) and the far ("old" part) region of the free vortex sheets. More specifically the vortex sheet assumption is retained only for the near region of every wake. On the contrary the rest, i.e. the "old" part, is transformed into free spatial vorticity, in the sense that a vortex particle approximation is introduced.

In this connection, let $S^N_{W_k}, S^W_{W_k}, k = 1(1)N_B$ denote the near and far part respectively of the vortex sheets of the $k$-th lifting surface (Figure Q.5). Accordingly the wake potential (term (II) in Q.7) is decomposed into two parts: the potential $\Phi^N_{W}(\cdot; t)$ induced by the near parts and the potential $\Phi^W_{W}(\cdot; t)$ induced by the far parts of all the vortex sheets:

\[
\Phi(\vec{x}; t) = \Phi + \Phi^N_{W} + \Phi^W_{W}
\]

\[
\Phi^N_{W} = \sum_{k=1}^{N_B} \Phi^N_{W_k}(\vec{x}; t)
\]

\[
\Phi^W_{W} = \sum_{k=1}^{N_B} \Phi^W_{W_k}(\vec{x}; t)
\]

(Q.18)

As regards the velocity calculations, it follows from Q.6 and Q.17 that $\nabla \Phi^W_{W}(\cdot; t)$ can be identified to the rotational part of the flow $\vec{U}_w(\cdot; t)$:

\[
\vec{U}_w(\vec{x}_0; t) = \int_{D_w(\vec{x}_0)} \frac{\vec{\omega}_W(\vec{x}; t) \land (\vec{x}_0 - \vec{x})}{4\pi |\vec{x}_0 - \vec{x}|} dD(\vec{x})
\]

(Q.19)

where $D_w(\cdot; t)$ denotes the support of the free vorticity $\vec{\omega}_W(\cdot; t)$ given by

\[
\vec{\omega}_W(\vec{x}; t) = \nabla \land \vec{U}_w(\vec{x}; t)
\]

\[
= \sum_{k=1}^{N_B} \sum_{\epsilon=1}^{E_{W_k}} \delta_{\vec{x}_0}(\vec{x} - \vec{x}_{W_k}) \cdot \mu_{W_k}(t, \vec{x}_{W_k}; t)
\]

(Q.20)

Note that in Q.20 only the line vorticity terms are included. This is due to the piecewise approximation of the dipole distributions $\mu_{W_k}(\cdot; t)$.

The above interpretation of $\nabla \Phi^W_{W}(\cdot; t)$ leads to some modifications of equations Q.7 and Q.11. At first in Q.7 $\Phi(\cdot; t)$ should be identified with the sum $(\Phi + \Phi^N_{W})(\cdot; t)$ of Q.18. This means that wherever the contribution of the wakes appears, it should be restricted only to the near parts $S^N_{W_k}$. Finally in Q.7 as well as in Q.11 $\vec{U}_w(\cdot; t)$ should be included into $\vec{U}_w(\cdot; t)$.

As regards the discrete problem, $\vec{U}_w(\cdot; t)$ as well as its evolution is approximated by means of the vortex particle approximations of the form:

\[
\vec{\omega}_W(\vec{x}; t) \cong \sum_{j \in J(t)} \vec{\omega}_j(t) \cdot \vec{\zeta}_j(x - \vec{z}_j(t))
\]

(Q.21)
where $\tilde{\Omega}_j(t)$ and $\tilde{Z}_j(t)$ denote the intensities and positions of the vortex particles, $J(t)$ the index set for the vortex particles and $\zeta_{\epsilon}(r)$ the cut-off function (Q.21):

$$\zeta_{\epsilon}(r) = \frac{1}{\epsilon^2} \exp(-\epsilon^2 r^2)$$  \hspace{1cm} (Q.22)

Using Q.21, $\tilde{U}_\omega(\cdot; t)$ takes the form:

$$\tilde{U}_\omega(\vec{x}; t) = \sum_{j \in J(t)} \frac{\tilde{\Omega}_j(t) \wedge [\vec{x} - \tilde{Z}_j(t)]}{4\pi |\vec{x} - \tilde{Z}_j(t)|^3} f_{\epsilon}(\vec{x} - \tilde{Z}_j(t))$$  \hspace{1cm} (Q.23)

where

$$f_{\epsilon}(r) = 1 - \epsilon^2 r^2$$  \hspace{1cm} (Q.24)

Thus instead of calculating the geometry of the vortex sheets and the dipole distributions they carry, we follow the evolution of the vortex particles defined by the following dynamic equations:

$$\frac{d\tilde{Z}_j(t)}{dt} = \tilde{u}(\tilde{Z}_j; t) - \tilde{U}_B(\tilde{Z}_j; t), \; j \in J(t)$$  \hspace{1cm} (Q.25)

$$\frac{d\tilde{\Omega}_j(t)}{dt} = (\tilde{\Omega}_j(t) \cdot \nabla) \tilde{u}(\tilde{Z}_j; t) - \frac{1}{2} \tilde{\Omega}_j(t) \wedge (\nabla \wedge \tilde{U}_B(\tilde{Z}_j; t))$$  \hspace{1cm} (Q.26)

Equations Q.25 and Q.26 concern the evolution of the far parts of the wakes. As the near parts still retain their character as vortex sheets their determination is different. Let $\bar{U}_{en}$ denotes the mean velocity at a point $\bar{x}_{en}$ along the vorticity emission line of a lifting body. The geometry of the near part of the corresponding wake $S_{ny}^N$ is determined kinematically through the following relation:

$$\bar{x}^N = \bar{x}_{en} + \Delta t \cdot \bar{U}_{en}$$  \hspace{1cm} (Q.27)

where $\bar{x}^N = x_{en}$ denotes the width of $S_{ny}^N$, in vectorial form (Figure Q.5). Finally, the intensity of the dipole distribution of $S_{ny}^N$ is determined by means of condition Q.14.

Due to the time dependent character of the problem, the wakes as well as the vortex particles they include in their far parts, will be constructed gradually. This means that vortex particles will be created as the near parts of the wakes evolve. In order to make this approach compatible with the dynamics of vortex sheets, $\tilde{\Omega}_j(t)$ and $\tilde{Z}_j(t)$ are defined as follows:

$$\tilde{\Omega}_j = \int_{p_j} \omega_w \, dS \quad \tilde{\Omega}_j \wedge \tilde{Z}_j = \int_{p_j} \omega_w \wedge \vec{S} \, dS$$  \hspace{1cm} (Q.28)

In the above relations, the integration covers for every vortex particle the surface of an element of the near part of the wake considered.

THE FLOW CHART OF THE GENUVP MODEL

For every time step ($H - I = n \cdot \Delta t$)

A. POTENTIAL CALCULATIONS

$$\vec{u} = \tilde{U}_{en} + \nabla \phi + \nabla \phi_{\omega}^N + \vec{U}_\omega$$
Joint investigation of dynamic inflow effects and implementation of an engineering method

0 Initialize $S^\theta_0$ and $\Phi^\theta_0$ (H-through the values of along the emission lines of the lifting bodies)

Iterative schemes for the near wake:
1 Calculate $\Phi$ (H-fulfillment of non-entry boundary conditions)
2 Calculate the emission velocities $\vec{U}_{em}$ along the emission lines
3.1 Correct $S^\theta_0$
3.2 Correct $[\Phi]^\theta_0$ (H-fulfillment of the Kutta condition)
4. Check for convergence: $\delta|\Phi|_0 < \epsilon$
5. FIRST OUTPUT: Force and velocity calculations.

B. VORTICITY CALCULATIONS
1. Create new vortex particles
2. SECOND OUTPUT: Wake structure and velocity profiles in the wake
3. Move and deform all the Vortex Particles
3.1 Calculate the velocities and deformations induced at the Vortex Particle locations
3.2 Check for Vortex-Solid surface interaction and correct accordingly
3.3 Produce the new far-wake

Q.1.3 The calculations of loads
The calculation of the unsteady loads on the rotor blades was based on Bernoulli's equation. For a thin lifting surface this equation takes the form:

$$\frac{\partial \phi}{\partial t}(\vec{x}; t) + \vec{U}_{m}(\vec{x}; t) \cdot [\vec{u}](\vec{x}; t) = -\frac{[p](\vec{x}; t)}{\rho}$$  \hspace{1cm} (Q.29)

where $-\phi(= \mu)$ denotes the surface dipole distribution, $\vec{U}_{m}(\cdot; t)$ denotes the mean velocity, $-\phi(\cdot; t) = \nabla \phi$ denotes the tangential velocity discontinuity and $[p](\cdot; t)$ the pressure jump. In the discrete problem equation Q.29 is applied to the control points. Due to the piecewise approximations used for the dipole distributions, $[\tilde{e}]$ is calculated through a zero-order difference scheme [NTUA.18]. Let $[p]^k_{e}, \ e = 1(1)E_k, \ k = 1(1)N_E$, denote the calculated values of the pressure jump. Integrating over the corresponding elements, the calculation of the loads is obtained as follows:

for every element, $\vec{F}^k_e(t) = -\vec{v}^e \cdot [p]^k_e \cdot |S^e_k|$  

for every body, $\vec{F}_e(t) = \sum_{e=1}^{N_E} \vec{F}^k_e \cdot [p]^k_e \cdot |S^e_k|$  

where $|S^e_k|$ denotes the area of $S^e_k$.

Due to the essentially inviscid character of the modelling only the "inviscid" part of the loadings can be predicted. The rest, i.e. the loads due to viscous effects must be superimposed. In this connection an approximate a-posteriori scheme, similar to the one used by the classical strip theory was applied. This scheme is based on the calculation of the radial distribution of the effective angle of attack. Next the drag coefficient $C_D$ is estimated from given airfoil data. Finally the drag forces are integrated over radial strips and thus an account of viscous effects is obtained.

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Q.2 Evaluation of the model

Q.2.1 Remarks on the computational characteristics of GENUVP

Vortex Particle Methods (VPM), as any other numerical method used in mechanics, are identified by the numerical characteristic scales of the simulated field they reproduce. In our case the "simulated field" is the velocity field of the flow. Since the whole approach is time dependent, we have as characteristic scales the time as well as the length scales of the flow $T_{ref}$ and $L_{ref}$ respectively.

The time scale $T_{ref}$ depends basically on the time step used in the time-marching integration scheme. The dependence of $L_{ref}$ on specific numerical parameters, is less straightforward to determine. In principle $L_{ref}$ is related to the "spacing" of the numerical points where the equations of the problem are approximated. For differential or grid-using methods $L_{ref}$ is a measure of the grid spacing whereas for Lagrangian or material methods that are grid-free, $L_{ref}$ is proportional to the mean distance between the "material" numerical points, i.e. the "pointers" that identify the flow. Keeping these remarks in mind, we shall try to give the various links that indirectly define $L_{ref}$ in the case of the VPM used by GENUVP.

The difficulty in defining $L_{ref}$ is due to the fact that GENUVP is in between of grid-free and grid-using methods. In fact it is grid-free as regards all the field equations of the problem. As field equations we consider the Laplace equation for the perturbation potential as well as the Helmholtz evolution equations for the vorticity field. It is worth noticing that the Laplace equation covers the kinematic part of the problem whereas the Helmholtz equations the dynamic one. Of course there are boundary and initial conditions that complete the formulation of the problem. It is for the approximation of both of them that spatial grids are used.\footnote{It is noted that as regards initial conditions, there is need of a spatial grid only if at t=0 the vorticity field is not zero. For Wind Energy applications this is the case if a Wind Turbine is placed in the wakes of a preceding one. In all the cases considered up to now with in the "Dynamic Inflow" project this situation was not encountered.}

As regards the non-entry boundary condition, a Boundary Element Grid is defined on the blades. In addition to this grid, a second grid for the "near" part of the wake is used that concerns the Kutta condition. The sizes of these grids are given for example by the mean value of the square roots of the areas of the elements they contain. Let $L_{B}$ and $L_{nw}$ denote the sizes of the grids defined on the blades and the near wakes respectively. The maximum of the two will provide a measure of the numerical length scale of the flow near the blades. Furthermore $L_{B}$ and $L_{nw}$ are closely related to $T_{ref}$. This is so because the geometry of the wakes is generated by shedding strips of vorticity (near wakes) from the blades with a "stream-wise" width equal to $\{\vec{u}(\vec{x}_{nw}) - \vec{U}(\vec{x}_{nw})\} \cdot \Delta t$. The spacing in the "span-wise" direction of the wake coincides with the discretisation of the grid on the blade.

Within the wake the length scale is given as the mean distance between the vortex particles it contains. Let $L_{jw}$ denote its length. Due to the Lagrangian description of the wake, for a given time-step $\Delta t$, $L_{jw}$ will give a measure of the velocity in
the wake. Because the wake region is a vortical flow that is solved as inviscid, the velocity must follow what is expected from the corresponding theory. It is well known that for three-dimensional inviscid but vortical flows the corresponding evolution equations (i.e., the Euler equations) blow-up. So it is expected that $L_{cut}$ will tend to infinity. That is why VPM makes use of regularizing functions, known as cut-off functions. These functions represent smooth interpolants of the Dirac functions. They are defined in order to give optimum error. The defining parameter is the cut-off length.

Concluding we see that $L_{cut}$ depends not only on the grid spacing but also the time step and the cut-off length.

Q.2.2 Computational effort of GENUVP

According to the above description and the flow chart of the computational model the arithmetic calculations are mainly composed of two parts:
- computations of induced velocities from distributions of singularities (vortex filaments, vortex particles)
- construction of the main matrix and solution of the linear system for the kinematic problem.

The later, for the case of wind turbine rotors is quite less computationally expensive than the former. The number of degrees of freedom that concern the kinematic problem are equal to the total number of the elements of the grid of the blades which for the cases analyzed in this project, never exceeded four hundreds. On the contrary the number of vortex particles is much greater. If $N_{emiss}$ is the total number of the free vorticity emission points, the number of velocity calculations from one particle to another that take place at time-step $NT^*$, is equal to $(NT^* N_{emiss})^2$. So, in order to perform runs $NT^*$ timesteps long, the total number of such elementary calculations comes up to the number of $(NT^* N_{emiss}^2/2)$, which is actually the parameter that places the upper limit of the computational power requirements of GENUVP.

Q.2.3 Testing of the accuracy of GENUVP

According to the above discussion, the arithmetic parameters that govern the computational behaviour of GENUVP are:
- the density of the grid defined on the solid boundaries
- the time step
- the cut-off length in the wake.

Detailed tests have shown that grids with chordwise dimension greater than 7 and spanwise dimension greater than 9 give grid independent predictions for the global loading characteristics on the rotor (Axial force, torque, etc.). Further refinement leads to approximations that are only locally more detailed. As an example, we give in Figure Q.6 the distribution of the bound vorticity for the blade of the DUT model. The finer grid just reproduces better the curve near the free edges i.e. the root and the tip.

Regarding the choice of the time step, the limit for satisfactory predictions is the division of the rotation into 18 steps. Smaller steps produce insignificant
As already mentioned, the cut-off function is a necessary ingredient of the vortex particle method that regularizes the evolution of the wake. In a more simplified approach, the cut-off function transforms point vortices into vortex blobs (or small spheres). The interior of the blob represents the region where the cut-off function really acts on the velocity field. The extent of this region is defined by the cut-off length. According to Beale and Majda [NTUA.21] for second order cut-off functions as the one used in GENUVP, the optimal numerical error is accomplished when the cut-off length is related to the length scale as: \( \epsilon = h^\alpha \) with \( 3/5 \leq \alpha \leq 1 \). \( h \) is a non-dimensionalized length scale and therefore it takes values less than 1. This means that the cut-off length must be thus chosen so as the vortex blobs overlap.

When chosen reasonably, the choice of the cut-off length little influences the results in an overall sense. Our experience showed differences only locally and at scales comparable to the cut-off length. This can be clearly seen in Figure Q.8 where the time-traces of the velocity downstream of the DUT rotor are given for different cut-off lengths. The predictions differ from one another only when a vortex particle comes close to the point of calculation.

Q.2.4 Testing of the physical consistency of GENUVP

Stability Characteristics of Vortex Rings

One of the classical problems in fluid mechanics is that of the behaviour of vortex rings and in particular their stability at circumferential perturbations. Vortex rings are in fact concentrations of vorticity around a circle. Their kinematic and dynamic behaviour has been extensively investigated by different researchers. Interesting experimental as well as analytic results exist that quantify the motion and stability of vortex rings. An interesting aspect of these results is the finding that the velocity field produced by vortex rings is to a large extent amenable to inviscid flow theory. That is one of the reasons that make the case of vortex rings a suitable test case for inviscid numerical models. Also important is the absence of any solid boundary which means that the testing is concentrated to the vortical part of GENUVP. Finally it is noted that vortex rings have a close geometrical connection with the wakes of wind turbines that makes the whole subject interesting from a practical point of view also. In order to evaluate the capability of GENUVP to predict the dominating physical mechanisms of vortex flows, the model was applied to the problem of the linear stability of a perturbed vortex ring and of the head-on collision of two vortex rings [NTUA.22]. Herein some indicative results are presented that concern the stability problem.
The vorticity distribution is initially concentrated around the centerline of a ring of radius \( R \) and on circular sections of radius \( \alpha \), so as a constant circulation \( \Gamma \) is found (Figure Q.9). We initially imposed on the ring a radial perturbation of amplitude \( \sigma \), wave number \( n \) and wave length \( L = (2\pi R)/2 \). Within the framework of the linear stability theory, the vortex core should be small compared to the wave length of the perturbation and to the vortex ring radius. The assumption of a constant vorticity distribution on the section of the ring permits to neglect the small local instabilities of the vortex core. The initial amplitude of perturbation was \( \sigma/R = 0.02 \) and the number of vortex particles was \( N = 120 \). According to theory [NTUA.23], \( n = \alpha = 6 \) is the neutrally stable mode, \( n = \alpha + 1 = 7 \) is the unstable mode and for all other wave number we get a periodic response (\( n = 8 \) was selected as being closer to the unstable mode). For \( \sigma/R = 0.2 \), \( \Gamma/R = 2 \) and \( n = 6, 7, 8 \) the evolution of the radial and axial perturbation of the centerline of the ring is presented in Figure Q.10. These results agree well with theory given by Widnall and Sullivan.

**Velocity time-traces behind a model rotor**

Most of the cases considered in the project concerned full scale measurements. Although extremely important for applications, full scale measurements have always the disadvantage of not allowing a tight control of the conditions of the experiment. On the contrary, in the case of wind tunnel tests, the conditions of the experiments are more or less controllable. That is why special attention was given to the wind tunnel tests that were performed by DUT.

**DUT provided** two sets of time series of the measurements performed in the wake of the two-bladed rotor. Both sets concern axi-symmetric cases that correspond to a steady state operation (no inflow shear, no yaw, no gust). The tip-speed-ratios for the two series were 6 and 8 respectively whereas measurements were recorded at four downstream positions: \( x/R = 0.053, 0.070, 0.103 \) and 0.137 and for ten radial positions \( y/R = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 1., 1.05 \) and 1.1. For these two tests, systematic calculations were performed with GENUVP that we collectively present in the next section. The calculations were performed with:

- a uniformly spaced grid of 20x8 panels (20 spanwise and 7 chordwise)
- a time step that divides each rotation into 48 steps (this choice was also suggested by other partners of the projects)
- a constant in time and space cut-off length, equal to 1/100 of the diameter of the rotor.

The grid was chosen relatively dense so as to alleviate any doubt regarding the space accuracy. The time step was chosen relatively small, although an even smaller time step could probably give more information. This was largely imposed by the time limits set for this piece of work.

Figure Q.11 gives the velocity time traces at the radial positions \( y/R = 0.8 \), \( y/R = 1.05 \) and at the downstream positions \( x/R = 0.053 \), \( x/R = 0.137 \) for tip-speed ratio equal to 6. In every diagram the sum of the axial and radial components of the velocity field is traced as a function of the azimuthal angle that corresponds to the current position of the "first" blade. The hot-wire is always fixed at the 90° azimuthal position. That is why the peaks always appear around the 90° and 270°.
As regards the results themselves and more specifically the comparison between measurements and predictions the following comments can be made:

Comment 1: In general, the predictions agree well with the trends of the measurements. However, there is a systematic phase delay of the predictions with respect to the experiments. For the numerical tests there is no doubt that the leading edge vortex induces a significant part of the total velocity. This is because the blade is represented by a thin lifting surface. If a thick blade approximation was used, then a slightly phase-shifted velocity field would result.

Comment 2: As regards the time step, it is clear that the specific choice cuts out the numerical peaks (see the region where the peaks appear). This suggests that a smaller time step would produce a better fit.

Comment 3: Separation takes place at the inboard stations near the root. This is clearly shown on one hand by the intense fluctuations of the velocity just after the blade has passed from the probe, and on the other by the calculation of the angles of attack. These fluctuations, which probably correspond to the passing of large scale eddies, are not predicted, simply because the numerical model assumes attached flow.

Q.3 Conclusions, limitations, possible extensions

The results show that GENUVP is a promising numerical model that can be trusted as a tool for investigation of unsteady phenomena related to Wind Turbines. However, there are two deficiencies that must be alleviated:

- The first concerns stall, an option that is currently in development.
- The second concerns thickness effects, which is an option already in GENUVP. Unfortunately, time did not permit to add results of this kind. Such an investigation is foreseen in the future.

Q.4 References


Joint investigation of dynamic inflow effects and implementation of an engineering method


NTUA.11 Maskew, B. (1980) "Predicting aerodynamic characteristics of vortical flows on three-dimensional configurations using a surface singularity panel method", AGARD C.P.208


NTUA.13 Wagner, S.N., Schout, Ch. (1990) "Validation of the aerelastic simulation program for horizontal axis wind energy converters", Proc. ECWE'90, p.296, Madrid, Spain (See also the references cited therein).


Figure Q.1  The basic notations

Figure Q.2  Equivalence between a dipole and a surface vorticity distribution
Joint investigation of dynamic inflow effects and implementation of an engineering method

Figure Q.3 The notations of the grid used

Figure Q.4 The notations for the wake of a lifting surface
Figure Q.5 *The hybrid scheme*

Figure Q.6 *Calculations for two different grid spacings*
Figure Q.7  **Step down calculations with three different time-steps**

Figure Q.8  **Velocity time-traces calculations with various cut-off function lengths**
Figure Q.9  Basic notations of vortex rings

Figure Q.10  History of the geometry of a vortex ring for several initial perturbations
Figure Q.11  Velocity time-traces behind DUT rotor
APPENDIX R. LINEARISATION OF INDUCED VELOCITY TERMS

The first preliminary calculations consisted from simple pitching steps and coherent wind gusts. A remarkable finding was that the dynamic inflow effect appeared to be much smaller for coherent wind gusts than for pitching steps, see section 8.3.2. This was explained by the relatively small change in induced velocities that result from the gusts which were considered. In order to get more insight into the reason why the change in induced velocity is so small, a linearized expression for the change in induced velocity as function of the change in wind speed is derived. Therefor, the blade element impulse relations are written in the following form, where the tangential induction factor, the profile drag and the tip losses are neglected for simplicity:

\[ 4u_i(V_\infty - u_i) = \sigma W^2 \alpha_1 \cos \phi \]  
\[ \tan \phi = (V_\infty - u_i)/\Omega \]  
\[ c_l = c_{l,0}(\alpha = 0) + c_{l,\alpha}(\phi - \theta) \]  
\[ W = (V_\infty - u_i)/\sin \phi \]

Then the step in wind speed is written as:

\[ V_\infty = V_1 + wV_1 \]  

The resulting induced velocity, inflow angle, lift coefficient and effective wind velocity are expressed as:

\[ u_i = u_1 + u_i u \]  
\[ \phi = \phi_1 + \phi \]  
\[ W^2 = W_1^2 + \nu W_1^2 \]  
\[ c_l = c_{l,1} + \Delta c_l \]  

By substituting the equations (R.5), (R.6), (R.7), (R.8), (R.9), into the equations (R.1), (R.2), (R.3), and (R.4) and neglecting higher order terms, the following expression for the change in induced velocity \((u)\) as function of the change in wind speed \((w)\) is obtained:

\[ u = \frac{V_1}{u_1} \frac{w}{1 - (V_1 - u_1)/(u_1 + F/4)} \]  

with:

\[ F = \frac{\sigma W_1^2}{\Omega r(1 + \tan^2 \phi_1)} \left( c_{l,0} \sin \phi_1 - c_{l,\alpha} \cos \phi_1 + 2c_{l,1} \frac{\cos \phi_1}{\sin \phi_1} \right) \]  
\[ - \frac{2\sigma \cos \phi_1}{\sin^2 \phi_1} (V_1 - u_1) c_{l,1} \]  

The expression for \(F\) can be further simplified by assuming that the inflow angle \(\phi_1\) is small:

\[ \tan \phi_1 = (V_1 - u_1)/(\Omega r) \approx \sin \phi_1 \ll 1 \]  
\[ \cos \phi_1 \approx 1 \]  

Then equation (R.11) gives:

\[ F = -\sigma c_{l,a} \Omega r \]
Substituting equation (R.14) in equation (R.10) yields:

\[ u = \frac{V_1}{u_1} \frac{w}{1 - G} \]  \hspace{1cm} (R.15)

with:

\[ G = \frac{1 - a_1}{a_1 - \sigma \lambda \varepsilon \alpha / \delta} \]  \hspace{1cm} (R.16)

Equation (R.15) can also be expressed in terms of absolute changes:

\[ \Delta u = \Delta V / (1 - G) \]  \hspace{1cm} (R.17)

in which \( \Delta u \) gives the absolute change in induced velocity and \( \Delta V \) gives the absolute change in wind speed.

In the figures R.1 to R.3 results from equation (R.15) and from a blade element impulse model are presented for a step on the wind speed from 8 to 10 m/s at \( \theta = 0^\circ \). Three different turbines are considered:

- Tjæreborg turbine;
- WPS-30 turbine;
- WEG-MS1 turbine.

![Graph](image)

**Figure R.1** Tjæreborg; Non-dimensional change in induced velocity for step on the wind speed

From the figures it can be seen that the agreement between the results from equation (R.15) and the blade element impulse model is acceptable. Furthermore it can be seen that in general the change in induced velocity for the WEG-MS1 and WPS-30 is larger than for the Tjæreborg turbine.

From the equations (R.17) and (R.16) it follows that the change in induced velocity is proportional to the value of the factor:

\[ a_1 - \sigma \lambda \varepsilon \alpha / \delta \]  \hspace{1cm} (R.18)

The value of this factor appears to be decisive for the change in induced velocity. For the Tjæreborg turbine this factor is very small leading to very small changes in induced velocity. For the WPS-30 and WEG-MS1 this factor is somewhat larger.
This explains why the change in induced velocity for the WPS-30 and WEG-MS1 is larger than for the Tjæreborg turbine.

The value of the factor given in (R.18) can be approximated from equation (R.1), when a small inflow angle is assumed. Then equation (R.2) becomes:

\[ \phi \approx (1 - \tilde{u})/\lambda_c \quad \text{(R.19)} \]

while

\[ \Omega_t \approx W \quad \text{(R.20)} \]

Substituting these expressions in equation (R.1) yields:

\[ a_1 - \sigma \lambda_c c_{l,\alpha}/d = -\frac{\sigma \lambda_c^2 c_{l,\alpha}(\theta + \alpha_0)}{d(1 - a_1)} \quad \text{(R.21)} \]

with:

\[ \alpha_0 = \frac{c_l(\alpha = 0)}{c_{l,\alpha}} \quad \text{(R.22)} \]
From equation (R.21) it follows that the change in induced velocity is proportional to the value of $\theta + \alpha_0$. 
APPENDIX S. DESCRIPTION OF DUT WIND TUNNEL MODEL

- Number of blades: 2
- Diameter: 1.2 m
- Hub height: 2.33 m.
- Aerodynamic part of blade: starts at 30% R
- Aerodynamic profile: Naca0012, see section S.1
- Tilt angle: 0 deg
- Cone angle: 0 deg
- Rotational speed: 12 Hz (for the case described in this report)
- Chord: 0.08 m (constant)
- Replaceable tip: 0.06 m span, no twist
- Twist: $0.3 < r/R < 0.9 : \theta(r/R) = (\theta_{tip} + 6) - 0.67r/R$ $(\text{deg})$
  \[0.9 < r/R < 1.0 : \theta = \theta_{tip}\]
- Main blade eigenfrequencies (at stand still):
  - First flap: 33.254 Hz
  - First lead-lag: 133.87 Hz
- Structural data of the blades: See section S.2
- Additional remarks: The aerodynamic as well as the structural data are available on floppy-disc or e-mail from the coordinator.
S.1 Aerodynamic profile coefficients of Naca0012 at Re = 1.5e5

(synthesized from various sources by W.A. Timmer and L.J. Vermeer from DUT)

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S.2 Geometric and material data 1.2 m rotor

- Airfoil: NACA0012 chord length 0.08 m
- elastic axis 0.0272 m from leading edge
- centroid (centre of gravity) 0.0337 m from leading edge
- moments of inertia with respect to centroid (principal moments of inertia)
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APPENDIX T. GLOBAL AND AERODYNAMIC DATA OF TJÆREBOG TURBINE

Rotor data:
- Number of blades: 3
- Diameter: 61.1 m
- Orientation: upwind
- Tilt angle: 3 deg
- Cone angle: 0 deg
- Hub height: 61 m
- Rotor overhang: 6.76 m
- Rated electrical power: 2MW
- Rated shaft power: 2.2MW
- Synchronous rotor speed: 21.93 rpm
- Rotor speed at rated power: 22.36 rpm
- Power control: full span pitch
- Operational pitch angle: 0-35 deg
- Idling pitch angle: 55 deg
- Stop pitch angle: 90 deg

Blade geometry
- Length incl. 0.1 m tip cap: 29.1 m
- Flange distance from rotor axis: 1.46 m
- Planform: see figure T.1
- Tip chord: 0.90 m
- Taper (linear): 0.1 m/m
- Thickness: see table T.1
- Twist (linear): 0.333 deg/m
- Airfoil family: NACA 44xx
- Airfoil data: see table T.2 to T.6

Tower geometry
- Height: 56 m
- Shape upper half: conical
- Diameter at h = 56 m: 4.25 m
- Diameter at h = 28 m: 4.75 m
- Diameter at base: 7.25 m
Figure T.1  Planform and thickness of the blade
Figure T.2  Definition of ref. axis and twist.

Table T.1  Blade geometry

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Table T.6  NACA 4424 airfoil data, smooth, Re = 6 × 10^6

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APPENDIX U. STRUCTURAL DYNAMICS
DATA OF TJÆREBORH
TURBINE

U.1 Blade stiffness and mass distribution
The blades are made of glass fibre reinforced polyester with a steel attachment fitting at the root. The sectional data are defined in figure U.1 and the values are shown in table U.1. The data are based on calculations done in the design phase. However, corrections have been added to the mass distribution in order to match the measured total mass and center of mass position of the finished blades. The stiffness values have been reduced by 6% relative to the calculated values in order to match the measured natural frequencies. Following are the measured data:
- Mass = 9000 kg
- \( R_{cg} = 8.75 \text{ m} (= 7.11 \text{ m from flange}) \)
- 1. flapwise frequency = 1.17 Hz (non-rotating)
- 1. chordwise frequency = 2.30 Hz

U.2 Hub and nacelle data
Figure U.2 shows the general arrangement of the hub and the nacelle. The mass and center of mass position of the two are shown in the figure. The hub mass includes all the rotating parts in front of the main shaft flange except the blades. The corresponding moment of inertia around the main shaft is estimated to:

\[ I_{\text{hub}} = 60000 \text{ kg} \cdot \text{m}^2 \]

U.3 Main shaft and drive train data
The masses of shaft, gearbox and generator are included in the nacelle mass. The effective stiffness of the shaft is condensed into two values, a bending stiffness located at the front bearing and a torsional stiffness which includes the gearbox and the couplings. The values are estimated from the measured natural frequencies of the first asymmetric flapwise rotor mode (1.10 Hz, pitch 0°) and the first rotational mode with locked high speed shaft (0.75 Hz, pitch 90°):

\[ K_{\text{bend}} = 1.0 \cdot 10^9 \text{ Nm/rad} \]
\[ K_{\text{tern}} = 1.1 \cdot 10^9 \text{ Nm/rad} \]

If the gearbox is ignored, the dynamics of the drive train can be modelled as a torsional spring with stiffness \( K_{\text{tern}} \) and with the following moment of inertia at the generator and of the shaft simulating the inertia of generator, brake disc and gearbox combined:

\[ I_{\text{gen}} = 8.0 \cdot 10^6 \text{ kgm}^2 \text{ (ref. low speed shaft)} \]

U.4 Tower mass and frequency
The tower is made of reinforced concrete with a constant wall thickness and a diameter variation as shown Appendix T. Assuming a density of 2600 kg/m³, the mass distribution can be estimated:
• h = 56 m: $m_{\text{tower}} = 8200 \text{ kg/m}$
• h = 28 m: $m_{\text{tower}} = 9200 \text{ kg/m}$

The tower bending stiffness should be modelled in such a way that the resulting tower frequency for the complete tower matches the measured value:

$$f_{\text{tower}} = 0.81 \text{Hz}$$

The simplest model is a spring with a lumped mass at the top. The following data should give approximately the right result:

$$M_{\text{tower}} = 1.0 \cdot 10^6 \text{kg}$$
$$K_{\text{tower}} = 8.5 \cdot 10^6 \text{N/m}$$

### U.5 Drive train efficiency

The combined mechanical and electrical loss of power $P_L$ has been estimated giving the following relation between mechanical input power $P_M$ on the main shaft and the electrical power $P_E$ from the generator:

<table>
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<tr>
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<th>$P_E$ [kW]</th>
<th>$P_L$ [kW]</th>
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<td>60</td>
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<tr>
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<tr>
<td>2180</td>
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### U.6 Pitch control data

Above rated wind speed, the power output of the turbine is limited by full span pitch control. This part of the control system is basically a digital PID-controller connected to a hydraulic pitch servo. The actual values of the P- and D-terms of the controller are so small that they can be neglected for most modelling purposes. The remaining integrating term can be expressed by the following relation between electrical power and pitch rate $\dot{\theta}_p$:

$$\dot{\theta}_p = 0.02 \cdot (P_E - 2000)/(1 + \theta_p/1.5) \quad \text{(deg/s)}$$

with $P_E$ in kW and $\theta_p$ in degrees. The last term of the relation reduces the gain of the controller as the wind and pitch angle increase. Integration over time of the pitch rate produces a control value of the pitch angle $\theta_{\text{inc}}$, which acts as input to the pitch servo. The values of $\theta_{\text{inc}}$ are limited towards $0^\circ$. The pitch servo response can be simplified to a first order lag with a time constant of:

$$\tau_{\text{servo}} = 0.25 \text{s}$$

A more detailed model of the control system should take into account the transfer function of the power transducer, a digital 3-P filter in the power signal, the processing delay of the digital controller and of course the P- and D- terms mentioned above.
Joint investigation of dynamic inflow effects and implementation of an engineering method

$E_A$  torsional stiffness
$E_{I_1}$ 1. principal bending stiffness
$E_{I_2}$ 2. principal bending stiffness
$G_{I_v}$ Torsional stiffness
$m$  mass per unit length
$I_{pm}$ Mass inertia around mass center per unit length
$x_se$ Location of stiffness center aft of c/4
$x_m$ Location of mass center aft of c/4
$x_s$ Location of shear center aft of c/4
$v$ Angle of 1. principal axis to local chord
$\beta + v$ Angle of 1. principal axis to tip chord

Figure U.1 Definitions
Table U.1  **Sectional data for 30 m blade**

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<th>GIₐ [MNm²]</th>
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<th>Iₚm [kgm]</th>
<th>xₑ [mm]</th>
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) average values for R = 1.46 m to 2.96 m including steel parts
$M_{\text{hub}} = 42500 \text{ kg}$

$M_{\text{nac}} = 154000 \text{ kg}$

Figure U.2  *Nacelle lay-out*