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Uncertainties in Cup Anemometer Calibrations

Type A and Type B uncertainties

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Acknowledgement/Preface

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Frank Ormel and Erik Korterink have performed the measurements in the DNW wind tunnel in 2003 (Chapter 3.6).

Abstract

ECN Wind Energy is accredited according ISO 17025 to perform power performance measurements of wind turbines following IEC 61400-12 and Measnet. The typical results of these measurements are measured power curves of the wind turbines. These curves show the electrical power versus the wind speed. Part of the power performance analysis is the uncertainty analysis. In power performance measurements according IEC 61400-12 or Measnet, the wind speed is measured using cup anemometers. The uncertainty of the wind speed measurements depends among others on the calibration uncertainty. The cup-anemometer calibration uncertainty is divided in Type A and Type B uncertainty. The Type A uncertainty is the statistical uncertainty of the wind speed measurement and can be calculated from the wind speed measurements. The Type B uncertainty includes all uncertainties that do not have a statistical background. For example, these are uncertainties due to temperature changes, air pressure changes, transducer gain influences, and digital conversion influences, etcetera.

This document presents an overview of the type B uncertainties of a cup anemometer calibration following the Measnet "Cup Anemometer Calibration Procedure, Version 1, September 1997" and the proposal for the IEC 61400-12 that is currently under vote. For the calculations, the numbers applying to the DEWI wind tunnel are used throughout the report.

It is shown that the cup anemometer in wind tunnels has unstable behaviour due to the turbulent wakes of the cups. An analysis of uncertainties due to regression in cup anemometer calibrations is presented and compared to the standard uncertainty.

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1. Introduction

The objective of the ACCUWIND project is to provide accurate wind speed measurements using cup and sonic anemometers. Part of the objective is to improve tools and methods to assess the accuracy of cup and sonic anemometers in wind energy measurements by development and implementation of procedures for calibration, field comparison, benchmark tests and classification. Another part of the objective is to define measured wind speed for power performance measurements in a consistent way, so that uncertainties of wind speed sensors can be reduced.

The ECN power performance measurements are carried out using cup anemometers that are calibrated at DEWI (Deutsches Windenergie Institüt in Wilhelmshaven). The uncertainty analysis is an essential part of the power performance analysis. Part of the ACCUWIND project is the analysis of the accuracy of cup anemometer calibrations. In 2003 ECN calibrated ten Risø cup anemometers at DEWI (following the MEASNET procedure). DEWI reported the combined calibration uncertainty, the combined type A and type B uncertainty. Following the ISO guide [4], there are two types of uncertainties: category A, the magnitude of which can be deduced from measurements, and category B, which are estimated by other means. In both categories, uncertainties are expressed as standard deviations and are denoted standard uncertainties. For the ten Risø cup anemometers calibrated at DEWI the reported extended values (2σ) range from 0.08 m/s to 0.14 m/s at 10 m/s wind speed. The mean value of the uncertainties of the ten calibrations is $u_{combined}$ = 0.05 m/s at 10 m/s wind speed.

ECN has performed an independent analysis of the type B uncertainty of the DEWI cup anemometer calibrations (see Chapter 2). The type B uncertainty (uncertainty with no statistical background) of a cup anemometer calibration is calculated following MEASNET "Cup Anemometer Calibration Procedure, Version 1, September 1997" [1]. The calculations are based on information received from the calibration institute DEWI. The results of the type B analyses using the example MEASNET data in [1] are compared to the type B analyses according MEASNET [1] using data provided by DEWI. The report includes recommendations to improve the present formulation of the IEC 61400-12 document. The value for the type B uncertainty that is found is approximately 0.03 m/s.

Given the reported uncertainties in the calibration reports that range from 0.4 (1σ) to 0.7 (1σ) at 10 m/s wind speed and given the above type B uncertainty, the type A uncertainties range from 0.2% to 0.6%. A detailed overview of the calibrations is presented in Table 1-1. During the calibrations, the air temperature ranges from 21ºC to 21.7ºC, air pressure ranges from 1016.9 hPa to 1018.3 hPa and relative humidity from 30.6% to 32.7%.

Table 1-1. Calibration information on ten riso cup anemometers calibrated in the DEWI wind tunnel on 27-3-2003 at 10m/s.

Cup Anemometer	Tunnel Wind Speed [m/s]	Utot (1 σ) [m/s]	type B uncertainty [m/s]	Derived type A uncertainty [m/s]
Riso cup 279_03	10.219	0.045	0.03	0.03
Riso cup 278_03	10.219	0.04	0.03	0.03
Riso cup 277_03	10.217	0.05	0.03	0.04
Riso cup 276_03	10.213	0.05	0.03	0.04
Riso cup 275_03	10.213	0.07	0.03	0.06
Riso cup 274_03	10.211	0.05	0.03	0.04
Riso cup 273_03	10.221	0.04	0.03	0.03
Riso cup 272_03	10.221	0.055	0.03	0.05
Riso cup 271_03	10.230	0.045	0.03	0.03
Riso cup 270_03	10.221	0.05	0.03	0.04

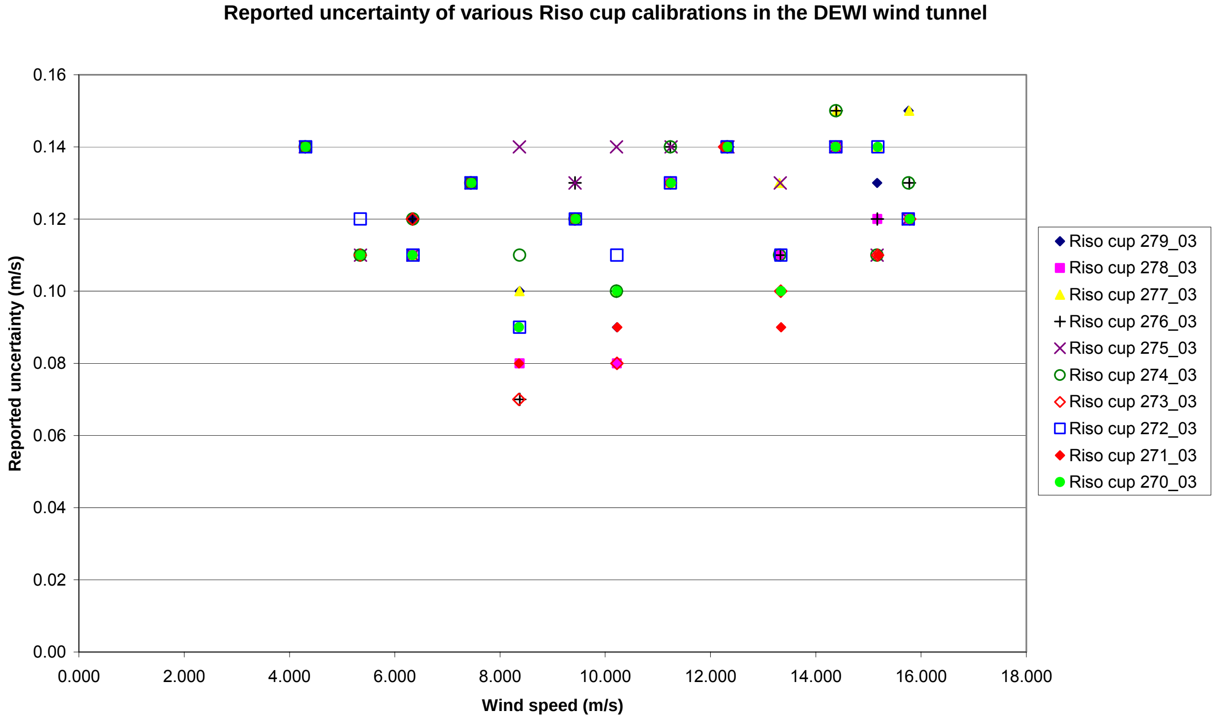

Figure 1-1. Reported uncertainty of 10 Risø cup anemometers calibrated at the same day. The extended uncertainty is presented here (2σ).

The report is organised as follows:
In Chapter 2 the analysis of the type B uncertainty of the DEWI wind tunnel is presented. Each correction factor is treated in a separate section. In order to get an overview on the cup anemometer calibration uncertainty analysis two analyses are presented. First the correction factor is shortly explained and the numbers of the example used in the Measnet guide Cup Anemometer Calibration Procedures [1] are applied. The same analysis is performed using the numbers provided by DEWI. Since the analyses are in some cases different, this will be explained in detail. The results in this report are the results of reviewed calculations and written information from DEWI. The report includes recommendations to the present version of the IEC 61400-12 document.

Since DEWI does not specify the type A and type B uncertainty separately with its calibration certificates, ECN has made estimates of the type A uncertainties. In Chapter 3 the type A uncertainty is reviewed. Special attention is paid to averaging times in wind tunnels. A source of instabilities of cup anemometer calibrations is presented. In Chapter 4 it is shown that a large part of the fluctuations is due to the cup anemometer itself instead of the wind tunnel.

2. CUP ANEMOMETER CALIBRATION UNCERTAINTY

2.1 Introduction

The following sections are organised as follows, first the general example from Measnet [1] is presented, after which the same analysis is presented using the numbers provided by DEWI.

MEASNET
All uncertainties referred to in this Chapter are type B uncertainties. The order of the listed calibration uncertainties follows the example of [1] page 18/26. All numbers used refer to these examples. The complete calibration is valid for a mean wind speed of 10 m/s.

DEWI
All uncertainties referred to in this Chapter are type B uncertainties. The order of the listed calibration uncertainties follows the example of [1] page 18/26.
Note: Most calculations presented in the report are based on verbally (telephone) given information from DEWI. The numbers have been verified by e-mail. In these calculations data from the DEWI cup anemometer calibration No 279_3 from 27.03.2003 were used. This calibration document has been added to this report in Appendix A.

2.1.1 Wind tunnel correction MEASNET

Wind tunnel correction				
Error source	u_f		0.0025	
Correction factor	k_f		1.005	
Calibration wind speed	v		10.000	m/s
Sensitivity value	c_f	$c_f = v/k_f$	9.95	m/s
Contributory uncertainty		$u_f c_f$	0.025	m/s

From the discussion on the example in the Measnet Cup Anemometer Calibration procedure [1]:

> A comparison with a good tunnel (e.g. the NLR facility) might show a correction factor of 0.5% on wind speed is needed, i.e. k_f=1.005. It is suggested that a standard uncertainty of half the difference between the corrected and uncorrected value should be applied.

The wind tunnel correction factor is $k_f = 1.005$. The correction is 0.5%. The standard uncertainty is half the difference between the corrected and uncorrected value:

$$u_f = \frac{0.005}{2} = 0.0025\,.$$

The sensitivity of the wind tunnel correction factor is

$$c_f = \frac{v}{k_f} = 9.95\,\mathrm{m/s}\,.$$

Now the contributory uncertainty of the wind tunnel correction can be calculated to:

$$u_f \cdot c_f = 0.025\,\mathrm{m/s}\,.$$

2.1.2 Wind tunnel correction DEWI

Wind tunnel correction				
Error source	u_f		0	
Correction factor	k_f		1	
Calibration wind speed	v		10.000	m/s
Sensitivity value	c_f	$c_f = v/k_f$		m/s
Contributory uncertainty		$u_f c_f$	0.0001	m/s

The uncertainty u_f of the wind tunnel correction factor k_f is given by DEWI and is $u_f = 0.00$.

The contributory uncertainty $u_f c_f$ of the wind tunnel correction is estimated by DEWI of being smaller than 0.0001 m/s.

2.1.3 Wind tunnel calibration MEASNET

Wind tunnel calibration				
Error source	u_t		0.01	
Correction factor	k_t		1.02	
Calibration wind speed	v		10.000	m/s
Sensitivity value	c_t	$c_t = \frac{v}{2k_t}$	4.90	m/s
Contributory uncertainty		$u_t c_t$	0.049	m/s

From the Measnet Cup Anemometer Calibration procedure [1]:

> Wind tunnel calibration can be carried out by using two Pitot tubes, one at the permanent reference position and one at the location to be occupied by the test anemometer. By swapping the two Pitot systems, all type B errors can be eliminated, and standard regression analysis can be applied to yield a correction factor (the intercept being forced through the origin) and a related type A standard uncertainty.
> Assume the correction has a value of 1.02 and the standard uncertainty is 0.01

The wind tunnel calibration factor is $k_t = 1.02$. The correction is 2%. The standard uncertainty is half the difference between the corrected and uncorrected value:

$$u_t = \frac{0.02}{2} = 0.01$$

The sensitivity of the wind tunnel calibration is

$$c_t = \frac{v}{2k_t} = 4.90\,\text{m/s}\,.$$

The contributory uncertainty of the wind tunnel correction is:

$$u_t \cdot c_t = 0.049\,\text{m/s}\,.$$

2.1.4 Wind tunnel calibration DEWI

Wind tunnel calibration				
Error source	u_t		0.001	
Correction factor	k_t		1.002	
Calibration wind speed	v		10.000	m/s
Sensitivity value	c_t	$c_t = \frac{v}{2k_t}$	4.990	m/s
Contributory uncertainty		$u_t c_t$	0.005	m/s

The wind tunnel calibration factor $k_t = 1.002$. The uncertainty u_t of the wind tunnel calibration is half of the difference to the calibration factor $k_t = 1$:

$$u_t = \frac{|1 - k_t|}{2} = \frac{|1 - 1.002|}{2} = 0.001$$

The sensitivity factor is $c_t = \frac{v}{2k_t} = 4.990\,\mathrm{m/s}$

Therefore, the contributory uncertainty of the wind tunnel calibration is
$u_t \cdot c_t = 0.005\,\mathrm{m/s}$

2.1.5 Pressure transducer sensitivity MEASNET

Pressure transducer sensitivity				
Error source	$u_{p,t}$		34	N/m^2
Transformation factor	$K_{p,t}$		5000	N/m^2
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{p,t}$	$c_{p,t} = 0.5\ v/K_{p,t}$	0.001	m^3/N.s
Contributory uncertainty		$u_{p,t}\ c_{p,t}$	0.034	m/s

From the Measnet Cup Anemometer Calibration procedure [1]:

> Assume the pressure transducer is rated at 500N/m^2. At 10m/s wind speed, the pressure will be about 60N/m^2. Assuming the ‘limits’ on error are quoted by the manufacturer to be 0.2% of full scale (1N/m^2), and assuming this to relate to a triangular uncertainty distribution, then the equivalent standard deviation can be derived as 1*1/√6 or 0.40N/m^2. Assuming also that the transducer sensitivity, $K_{p,t}$ is 5000N/m^2 per V (100mV max output), then the standard uncertainty at 60N/m^2 $u_{p,t}$ equates to 33N/m^2 per V.

The correction factor (gain) is $K_{p,t}$ = 5000 N/m^2. The rated pressure of the transducer is p_r = 500 N/m^2. The output voltage at rated pressure is U_r = 0.1 V. The uncertainty of the pressure transducer is $u_{p,t}$ = 0.2% = 0.002.

The error in pressure at full scale is:

$$u_{fullscale} = p_r \cdot u_{p,t} = 1\,\text{N/m}^2 .$$

Assuming a triangular distribution the standard uncertainty of the pressure transducer is

$$u_p = \frac{u_{fullscale}}{\sqrt{6}} = 0.41\,\text{N/m}^2 .$$

The pressure value at reference calibration point (here 10.000 m/s) is p_{cal} = 60 N/m^2.
The output voltage at calibration point can be calculated to:

$$U_{cal} = \frac{p_{cal} \cdot U_r}{p_r} = 0.012\,\text{V} .$$

The standard uncertainty at 0.012V output signal is ca. 0.4 N/m^2. Now the standard uncertainty of U_{out} = 1V output signal and therefore at calibration point $u_{p,t}$ is searched:

$$u_{p,t} = \frac{U_{out} \cdot u_p}{U_{cal}} = 34\,\text{N/m}^2 .$$

The sensitivity of the pressure transducer can be calculated to:

$$c_{p,t} = \frac{v}{2K_{p,t}} = 0.001 \frac{\text{m/s}}{\text{N/m}^2} .$$

The contributory uncertainty of the pressure transducer is:

$$u_{p,t} \cdot c_{p,t} = 0.034\,\text{m/s} .$$

NOTE: In the Measnet procedure [1], the numbers are slightly wrong, they find $u_{p,t} = 33\,\text{N/m}^2$. As a result, Measnet [1] finds $u_{p,t} \cdot c_{p,t} = 0.033\,\text{m/s}$.

2.1.6 Pressure transducer sensitivity DEWI

Pressure transducer sensitivity				
Error source	$u_{p,t}$		0.212	Pa
Transformation factor	$K_{p,t}$		--	
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{p,t}$	$c_{p,t} = 0.5\ v/K_{p,t}$	0.08163	m/s.Pa
Contributory uncertainty		$u_{p,t}\ c_{p,t}$	0.017	m/s

The pressure transducer has 3 possible uncertainty influences. These are

- pressure transducer sensitivity
- pressure transducer signal conditioning gain
- pressure transducer data sampling conversion

DEWI calculates only on the first and the third of these uncertainty influences. Because DEWI uses no amplifier to amplify the pressure signal, there is no uncertainty contribution.

The pressure transducer sensitivity is not calculated as described in the Measnet example [1]. Instead, the wind speed uncertainty is calculated by the following:

- The pressure at v = 10.000 m/s is p_d = 61.25 Pa.
- The uncertainty of one single pressure sensor is 0.3 Pa. The uncertainty by using two pressure sensors is $u_p = \frac{0.3\,\mathrm{Pa}}{\sqrt{2}} = 0.212\,\mathrm{Pa}$.
- The sensitivity factor is $c_{p,t} = \frac{v}{2p_d} = 0.08163\frac{\mathrm{m/s}}{\mathrm{Pa}}$.

The combined uncertainty is calculated using:

$u_{p,t} \cdot c_{p,t} = 0.017\,\mathrm{m/s}$.

2.1.7 Pressure transducer signal conditioning gain MEASNET

Pressure transducer signal conditioning gain				
Error source	$u_{p,s}$		0.00002	
Transformation factor	$K_{p,s}$		0.01	
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{p,s}$	$c_{p,s}= 0.5\ v/K_{p,s}$	500	m/s
Contributory uncertainty		$u_{p,s}c_{p,s}$	0.010	m/s

From the Measnet Cup Anemometer Calibration procedure [1]:

> Assume that the signal conditioning is designed to raise the maximum transducer output voltage (100mV) to the full scale range of the data system (10V), then the required gain is 100. Thus $K_{p,s}$=0.01. Assuming a standard uncertainty of 0.2%, this gives a value of $u_{p,s}$ of 0.00002

The maximum transducer output is $U_{t,max} = 0.1$ V.
The full-scale range of the data system is $U_{d,max} = 10$ V.

This gives a gain respectively a correction factor of $K_{p,s} = \frac{U_{t,\max}}{U_{d,\max}} = 0.01$.

Assuming a standard uncertainty of $u_{gain\ p,s} = 0.2\% = 0.002$, this leads to a gain standard uncertainty of $u_{p,s} = \frac{K_{p,s}}{u_{gain\ p,s}} = 0.00002$.

The sensitivity value at the calibration point $v = 10$ m/s can be calculated to

$$c_{p,s} = \frac{v}{2K_{p,s}} = 500 \text{ m/s} .$$

The contributory uncertainty of the pressure gain is:

$$u_{p,s} \cdot c_{p,s} = 0.01 \text{m/s} .$$

2.1.8 Pressure transducer signal conditioning gain DEWI

Pressure transducer signal conditioning gain				
Error source	$u_{p,s}$			
Transformation factor	$K_{p,s}$			
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{p,s}$	$c_{p,s}= 0.5\ v/K_{p,s}$		m/s
Contributory uncertainty		$u_{p,s}c_{p,s}$		

Here the uncertainty of the transducer gain is asked. The transducer is normally used to bring the output signal of the Pitot tube up to the nominal range of the data sampling system. DEWI has a nominal voltage input of 2.5 V for the data system. The Pitot tube delivers this voltage. Therefore, no transducer is needed and used. There is no combined contributory uncertainty of the signal conditioning gain.

2.1.9 Pressure transducer data sampling conversion MEASNET

Pressure transducer data sampling conversion				
Error source	$u_{p,d}$		0.000002	V/bit
Transformation factor	$K_{p,d}$		0.00244	V/bit
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{p,d}$	$c_{p,d}= 0.5\ v/K_{p,d}$	2048	m/s
Contributory uncertainty		$u_{p,d}c_{p,d}$	0.0029	m/s

From the Measnet Cup Anemometer Calibration procedure [1]:

> The resolution of the data system is defined by the full scale values, e.g. 4096 bits for 10 V or *Kp,d* of 0.00244 V per bit. The quantisation limits are half of this ie 0.00122 V per bit, and since a rectangular distribution is appropriate, the related standard uncertainty is 0.00122/√3 V or 0.00704 V.
> For 10m/s wind speed, the voltage seen by the d/a system will be in the region of 1.2 V, giving a nominal bit value of 490. The conversion uncertainty $u_{p,d}$ is then no more than 0.000002 V/bit

The output voltage at full scale of the data sampling conversion is $U_{max} = 10$ V.
The bits for full scale are $bit_{max} = 4096$. The calibration wind speed is $v = 10$ m/s.

The correction factor can now be calculated to $K_{p,d} = \frac{U_{max}}{bit_{max}} = 0.00244\ \text{V/bit}$.

The quantisation limit l_q is half of the correction factor: $l_q = \frac{K_{p,d}}{2} = 0.00122\,\text{V/bit}$.

When a rectangular distribution is assumed the related standard uncertainty is[1]:

$$u_{max} = \frac{l_q}{\sqrt{3}} = 0.000705\,\text{V/bit}\,.$$

Voltage for the data acquisition system at calibration point $v = 10$ m/s is $U_{cal} = 1.2$ V.
The bits for voltage at calibration point can be calculated to:

$$bit_{cal} = \frac{U_{cal}}{U_{max}} \cdot bit_{max} \cong 492\,\text{bit}\,.$$

Now the conversion uncertainty at calibration point is:

$$u_{p,d} = \frac{1bit \cdot u_{max}}{bit_{cal}} \cong 0.0000014\ \text{V/bit}\,.$$

The sensitivity of the pressure transducer data sampling conversion is:

$$c_{p,d} = \frac{v}{2K_{p,d}} = \frac{v.bit_{\max}}{2U_{\max}} = 2048\frac{\text{m/s}}{\text{V/bit}}\,.$$

The contributory uncertainty can be calculated to:

$$u_{p,d} \cdot c_{p,d} \cong 0.0029\,\text{m/s}\,.$$

NOTE: In the Measnet procedure [1], the numbers are slightly wrong, they find $bit_{cal} = 490$ and $u_{p,d} = 0.000002\ \text{V/bit}$. As a result, Measnet [1] finds $c_{p,d} = 2049$ and $u_{p,d}c_{p,d} = 0.004$.

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[1] Note that in the example of the Measnet Cup Anemometer Calibration Procedure [1] a value of 0.00704V is mentioned.

2.1.10 Pressure transducer data sampling conversion DEWI

Pressure transducer data sampling conversion				
Error source	$u_{p,d}$		0.002	
Transformation factor	$K_{p,d}$		1	
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{p,d}$	$c_{p,d}= 0.5\ v/K_{p,d}$	5	m/s
Contributory uncertainty		$u_{p,d}c_{p,d}$	0.01	m/s

DEWI applies the following coefficients for the uncertainty factors in the pressure transducer data sampling conversion: $K_{p,s}$, $u_{p,s}$ and $c_{p,s}$. To stay compatible with [1] it is assumed that $K_{p,d}$, $u_{p,d}$ and $c_{p,d}$ are meant.

The correction factor is $K_{p,d} = 1$. The uncertainty of the whole measuring system is $u_{p,d} = 0.002$. The sensitivity value $c_{p,d} = 5$.
Therefore the contributory uncertainty can be calculated to:

$$u_{p,d} \cdot c_{p,d} = 0.01\,\mathrm{m/s}$$

2.1.11 Ambient temperature transducer MEASNET

Ambient temperature transducer				
Error source	$u_{T,t}$		n/a	
Transformation factor	$K_{T,t}$			
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{T,t}$	$c_{T,t}= 0.5\ v/K_{T,t}$	n/a	
Contributory uncertainty		$u_{T,t}c_{T,t}$	0.0014	m/s

From the Measnet Cup Anemometer Calibration procedure [1]:

> Temperature may appear to be somewhat difficult to handle, because whereas the foregoing theory assumed a zero offset in the relationship connecting temperature to transducer output, in reality a very high offset exists. Typically a temperature system might be quoted as giving a 4 to 20mA current range for a -20 to 30C temperature range. Rather than trying to restructure the mathematics, it is possible to take a lateral approach. Assume the transducer is quoted as being good to 0,2C. Assuming a triangular distribution, this relates to a standard uncertainty of 0,08C. We know this is the temperature error attributable to the transducer, rather than the complete temperature chain. Going back to the basic equation for wind speed in terms of the physical T, B and p parameters, it is easy by varying T (from say 15C, 288K up to 15,08C, 288,08K) to determine the corresponding change in wind speed. This comes out, for 10m/s, as 0,001m/s. This value can be inserted directly in the last column of the table without reference to the third and fourth columns, which were based on the more general analytical approach.

Here we use the basic equation for wind speed in terms of the physical T, B and p parameters. The transducer uncertainty due to temperature is calculated by using formula (6) from [1] page 12. The mean flow wind speed $\bar{v}$ at anemometer position can be calculated with

$$\bar{v} = k_f \cdot \frac{1}{n} \cdot \sum_{k=1}^{n} v_k = k_f \cdot \frac{1}{n} \cdot \sum_{k=1}^{n} \sqrt{\frac{2 \cdot k_c \cdot p_k \cdot R \cdot T_k}{C_h \cdot B_k \cdot k_\rho}}$$

For n = 1 follows:

$$\bar{v} = k_f \cdot \sqrt{\frac{2 \cdot k_c \cdot p \cdot R \cdot T}{C_h \cdot B \cdot k_\rho}} = k_f \cdot \sqrt{\frac{2 \cdot k_c \cdot p \cdot R}{C_h \cdot B \cdot k_\rho}} \cdot \sqrt{T} = C \cdot \sqrt{T}$$

The sensor in the example is considered to be good to 0.2 °C. Assume a triangular distribution, relating to a standard uncertainty of 0.08 °C (=0.2/√6).
The temperature at calibration point is $T_1 = 288.00$ K.
Due to the temperature sensor uncertainty a second temperature value is
$T_2 = 288.00K + 0.08K = 288.08K$.

The wind speed difference due to this temperature uncertainty is $\Delta\bar{v} = \left|v_2 - v_1\right|$.

$$\frac{\Delta\bar{v}}{v_1} = \frac{\bar{v}_2}{\bar{v}_1} - 1 = \frac{C \cdot \sqrt{T_2}}{C \cdot \sqrt{T_1}} - 1 = \frac{\sqrt{T_2}}{\sqrt{T_1}} - 1 = 0.00013888 \text{, hence } \Delta\bar{v} = 0.0014 \text{ m/s if } v_1 = v.$$

This is taken as the contributory uncertainty $u_{T,t}c_{T,t}$ due to the temperature uncertainty of the temperature sensor.

2.1.12 Ambient temperature transducer DEWI

Ambient temperature transducer				
Error source	$u_{T,t}$		n/a	
Transformation factor	$K_{T,t}$		n/a	
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{T,t}$	$c_{T,t}= 0.5\ v/K_{T,t}$	n/a	m/s
Contributory uncertainty		$u_{T,t}c_{T,t}$	0.0034	m/s

The temperature transducer has 3 possible uncertainty influences. These are
- ambient temperature transducer
- temperature signal conditioning gain
- temperature signal digital conversion

DEWI calculates only on the first of these uncertainty influences. But for the uncertainty of the temperature transducer they use a accuracy of 0.5 degrees C while the real accuracy of the temperature transducer is much better. They neglect the uncertainties of signal conditioning gain and data sampling conversion, because they are small and the first uncertainty factor contains already some extra uncertainty.

The ambient temperature transducer uncertainty is not calculated as described in the example [1]. Instead the transducer uncertainty due to temperature is calculated by using formula (6) from [1] page 12.

The mean flow wind speed $\bar{v}$ at anemometer position can be calculated with:

$$\bar{v} = k_f \cdot \frac{1}{n} \cdot \sum_{k=1}^{n} v_k = k_f \cdot \frac{1}{n} \cdot \sum_{k=1}^{n} \sqrt{\frac{2 \cdot k_c \cdot p_k \cdot R \cdot T_k}{C_h \cdot B_k \cdot k_\rho}}$$

For n = 1 follows:

$$\bar{v} = k_f \cdot \sqrt{\frac{2 \cdot k_c \cdot p \cdot R \cdot T}{C_h \cdot B \cdot k_\rho}} = k_f \cdot \sqrt{\frac{2 \cdot k_c \cdot p \cdot R}{C_h \cdot B \cdot k_\rho}} \cdot \sqrt{T} = C \cdot \sqrt{T}$$

The DEWI sensor is considered to be good to 0.5 ºC. Assuming a triangular distribution like the example in [1], the standard uncertainty will be 0.2 °C (=0.5/√6).
Assume that the temperature at calibration point is $T_1 = 21.7\,°\mathrm{C} \equiv 294.85\,\mathrm{K}$
Due to the temperature sensor uncertainty a second temperature value is obtained of
$T_2 = 21.7\,°\mathrm{C} + 0.2\,°\mathrm{C} \equiv 295.05\,\mathrm{K}$
The wind speed difference due to this temperature uncertainty is $\Delta\bar{v} = \left|v_2 - v_1\right|$.

$$\frac{\Delta\bar{v}}{v_1} = \frac{\bar{v}_2}{\bar{v}_1} - 1 = \frac{C \cdot \sqrt{T_2}}{C \cdot \sqrt{T_1}} - 1 = \frac{\sqrt{T_2}}{\sqrt{T_1}} - 1 = 0.00034$$, hence $\Delta\bar{v} = 0.0034\,\mathrm{m/s}$ if $v_1 = v$.

This is taken as the contributory uncertainty $u_{T,t}c_{T,t}$ due to the temperature uncertainty of the temperature sensor.

2.1.13 Temperature signal conditioning gain MEASNET

Temperature signal conditioning gain				
Error source	$u_{T,s}$		0.0008	mA/V
Transformation factor	$K_{T,s}$		2	mA/V
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{T,s}$	$c_{T,s}= 0.5\ v/K_{T,s}$	2.5	m.V/mA.s
Contributory uncertainty		$u_{T,s}c_{T,s}$	0.002	m/s

From the Measnet Cup Anemometer Calibration procedure [1]:

> Assume the current output from the temperature sender unit is fed to a 500Ω precision resistor, to give a 2V to 10V output for the temperature range. The gain $K_{T,s}$ is thus 2mA/V. Assuming the resistor has a standard uncertainty of 0,2Ω, then the gain will have a corresponding uncertainty of 0,0008mA/V.

The current output from the temperature sensor is fed to a R = 500Ω precision resistor. To give a minimum output signal of U_{min} = 2V and a maximum output signal of U_{max} = 10V a currents are used of

$I_{min} = U_{min} / R = 4$ mA and

$I_{max} = U_{max} / R = 20$ mA.

Herewith the gain $K_{T,s}$ of the temperature sensor is calculated:

$$K_{T,s} = \frac{I_{\max} - I_{\min}}{U_{\max} - U_{\min}} = 2\,\text{mA/V}\,.$$

The uncertainty of the resistor is $u_{resistor}$ = 0.2 Ω.
The standard uncertainty of the resistor at 500 Ω is:

$$u = \frac{u_{resistor}}{R} = 0.0004\ .$$

Now the uncertainity of the gain can be calculated to:

$$u_{T,s} = K_{T,s} \cdot u = 0.0008\,\text{mA/V}\,.$$

The sensivity can be calculated to:

$$c_{T,s} = 0.5 \cdot \frac{v}{K_{T,s}} = 2.5 \frac{\text{m/s}}{\text{mA/V}}\,.$$

Now the contributory uncertainty can be calculated to:

$$u_{T,s} \cdot c_{T,s} = 0.002\,\text{m/s}\,.$$

2.1.14 Temperature signal conditioning gain DEWI

Temperature signal conditioning gain				
Error source	$u_{T,s}$			
Transformation factor	$K_{T,s}$			
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{T,s}$	$c_{T,s} = 0.5\ v/K_{T,s}$		
Contributory uncertainty		$u_{T,s}c_{T,s}$	0.001	m/s

Because by the ambient temperature transducer the calculated uncertainty of 0.5ºC is used and the real uncertainty of the temperature sensor is much smaller (smaller than 0.1ºC), this standard uncertainty is estimated by DEWI of being smaller than 0.001 m/s.

2.1.15 Temperature signal digital conversion MEASNET

Temperature signal digital conversion				
Error source	$u_{T,d}$		0.00000023	V/bit
Transformation factor	$K_{T,d}$		0.00244	V/bit
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{T,d}$	$c_{T,d}= 0.5\, v/K_{T,d}$	2049	m.bit/V.s
Contributory uncertainty		$u_{T,d}c_{T,d}$	0.0046	m/s

From the Measnet Cup Anemometer Calibration procedure [1]:

> As for the pressure transducer signal line in the case above, the standard uncertainty of the quantisation is 0,00704V.
> For 15C temperature, the voltage seen by the d/a system will be in the region of 7,6 V, giving a nominal bit value of 3113.
> The conversion uncertainty $u_{T,d}$ is then no more than 0,0000023 V/bit

The output voltage at full scale of the data sampling conversion is U_{max} = 10 V.
The bits for full scale are bit_{max} = 4096. The calibration wind speed is v = 10 m/s.

The correction factor can now be calculated to $K_{T,d} = \frac{U_{max}}{bit_{max}} = 0.00244 \text{ V/bit}$.

The quantisation limit l_q is half of the correction factor:

$$l_q = \frac{K_{T,d}}{2} = 0.00122 \,\text{V/bit}.$$

When we assume a rectangular distribution the related standard uncertainty can be calculated to:

$$u_{max} = \frac{l_q}{\sqrt{3}} = 0.000705 \text{ V/bit}.$$

The voltage for the data acquisition system for a temperature of 15 °C during the calibration was $\underline{U_{cal}}$ = 7.6 V. Now the bits during the calibration can be calculated:

$$bit_{cal} = \frac{U_{cal}}{U_{max}} \cdot bit_{max} \cong 3113 \,\text{bit}.$$

Now the conversion uncertainty is:

$$u_{T,d} = \frac{1bit \cdot u_{max}}{bit_{cal}} \cong 0.00000023 \text{ V/bit}.$$

The sensitivity of the temperature digital conversion is:

$$c_{T,d} = \frac{v}{2K_{T,d}} = \frac{v.bit_{\max}}{2U_{\max}} = 2048 \frac{\text{m/s}}{\text{V/bit}}.$$

The contributory uncertainty can be calculated to:

$$u_{T,d} \cdot c_{T,d} = 0.00046 \,\text{m/s}.$$

NOTE: In [1] the erroneous numbers $u_{\max} = 0.00704 \text{ V/bit}$, $u_{T,d} \cong 0.0000023 \text{ V/bit}$, $c_{T,d} = 2049 \frac{\text{m/s}}{\text{V/bit}}$ and $u_{T,d} \cdot c_{T,d} = 0.0046 \,\text{m/s}$ are obtained.

2.1.16 Temperature signal digital conversion DEWI

Temperature signal digital conversion				
Error source	$u_{T,d}$			
Transformation factor	$K_{T,d}$			
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{T,d}$	$c_{T,d} = 0.5\ v/K_{T,d}$		
Contributory uncertainty		$u_{T,d}c_{T,d}$	0.01	m/s

This standard uncertainty is estimated by DEWI of being smaller than 0.01 m/s.

2.1.17 Pitot tube head coefficient MEASNET

Pitot tube head coefficient				
Error source	u_h		0.000997	
Transformation factor	C_h		0.997	
Calibration wind speed	v		10.000	m/s
Sensitivity value	c_h	$c_h = -0.5\ v/C_h$	-5.015	m/s
Contributory uncertainty		$u_h c_h$	-0.005	m/s

From the Measnet Cup Anemometer Calibration procedure [1]:

> The head coefficient of a pitot tube depends upon the angle of attack of the wind. Two error sources are possible, one related to the accuracy with which the pitot tube is set up in alignment with the mean flow direction, and the other due to turbulent variations in instantaneous flow direction. Assume the nominal head coefficient, C_h, is 0,997, and assume also that it is possible to deduce that the standard deviation on angle of attack is 2°. Relevant ISO standards suggest this will give rise to a 0,1% change in head coefficient.

The pitot tube head coefficient and therefore the correction factor is $C_h = 0.997$.
The standard deviation on the angle of attack is 2 degrees.
ISO Guide 3996 gives a value of uncertainty = 0.1% = 0.001 for the change in head coefficient due to the angle of attack.
The uncertainty of the pitot tube head coefficient can now be calculated to:

$$u_h = \text{uncertainty} \cdot C_h = 0.000997\,.$$

The sensitivity of the pitot tube head coefficient is:

$$c_h = -\frac{v}{2C_h} = -5.015\,\text{m/s}\,.$$

Now the contributory uncertainty can be calculated to:

$$u_h \cdot c_h = -0.005\,\text{m/s}\,.$$

2.1.18 Pitot tube head coefficient DEWI

Pitot tube head coefficient				
Error source	u_h		0.001	
Transformation factor	C_h		1	
Calibration wind speed	v		10.00	m/s
Sensitivity value	c_h	$c_h = -0.5\ v/C_h$	-5	m/s
Contributory uncertainty		$u_h c_h$	-0.005	m/s

The DEWI Pitot tube head coefficient is $C_h = 1$.

Assume that the standard deviation on angle of attack is 2 degrees. Relevant ISO standards (ISO guide 3996) suggest this will give rise to a 0.1 % change in head coefficient ($u_h = 0.001$). The sensitivity factor can now be calculated as

$$c_h = -\frac{v}{2C_h} = -5\,\text{m/s}\,.$$

Then the contributory uncertainty can be calculated as

$$u_h \cdot c_h = -0.005\,\text{m/s}\,.$$

2.1.19 Sensitivity of barometer MEASNET

Sensitivity of barometer				
Error source	$u_{B,t}$		n/a	
Transformation factor	$K_{B,t}$			
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{B,t}$	$c_{B,t} = -0.5\ v/K_{B,t}$		
Contributory uncertainty		$u_{B,t}c_{B,t}$	0.0014	m/s

From the Measnet Cup Anemometer Calibration procedure [1]:

> The barometer can be treated in much the same way as the temperature probe, since it to will have a large physical offset.

Here no values are given. The barometer can be treated in the same way as the temperature probe is given. Therefore the uncertainty value of the temperature probe (see section 2.1.11) is copied.

2.1.20 Sensitivity of barometer DEWI

Sensitivity of barometer				
Error source	$u_{B,t}$		2	hPa
Transformation factor	$K_{B,t}$		101300	Pa
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{B,t}$	$c_{B,t}$= - 0.5 $v/K_{B,t}$	-0.000049	m/s.Pa
Contributory uncertainty		$u_{B,t}c_{B,t}$	-0.01	m/s

The transformation factor of the barometer is $K_{B,t} = 101300\,\mathrm{Pa}$.

The sensor uncertainty according to the manufacturer is 0.3 hPa, but the sensor uncertainty is assumed to be $u_{B,t} = 2\,\mathrm{hPa}$ by DEWI.

The sensitivity factor can be calculated to

$$c_{B,t} = -\frac{v}{2K_{B,t}} = -0.000049\frac{\mathrm{m}}{\mathrm{s}\cdot\mathrm{Pa}}.$$

Now the contributory uncertainty can be calculated to

$$u_{B,t}\cdot c_{B,t} = -0.01\,\mathrm{m/s}.$$

2.1.21 Signal conditioning gain on barometer MEASNET

Signal conditioning gain on barometer				
Error source	$u_{B,s}$			
Transformation factor	$K_{B,s}$			
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{B,s}$	$c_{B,s} = -0.5\, v/K_{B,s}$		
Contributory uncertainty		$u_{B,s}c_{B,s}$	0.002	m/s

From the Measnet Cup Anemometer Calibration procedure [1][2]:

Similar approach as for other signal processing parameters

Here no values are given. The barometer can be treated in the same way as the temperature probe is given (see 2.1.13). Therefore the uncertainty value of the temperature gain is copied.

---

[2] Note that the minus sign for $c_{B,s}$ in the formula is missing in the MEASNET example.

2.1.22 Signal conditioning gain on barometer DEWI

Signal conditioning gain on barometer				
Error source	$u_{B,s}$			
Transformation factor	$K_{B,s}$			
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{B,s}$	$c_{B,s} = -0.5\ v/K_{B,s}$		
Contributory uncertainty		$u_{B,s}c_{B,s}$	0.01	m/s

The uncertainty of the barometer according to the manufacturer is 0.3 hPa while DEWI assumes the uncertainty is 2 hPa (see also section 2.1.20). Since calculations are done with a higher uncertainty, no additional calculation concerning the contributory uncertainty of the signal conditioning gain on barometer is performed. The standard contributory uncertainty of the signal conditioning gain on barometer is estimated by DEWI of being smaller than 0.01 m/s.

2.1.23 Digital conversion of barometer signal MAESNET

Digital conversion of barometer signal				
Error source	$u_{B,d}$			
Transformation factor	$K_{B,d}$			
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{B,d}$	$c_{B,d} = -0.5\ v/K_{B,d}$		
Contributory uncertainty		$u_{B,d}c_{B,d}$	(-)0.00046	m/s

From the Measnet Cup Anemometer Calibration procedure [1][3]:

Similar approach as for other data acquisition channels

Here no values are given. Only the hint that the barometer can be treated in the same way as the temperature probe is given. Therefore the (corrected) uncertainty value of the temperature digital conversion is copied (see Section 2.1.15).

---

[3] Note that the minus sign for $c_{B,d}$ in the formula is missing in the MEASNET example.

2.1.24 Digital conversion of barometer signal DEWI

Digital conversion of barometer signal				
Error source	$u_{B,d}$			
Transformation factor	$K_{B,d}$			
Calibration wind speed	v		10.000	m/s
Sensitivity value	$c_{B,d}$	$c_{B,d} = -0.5\ v/K_{B,d}$		
Contributory uncertainty		$u_{B,d}c_{B,d}$	(-)0.01	m/s

This standard uncertainty is estimated by DEWI of being smaller than 0.01 m/s.

2.1.25 Statistical uncertainty in the mean of the wind speed time series MEASNET

s_A statistical uncertainty in the mean of the wind speed time series				
Error source	u_{sA}	$\frac{1}{\sqrt{samples}} \cdot T_I \cdot \bar{v}$	0.026	m/s
Calibration wind speed	v		10.000	m/s
Sensitivity value	c_{sA}		1	
Contributory uncertainty		$u_{s,A}c_{s,A}$	0.026	m/s

From the Measnet Cup Anemometer Calibration procedure [1]:

> Assume the turbulence intensity is 2%, and that 2Hz sampling over 30 seconds is used, giving 60 samples. The standard uncertainty in the mean value of 10m/s is then given by √1/60 . 0,02 . 10

Assume that the turbulence intensity is $T_I = 2\% = 0.02$. The sampling frequency is $f_s = 2$ Hz. The sampling interval is $t_s = 30$ seconds. Then the number of samples during one sampling interval is:

$$n_s = f_s \cdot t_s = 60 \ .$$

The calibration wind speed is $v = 10$ m/s.

Then the standard uncertainty of the time series can be calculated to

$$u_{s,A} = \frac{1}{\sqrt{n_s}} \cdot T_I \cdot \bar{v} = 0.026\,\mathrm{m/s}\,.$$

Because the sensitivity factor $c_{s,A} = 1$ this also is the contributory uncertainty of s_A.

$$u_{s,A} \cdot c_{s,A} = 0.026\,\mathrm{m/s}\,.$$

2.1.26 s_A statistical uncertainty in the mean of the wind speed time series DEWI

s_A statistical uncertainty in the mean of the wind speed time series				
Error source	u_{sA}	$\frac{1}{\sqrt{samples}} \cdot T_I \cdot \bar{v}$	0.004	m/s
Calibration wind speed	v		10.000	m/s
Sensitivity value	c_{sA}		1	
Contributory uncertainty		$u_A c_A$	0.004	m/s

The DEWI turbulence intensity is 0.2 %.
1 Hz sampling over 30 seconds gives 30 samples. Then the standard uncertainty of the time series can be calculated to

$$u_{s,A} = \frac{1}{\sqrt{samples}} \cdot T_I \cdot \bar{v} = 0.004\,\mathrm{m/s}\,.$$

Because the sensitivity factor $c_{s,A}$ = 1 this also is the contributory uncertainty of s_A.

$$u_{sA} \cdot c_{s,A} = 0.004\,\mathrm{m/s}\,.$$

2.1.27 Humidity correction to density MEASNET

Humidity correction to density				
Error source	u_ρ		n/a	m/s
Correction factor	k_ρ	$k_\rho \approx 1 - 0.378 \cdot \frac{\varphi_k \cdot P_w}{B_k}$	0.997	
Relative humidity	φ_k		50	%
Accuracy of relative humidity	d_φ		5.0	%
Barometric pressure	B_k		101300	Pa
Vapour pressure	P_w	$P_w = 0.0000205 \exp(0.0631846\ T)$	1655	Pa
Temperature	T		15	°C
Calibration wind speed	v		10	m/s
Error source relative humidity	u_φ	$u_\varphi = d_\varphi \varphi_k$	0.025	m/s
Sensitivity value relative humidity	c_φ	$c_\varphi = 0.5 \cdot \frac{\bar{v}}{k_\rho} \cdot 0.378 \cdot \frac{P_w}{B_k}$	0.031	m/s
Contributory uncertainty in relative humidity		$u_\varphi c_\varphi$	0.00077	m/s
Assumption		$u_\rho^2 c_\rho^2 = u_\varphi^2 c_\varphi^2$		
Contributory uncertainty in humidity correction to density		$u_\rho c_\rho$	0.00077	m/s

From the Measnet Cup Anemometer Calibration procedure [1]:

> It is possible to show that $c_\rho^2 u_\rho^2$ is equivalent to $c_\varphi^2 u_\varphi^2$ (where u_ρ is the uncertainty in relative humidity and c_ρ is the sensitivity of derived wind speed to humidity) if c_ρ is dominated by c_φ rather than cB or cT. This is normally the case.
> Assume relative humidity, φ, is measured from a hand-held meter as 50% to an accuracy of 5% within 95% confidence. φ= 0,5 and u_φ = 0,025
>
> $$c_\varphi = \frac{\partial \bar{v}}{\partial k_\rho} \cdot \frac{\partial k_\rho}{\partial \varphi} = \frac{1}{2} \cdot \frac{\bar{v}}{k_\rho} \cdot 0.378 \cdot \frac{P_w}{B}$$
>
> At 15°C, P_W = 1700 Pa and assuming B = 1013 mbar = 101300 Pa k_ρ is evaluated as 0,997 and c_φ (at 10m/s) is 0,032

To calculate the humidity correction to density k_ρ formula (7) from [1] page 13 is used:

$$k_\rho \approx 1 - 0.378 \cdot \frac{\varphi_k \cdot P_w}{B_k}$$

During the calibration the value of the relative humidity was $\varphi_k = 50\,\%$. The barometric pressure B_k during the calibration was B_k = 101300 Pa. The temperature during the calibration was 15 °C ≡ 288.15 K. The vapour pressure for the prevailing temperature can be calculated by $P_w = 0.0000205 \cdot \exp(0.0631846 \cdot T) \cong 1655\,\text{Pa}$

Now k_ρ can be calculated to $k_\rho \approx 1 - 0.378 \cdot \frac{\varphi_k \cdot P_w}{B_k} = 0.997$.

To calculate c_φ the formula on page 21 from [1] is used:

$$c_\varphi = \frac{\partial \bar{v}}{\partial k_\rho} \cdot \frac{\partial k_\rho}{\partial \varphi} = \frac{\bar{v}}{2k_\rho} \cdot 0.378 \cdot \frac{P_w}{B} = 0.031\,\text{m/s}\,.$$

The accuracy of the relative humidity is given to 5% = 0.005.
Therefore the standard uncertainty u_φ of the relative humidity is:

$u_\varphi = 0.05 \cdot \varphi_k = 0.025\,\text{m/s}$.

The contributory uncertainty of the humidity correction to density than is

$u_\varphi \cdot c_\varphi = 0.00077\,\text{m/s}$.

NOTE: In [1] erroneous numbers are used: $P_w = 1700\,\text{Pa}$, $c_\varphi = 0.032\,\text{m/s}$ and $u_\varphi \cdot c_\varphi = 0.001\,\text{m/s}$

2.1.28 Humidity correction to density DEWI

Humidity correction to density				
Error source	u_ρ		--	m/s
Correction factor	k_ρ	$k_\rho \approx 1 - 0.378 \cdot \frac{\varphi_k \cdot P_w}{B_k}$	0.997	
Relative humidity	φ_k		31.7	%
Accuracy of rel. humidity	d_φ		5.0	%
Barometric pressure	B_k		101690	Pa
Vapour pressure	P_w	$P_w = 0.0000205 \exp(0.0631846\ T)$	2527	Pa
Temperature	T		294.85	K
Calibration wind speed	v		10.000	m/s
Error source rel. humidity	u_φ	$u_\varphi = d_\varphi\, \varphi_k$	0.016	m/s
Sensitivity value relative humidity	c_φ	$c_\varphi = 0.5 \cdot \frac{\bar{v}}{k_\rho} \cdot 0.378 \cdot \frac{P_w}{B_k}$	0.0471	m/s
Contributory uncertainty in relative humidity		$u_\varphi c_\varphi$	0.00075	m/s
Assumption		$u_\rho^2 c_\rho^2 = u_\varphi^2 c_\varphi^2$		
Contributory uncertainty in humidity correction to density		$u_\rho c_\rho$	0.00075	m/s

To calculate the humidity correction to density k_ρ formula (7) from [1] page 13 is used:

$$k_\rho \approx 1 - 0.378 \cdot \frac{\varphi_k \cdot P_w}{B_k}$$

During the calibration the value of the relative humidity was $\varphi_k = 31.7\,\%$. [3]

The barometric pressure B_k during the calibration was $B_k = 101690$ Pa. [3]

The temperature during the calibration was $T = 21.7$ °C ≡ 294.85 K. [3]

The vapour pressure for the prevailing temperature can be calculated by

$$P_w = 0.0000205 \cdot \exp(0.0631846 \cdot T) = 2527\,\text{Pa}$$

Now k_ρ can be calculated to $k_\rho \approx 1 - 0.378 \cdot \frac{\varphi_k \cdot P_w}{B_k} = 0.997$.

To calculate c_φ the formula on page 21 from [1] is used:

$$c_\varphi = \frac{\partial \bar{v}}{\partial k_\rho} \cdot \frac{\partial k_\rho}{\partial \varphi} = \frac{\bar{v}}{2k_\rho} \cdot 0.378 \cdot \frac{P_w}{B} = 0.0471\,\text{m/s}.$$

The accuracy of the relative humidity is given to 5% = 0.005.

Therefore the standard uncertainty u_φ of the relative humidity is:

$u_\varphi = 0.05 \cdot \varphi_k = 0.01585\,\text{m/s}$.

The contributory uncertainty of the humidity correction to density is:

$u_\varphi \cdot c_\varphi = 0.00075\,\text{m/s}$.

2.2 Total type B uncertainty MEASNET

Contributory type B uncertainty u_i		Value [m/s]
1.	Wind tunnel correction	0.025
2.	Wind tunnel calibration	0.049
3.	Pressure transducer sensitivity	0.034
4.	Pressure transducer signal conditioning gain	0.01
5.	Pressure transducer data sampling conversion	0.0029
6.	Ambient temperature transducer	0.0014
7.	Temperature signal conditioning gain	0.002
8.	Temperature signal digital conversion	0.00046
9.	Pitot tube head coefficient	-0.005
10.	Sensitivity of barometer	0.0014
11.	Signal conditioning gain on barometer	0.002
12.	Digital conversion of barometer signal	0.00046
13.	s_A statistical uncertainty in the mean of the wind speed time series	0.026
14.	Humidity correction to density	0.00077
	Total type B uncertainty $u_B = \sqrt{\sum_i u_i^2}$	0.07

2.3 Total type B uncertainty DEWI

Contributory type B uncertainty u_i		Value [m/s]
1.	Wind tunnel correction	0.0001
2.	Wind tunnel calibration	0.005
3.	Pressure transducer sensitivity	0.017
4.	Pressure transducer signal conditioning gain	0
5.	Pressure transducer data sampling conversion	0.01
6.	Ambient temperature transducer	0.0034
7.	Temperature signal conditioning gain	0.001
8.	Temperature signal digital conversion	0.01
9.	Pitot tube head coefficient	-0.005
10.	Sensitivity of barometer	-0.01
11.	Signal conditioning gain on barometer	0.01
12.	Digital conversion of barometer signal	0.01
13.	s_A statistical uncertainty in the mean of the wind speed time series	0.004
14.	Humidity correction to density	0.00075
	Total type B uncertainty $u_B = \sqrt{\sum_i u_i^2}$	0.03

2.4 Comparisons

In the table below the differences between Version 1 of the Measnet cup calibration procedure [1] and the power performance procedure under proposal [2] are summarised, together with differences found to the analysis presented in this report in Section 2.1.

Error source, item	Previous [1] procedure	Proposed [2] procedure	Derived
$u_{p,t}$, standard uncertainty calibr. point	33	33	34
$u_{p,t}$, contributory uncertainty	0.033	0.033	0.034
$u_{p,d}$, standard uncertainty quantisation limit	0.00704	0.000704	0.000705
$u_{p,d}$, no. of bits for voltage at cal. point	490	490	492
$u_{p,d}$, conversion uncertainty	0.000002	0.000002	0.0000014
$u_{p,d}$, sensitivity value $c_{p,d}$	2049	2049	2048
$u_{p,d}$, contributory uncertainty	0.004	0.004	0.0029
$u_{T,d}$, st. uncertainty quantisation limit	0.00704	0.000704	0.000705
$u_{T,d}$, conversion uncertainty	0.0000023	0.0000023	0.00000023
$u_{T,d}$, sensitivity value $c_{T,d}$	2049	2049	2048
$u_{T,d}$, contributory uncertainty	0.004	0.004	0.00046
$u_{B,s}$, sensitivity value $c_{B,s}=$	$0.5\ v/K_{B,s}$	$0.5\ v/K_{B,s}$	$-\ 0.5\ v/K_{B,s}$
$u_{B,d}$, sensitivity value $c_{B,d}=$	$0.5\ v/K_{B,d}$	$0.5\ v/K_{B,d}$	$-\ 0.5\ v/K_{B,d}$
u_{ρ} , vapour pressure	1700	1700	1655
u_{ρ} , sensitivity value relative humidity	0.032	0.032	0.031
u_{ρ} , contributory uncertainty	0.001	0.001	0.00077

This list summarises the (minor) mistakes in the MEASNET document for cup-anemometer calibrations [1] and the proposal for the IEC document on power performance measurements [2]. This list has been handed over to the IEC with the recommendation to correct these.

3. Cup anemometer calibration uncertainty: TYPE A

3.1 Introduction

The type A uncertainty is defined by ISO [4] as a method of evaluation of uncertainty by the statistical analysis of series of observations. Basically it is an uncertainty analysis based on statistics.

The MEASNET document [1] gives the formulas how to calculate these type A uncertainties on pages 34 to 36. This calculation was analysed in detail to understand the background. In this report it is assumed that all necessary requirements regarding the statistical model for linear calibration are valid and that therefore the general theory for linear calibration [8] can be used.

3.2 Standard MEASNET calibration

In the standard calibration for cup anemometers according to MEASNET the main steps of the calibration are:

- 5 minute run in for the cup anemometer
- Take calibration points at rising and falling wind speeds between 4 to 16 m/s (a suggested sequence is 4, 6, 8, 10, 12, 14, 16, 15, 13, 11, 9, 7, 5 m/s)
- Sampling frequency shall be at least 1 Hz and the sampling interval shall be at least 30 seconds
- Data collection can only start after stable flow conditions have established (this takes normally 1 minute at each wind speed)

In the data analysis an inverse regression is done by regressing the wind speed on the anemometer output. The data is averaged to a 30 second average and then the regression analysis is done, together with the type A uncertainty analysis for the regression.

However, an important point is that the regression is done over 30 second intervals, whereas for power performance measurements data are averaged over 10-minute intervals. Since a main characteristic of type A uncertainty is that it can be reduced by extra measurements (it is based on statistics of the data set), this raises the question if the calculated uncertainty for the regression is the best estimate for the type A uncertainty for calibration, when data is taken with 10-minute intervals.

When doing the regression analysis we can distinguish between the number of data points at each wind speed (N) and the time each data point represents (M). The standard calibration would have N=1 and M=30 seconds. In this Chapter we will investigate whether the calibration uncertainty depends on N or M.

3.3 DNW wind tunnel measurements

To answer this question data taken in a wind tunnel have been analysed. For the measurements the DNW-LST wind tunnel has been used, which is a very accurate, low turbulence wind tunnel in the Netherlands.

Instead of taking an average over 30 seconds at each wind speed (with a sample frequency of at least 1 Hz) 1 second measurements are done at each wind speed, with 100 samples at each wind speed. This way various calibrations with various combinations of the values for N and M can be performed. An overview of the calibrations that have been performed is presented in Table 3-1.

Table 3-1. Overview of the various combinations of the values for N and M

Calibration cycle 1 4(2)16, 15(-2)5 13 wind speeds
Uncertainty interval $\alpha = 0.05$

Run no.		N	M	No. of points n in regression	n-2 used for $t_{\alpha/2}$	$t_{\alpha/2,n-2}$
0	1 * 1sec per WS	1	1 sec	13	11	2.201
1	2 * 1sec per WS	2	1 sec	26	24	2.064
2	5 * 1sec per WS	5	1 sec	65	60	2.000
3	10 * 1sec per WS	10	1 sec	130	120	1.980
4	20 * 1sec per WS	20	1 sec	260	infinite	1.960
5	50 * 1sec per WS	50	1 sec	650	infinite	1.960
6	100 * 1sec per WS	100	1 sec	1300	infinite	1.960

Calibration cycle 2 4(2)16, 15(-2)5 13 wind speeds
Uncertainty interval $\alpha = 0.05$

Run no.		N	M	No. of points n in regression	n-2 used for $t_{\alpha/2}$	$t_{\alpha/2,n-2}$
0	1 * 1 sec per WS	1	1 sec	13	11	2.201
1	1 * 2 sec per WS	1	2 sec	13	11	2.201
2	1 * 5 sec per WS	1	5 sec	13	11	2.201
3	1 * 10 sec per WS	1	10 sec	13	11	2.201
4	1 * 20 sec per WS	1	20 sec	13	11	2.201
5	1 * 50 sec per WS	1	50 sec	13	11	2.201
6	1 * 100 sec per WS	1	100 sec	13	11	2.201

Calibration cycle 3 4(2)16, 15(-2)5 13 wind speeds
Uncertainty interval $\alpha = 0.05$

Run no.		N	M	No. of points n in regression	n-2 used for $t_{\alpha/2}$	$t_{\alpha/2,n-2}$
0	1 * 10 sec per WS	1	10 sec	13	11	2.201
1	2 * 10 sec per WS	2	10 sec	26	24	2.064
2	5 * 10 sec per WS	5	10 sec	65	60	2.000
3	10 * 10 sec per WS	10	10 sec	130	120	1.980

Calibration cycle 4 4(2)16, 15(-2)5 13 wind speeds
Uncertainty interval $\alpha = 0.05$

Run no.		N	M	No. of points n in regression	n-2 used for $t_{\alpha/2}$	$t_{\alpha/2,n-2}$
0	2 * 1 sec per WS	2	1 sec	26	24	2.064
1	2 * 2 sec per WS	2	2 sec	26	24	2.064
2	2 * 5 sec per WS	2	5 sec	26	24	2.064
3	2 * 10 sec per WS	2	10 sec	26	24	2.064
4	2 * 20 sec per WS	2	20 sec	26	24	2.064
5	2 * 50 sec per WS	2	50 sec	26	24	2.064

Calibration cycle 5 4(2)16, 15(-2)5 13 wind speeds
Uncertainty interval $\alpha = 0.05$

		N	M	No. of points n in regression	n-2 used for $t_{\alpha/2}$	$t_{\alpha/2,n-2}$
IEA	2 * 30 sec per WS	2	30 sec	26	24	2.064
MEASNET	1 * 30 sec per WS	1	30 sec	13	24	2.064

N.B. the notation 4(2)16, 15(-2)5 designates the wind tunnel speed cycle, from 4 to 16 with steps of 2 m/s and then from 15 to 5 with steps of -2 m/s. The resulting cycle is 4, 6, 8, 10, 12, 14, 16, 15, 13, 11, 9, 7, 5 m/s.

The uncertainty that is reported is calculated according to Montgomery [8] and it is the standard deviation of the Y-estimate. From Table 3-1 can be seen that calibrations series have been done where the number of data points per wind speed have been kept constant while the time per data point varied, as well as vice-versa.

3.4 Linear regression analysis

The data from the calibration measurements will be analysed using a simple linear regression model [8]:

$$y = \beta_0 + \beta_1 x + \varepsilon \tag{1}$$

with the intercept β_0 and the slope β_1 as unknown constants called *regression coefficients* and a random error component ε. The error components are assumed to have mean zero, $E(\varepsilon)=0$, and unknown variance σ^2. Furthermore it is assumed that the errors are uncorrelated. Formula (1) is called the *population regression model*.

3.4.1 Estimate of the regression coefficients

When n pairs of data (y_i, x_i) are available, formula (1) can be rewritten to a *sample regression model*:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \tag{2}$$

The method of least squares is used to estimate β_0 and β_1 i.e. the sum of the squares of the differences between the observations y_i and the straight regression line is a minimum. The estimators of β_0 and β_1 are denoted as $\hat{\beta}_0$ and $\hat{\beta}_1$. The solution of the least squares minimisation of (2) is:

$$\bar{y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x} \tag{3}$$

with

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{4}$$

and

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} y_i x_i - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)\left(\sum_{i=1}^{n} x_i\right)}{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}} \tag{5}$$

The estimator of the slope $\hat{\beta}_1$ can be rewritten to

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} y_i (x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} = \frac{S_{xy}}{S_{xx}} \tag{6}$$

The residual e_i is defined as the difference between the observed value y_i and and the corresponding fitted value $\hat{y}_i$, $e_i = y_i - \hat{y}_i = y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_i\right)$.

3.4.2 Estimation of uncertainties

First an estimation of σ^2 is determined following [8]. The estimate of σ^2 is obtained using the residual, or error sum of squares:

$$SS_{\mathrm{Res}}=\sum_{i=1}^{n}e_i^2=\sum(y_i-\hat{y}_i)^2 \tag{7}$$

where y_i are the observed values and $\hat{y}_i$ the fitted values. Using $\hat{y}_i=\hat{\beta}_0+\hat{\beta}_1x_i$ equation (7) can be rewritten to

$$SS_{\mathrm{Res}}=\sum_{i=1}^{n}y_i^2-n\bar{y}^2-\hat{\beta}_1S_{xy}$$

or to

$$SS_{\mathrm{Res}}=SS_T-\hat{\beta}_1S_{xy}$$

with

$$SS_T=\sum_{i=1}^{n}y_i^2-n\bar{y}^2=\sum_{i=1}^{n}(y_i-\bar{y})^2$$

The expected value of SS_{Res} is

$$E(SS_{\mathrm{Res}})=(n-2)\sigma^2 \tag{8}$$

since the residual sum of squares has (n-2) degrees of freedom, because two degrees of freedom are associated with the estimates $\hat{\beta}_0$ and $\hat{\beta}_1$. Hence an *unbiased estimator* of σ^2 is

$$\hat{\sigma}^2=\frac{SS_{\mathrm{Res}}}{(n-2)}=MS_{\mathrm{Res}} \tag{9}$$

The quantity MS_{Res} is called the *residual mean square*. The quantity $\sqrt{\hat{\sigma}^2}$ is also called the *standard error of regression*. Any violation of the model assumptions or misspecification of the model form may however affect the usefulness of $\hat{\sigma}^2$ as an estimate of σ^2, therefore it is a *model-dependent* estimate of σ^2.

Next an estimation of the uncertainty in the regression parameters is determined. The variance in the parameters $\hat{\beta}_1$ and $\hat{\beta}_0$ is given by [8]:

$$Var(\hat{\beta}_1)=\frac{\sigma^2}{S_{xx}} \tag{10}$$

and, since the covariance between $\bar{y}$ and $\hat{\beta}_1$ is zero,

$$Var(\hat{\beta}_0)=Var(\bar{y})+\bar{x}^2Var(\hat{\beta}_1)=\sigma^2\left(\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right) \tag{11}$$

with

$$Var(\bar{y})=\frac{\sigma^2}{n} \tag{12}$$

The covariance between the regression parameters $\hat{\beta}_0$ and $\hat{\beta}_1$ is given by:

$$Cov(\hat{\beta}_0,\hat{\beta}_1)=\frac{-\bar{x}\sigma^2}{S_{xx}} \tag{13}$$

The variance of a single data point y_i given the n measurements (x_i, y_i) is given by [8]:

$$Var(y_i)=x_i^2 Var(\hat{\beta}_1)+Var(\hat{\beta}_0)+2x_i Cov(\hat{\beta}_0,\hat{\beta}_1) \tag{14}$$

3.4.3 Uncertainty in the prediction of new observations

With the analysis shown in section 3.4.1 it is possible to predict new observations using the regression coefficients. Therefore the following regression equation can be used:

$$\hat{y}_0=\hat{\beta}_0+\hat{\beta}_1 x_0 \tag{15}$$

with x_0 the value of the regressor variable of interest and $\hat{y}_0$ the point estimate of the new value of response y_0. For the prediction of the value of new observations, a prediction interval must be determined from the distribution.

A (1-α) prediction interval on a future observation y_0 is now [8]:

$$y_0=\hat{y}_0 \pm t_{\alpha/2,n-2}\sqrt{MS_{\mathrm{Res}}\left(1+\frac{1}{n}+\frac{(x_0-\bar{x})^2}{S_{xx}}\right)} \tag{16}$$

with $t_{\alpha/2,\,n-2}$ the percentage point of the t-Distribution with n-2 degrees of freedom. The prediction interval (16) widens as $|x_0-\bar{x}|$ increases.

3.5 Methodology

3.5.1 Averaging

To perform the regression analysis with the calibration data as shown in Table 3-1 in case $M > 1$ seconds the following additions to section 3.4 can be made:

$$y_i=\frac{1}{M}\sum_{j=1}^{M} s_{k,j} \qquad x_i=\frac{1}{M}\sum_{j=1}^{M} f_{k,j} \tag{17}$$

with $s_{k,j}$ the wind tunnel speed and $f_{k,j}$ the cup anemometer frequency at sample j (seconds) and wind speed k (m/s). The range of k is listed in Table 3-1 and is from 4 to 16 m/s in steps of 2 m/s and from 15 to 5 m/s in steps of -2 m/s, these are 13 wind speeds in total. The range of i therefore is [13N], this equals [n]. The range of M is also listed in Table 3-1 and is [1, 2, 5, 10, 20, 30, 50, 100] seconds. Note that N is the number of data points at each wind speed in the linear regression analysis and M the time each data point represents (see section 3.2).

3.5.2 Prediction table and uncertainty calculation

After the regression analysis and the determination of the regression coefficients (equations 3 & 6) a prediction table is made using these coefficients and the uncertainty calculation in the prediction of new observations (equation 10). The prediction for new observations is done using frequencies belonging to 13 wind speeds of $\hat{y}_0$ = 4 to 16 m/s in steps of 1 m/s. The resulting uncertainties are averaged per run in a cycle (see Table 3-1):

$$U_{c,r} = \frac{1}{13}\sum_{\hat{y}_0=4}^{16} u_{c,r,\hat{y}_0} = \frac{1}{13}\sum_{\hat{y}_0=4}^{16} t_{a/2,n-2}\sqrt{MS_{\mathrm{Res}}\left(1+\frac{1}{n}+\frac{(\hat{y}_0-\hat{\beta}_0-\bar{x}\hat{\beta}_1)^2}{\hat{\beta}_1^{\,2}\, S_{xx}}\right)} \qquad (18)$$

with $U_{c,r}$ the averaged uncertainty for cycle c and run r.

3.5.3 Examples of uncertainty calculation

Two examples will be given here of the uncertainty analysis using inverse regression for the Risø cup, for N=2 and M=1 and for N=1 and M=2.

Example with N=2 and M=1

This example is equivalent to cycle 1 run 1, with 2 regression data points per wind speed over 1 second per regression point. Note that cycle 1 run 0 and cycle 2 run 0 are actually equivalent.

Table 3-2. Regression data cycle 1 run 1

s [m/s]	f [Hz]	y_i	x_i
4.01366	6.11	4.01366	6.11
4.02924	6.11	4.02924	6.11
5.00103	7.72	5.00103	7.72
5.00125	7.72	5.00125	7.72
6.04819	9.42	6.04819	9.42
6.04819	9.34	6.04819	9.34
7.02091	11.07	7.02091	11.07
7.02091	11.07	7.02091	11.07
8.05948	12.68	8.05948	12.68
8.06205	12.68	8.06205	12.68
8.99809	14.11	8.99809	14.11
9.00096	14.11	9.00096	14.11
10.07100	15.91	10.07100	15.91
10.07330	15.85	10.07330	15.85
11.00340	17.41	11.00340	17.41
11.00340	17.41	11.00340	17.41
12.03890	19.11	12.03890	19.11
12.03890	19.08	12.03890	19.08
12.98760	20.72	12.98760	20.72
12.98810	20.47	12.98810	20.47
14.01550	22.32	14.01550	22.32
14.01410	22.12	14.01410	22.12
15.01870	23.84	15.01870	23.84
15.02280	23.70	15.02280	23.70
15.94700	25.25	15.94700	25.25
15.94830	25.25	15.94830	25.25

Table 3-3. Prediction table cycle 1 run 1

x_0 [Hz]	$\hat{y}_0$ [m/s]	$u_{1,1,\hat{y}_0}$ [m/s]
6.13	4	0.0991
7.74	5	0.0978
9.34	6	0.0967
10.95	7	0.0958
12.55	8	0.0951
14.16	9	0.0948
15.76	10	0.0946
17.37	11	0.0948
18.97	12	0.0951
20.58	13	0.0958
22.18	14	0.0966
23.79	15	0.0977
25.39	16	0.0990

The average uncertainty is:
$U_{1,1}$ = 0.0964 m/s

In this case $t_{\alpha/2,n-2}$ = 2.064 (see Table 3-1) with a confidence interval of (1-α) = 95%. The regression coefficients are: slope $\hat{\beta}_1$ = 0.62288 m and intercept $\hat{\beta}_0$ = 0.18200 m/s. The prediction table is shown in Table 3-3.

Example with N=1 and M=2

This example is equivalent to Cycle 2 run 1, with 1 regression data point per wind speed over 2 seconds averaged per regression point. In this case $t_{\alpha/2,n-2}$ = 2.201 (see Table 3-1) with a confidence interval of (1-α) = 95%. The regression coefficients are: slope $\hat{\beta}_1$ = 0.62293 m and intercept $\hat{\beta}_0$ = 0.18129 m/s. The prediction table is presented in Table 3-5.

Table 3-4. Regression data cycle 2 run 1

s [m/s]	f [Hz]	y_i	x_i
4.01366	6.11	4.02145	6.11
4.02924	6.11		
5.00103	7.72	5.00114	7.72
5.00125	7.72		
6.04819	9.42	6.04819	9.38
6.04819	9.34		
7.02091	11.07	7.02091	11.07
7.02091	11.07		
8.05948	12.68	8.06077	12.68
8.06205	12.68		
8.99809	14.11	8.99953	14.11
9.00096	14.11		
10.07100	15.91	10.07215	15.88
10.07330	15.85		
11.00340	17.41	11.00340	17.41
11.00340	17.41		
12.03890	19.11	12.03890	19.10
12.03890	19.08		
12.98760	20.72	12.98785	20.60
12.98810	20.47		
14.01550	22.32	14.01480	22.22
14.01410	22.12		
15.01870	23.84	15.02075	23.77
15.02280	23.70		
15.94700	25.25	15.94765	25.25
15.94830	25.25		

Table 3-5. Prediction table cycle 2 run 1

x_0 [Hz]	$\hat{y}_0$ [m/s]	$u_{1,1,\hat{y}_0}$ [m/s]
6.13	4	0.0792
7.74	5	0.0773
9.34	6	0.0757
10.95	7	0.0744
12.55	8	0.0735
14.16	9	0.0729
15.76	10	0.0727
17.37	11	0.0729
18.97	12	0.0734
20.58	13	0.0744
22.18	14	0.0756
23.79	15	0.0772
25.39	16	0.0791

The average uncertainty is
$U_{2,1}$ = 0.0753 m/s

3.6 Uncertainty of the regression

In this Section, the uncertainty of the regression applied to the cup anemometer calibration data is investigated for the same cup anemometer calibrated in the DEWI wind tunnel and the DNW wind tunnel. The uncertainties of the regression for the Risø cup anemometer in the DEWI wind tunnel are presented in Table 3-6, those of the DNW wind tunnel are presented in Table 3-7. Sya is the standard deviation for y according to the MEASNET document (eq. 14), Ua is the resulting type A standard uncertainty, with a confidence interval of 95% (k = 2). The resulting uncertainty of the applied regression is quite similar for the DNW and the DEWI wind tunnel. The type A uncertainties according [1] are larger than these uncertainties attached to the regression (See also Appendix A).

Table 3-6. Calibration data and uncertainties of a Risø cup in the DEWI wind tunnel

Tunnel speed [m/s]	Cup freq [Hz]	Sya [m/s]	Ua [m/s]
4.301	6.515	0.0091	0.0181
5.351	8.188	0.0079	0.0159
6.343	9.750	0.0070	0.0139
7.448	11.500	0.0060	0.0120
8.372	13.063	0.0053	0.0106
9.425	14.719	0.0049	0.0097
10.219	16.032	0.0047	0.0095
11.230	17.656	0.0049	0.0098
12.322	19.355	0.0054	0.0108
13.298	20.909	0.0061	0.0122
14.382	22.688	0.0071	0.0142
15.167	23.938	0.0079	0.0158
15.762	24.903	0.0085	0.0171

Table 3-7. Calibration data and uncertainties of a Risø cup in the DNW wind tunnel

Tunnel speed [m/s]	Cup freq [Hz]	Sya [m/s]	Ua [m/s]
4.0152627	6.1807	0.00911	0.01823
4.9973663	7.7497	0.00807	0.01613
6.0331980	9.4390	0.00702	0.01404
7.0183713	10.9727	0.00618	0.01236
8.0561847	12.6767	0.00542	0.01085
9.0007093	14.1047	0.00500	0.01000
10.0748600	15.8923	0.00481	0.00963
11.0041933	17.3157	0.00497	0.00994
12.0367333	18.9813	0.00546	0.01093
12.9926767	20.5317	0.00615	0.01231
14.0130000	22.1910	0.00707	0.01413
15.0214667	23.7813	0.00805	0.01611
15.9471967	25.2830	0.00905	0.01811

3.7 Results of the analysis

These calibrations have been done for six different cup anemometers: three different Mierij cup anemometers, a Risø cup anemometer, a Lambrecht cup anemometer and an NRG cup anemometer. The results from the Risø cup anemometer and the Mierij cup anemometer (ECN identification DEST0005) for cycle 1 and 4 can be seen in Figure 3-1 and Figure 3-2. In these figures no clear dependency can be observed of the number of points per wind speed and the calibration uncertainty.

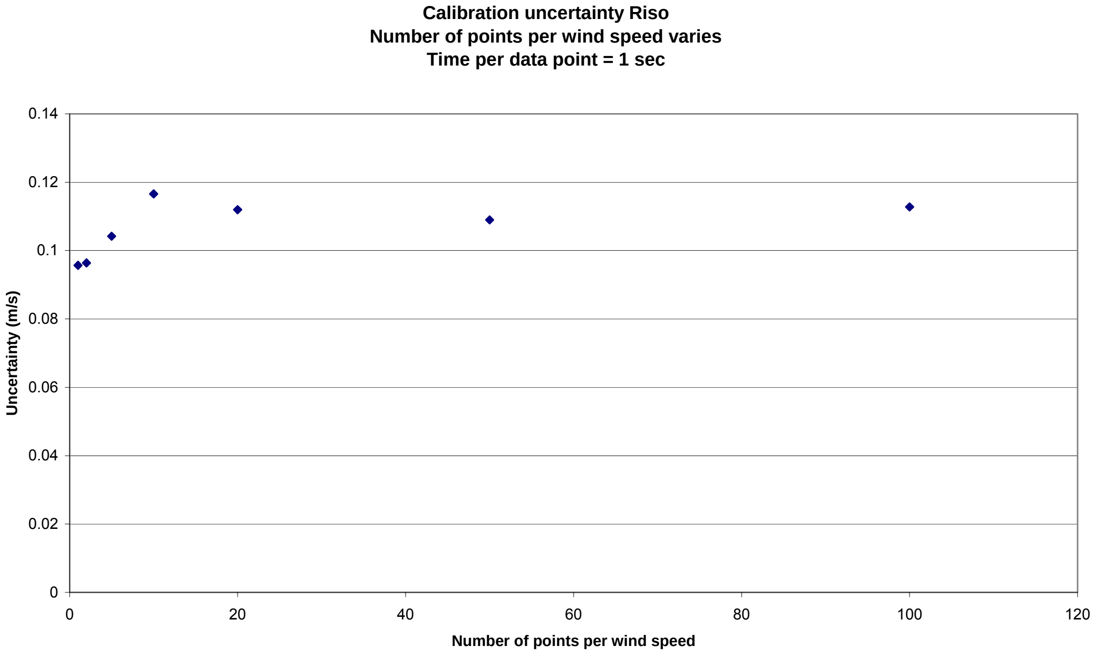

Figure 3-1 Calibration uncertainty versus the number of data points per wind speed for the Risø cup anemometer, cycle 1

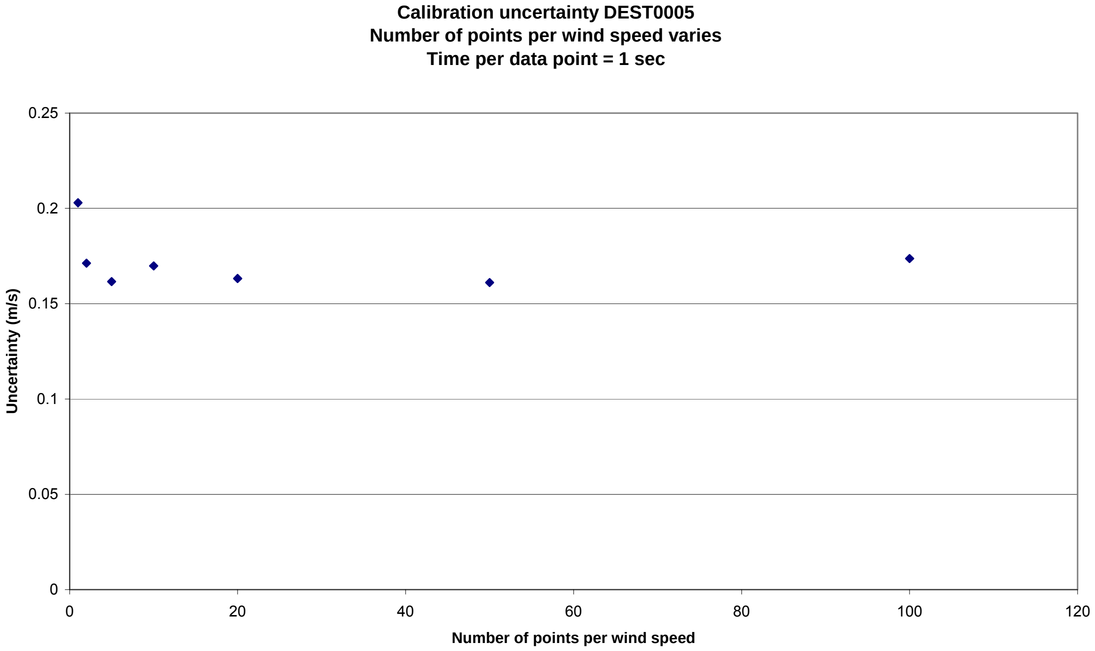

Figure 3-2 Calibration uncertainty versus the number of data points per wind speed for the Mierij cup anemometer DEST0005, cycle 1

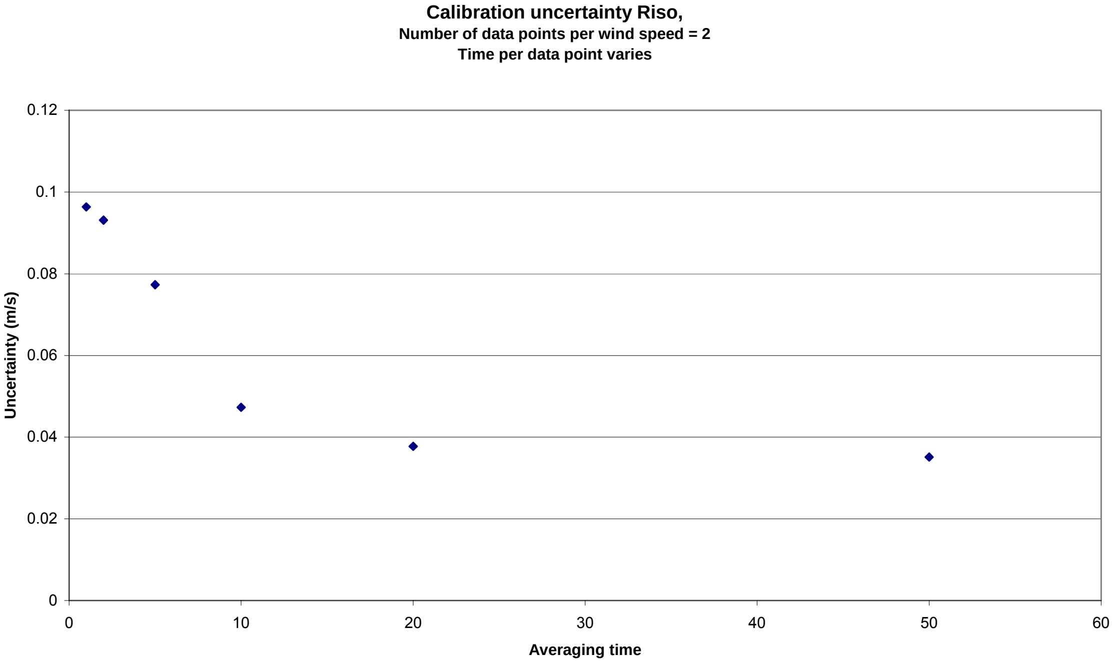

Figure 3-3 Calibration uncertainty versus the time per data point for the Risø cup anemometer, cycle 4

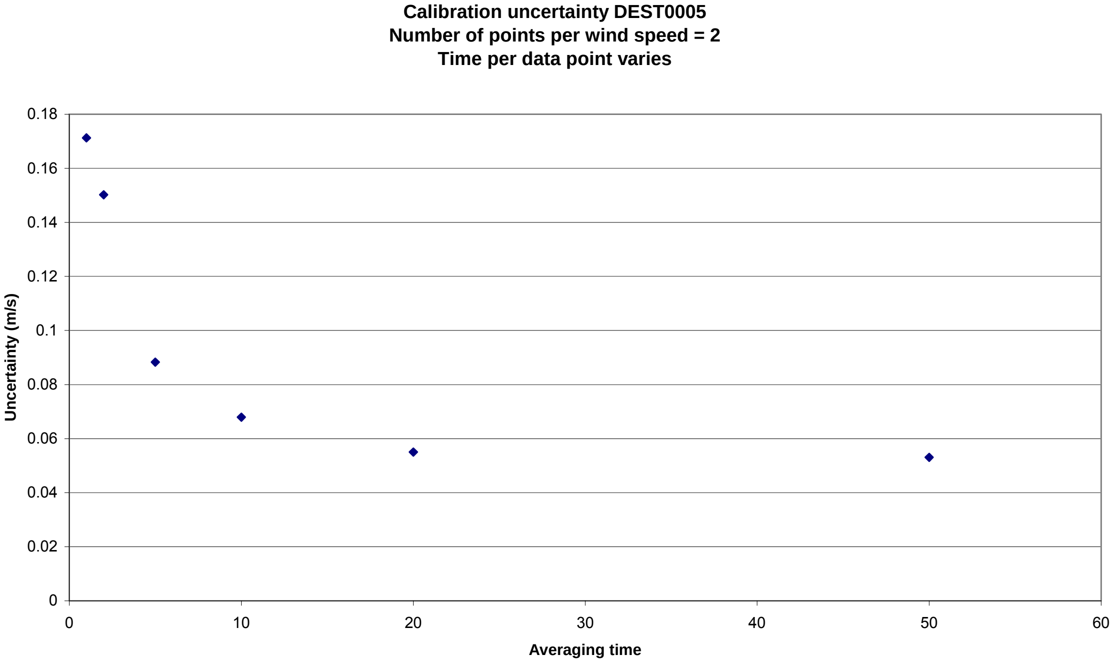

Figure 3-4 Calibration uncertainty versus the time per data point for the Mierij cup anemometer DEST0005, cycle 4

From Figure 3-3 and Figure 3-4, a dependency is observed of the time per data point on the calibration uncertainty. From these results the conclusion is drawn that the number of data points per wind speed does not have a noticeable influence on the calibration uncertainty of these two cups. For the time per data point the picture is completely different. It is observed that the uncertainty decreases with increasing averaging time.

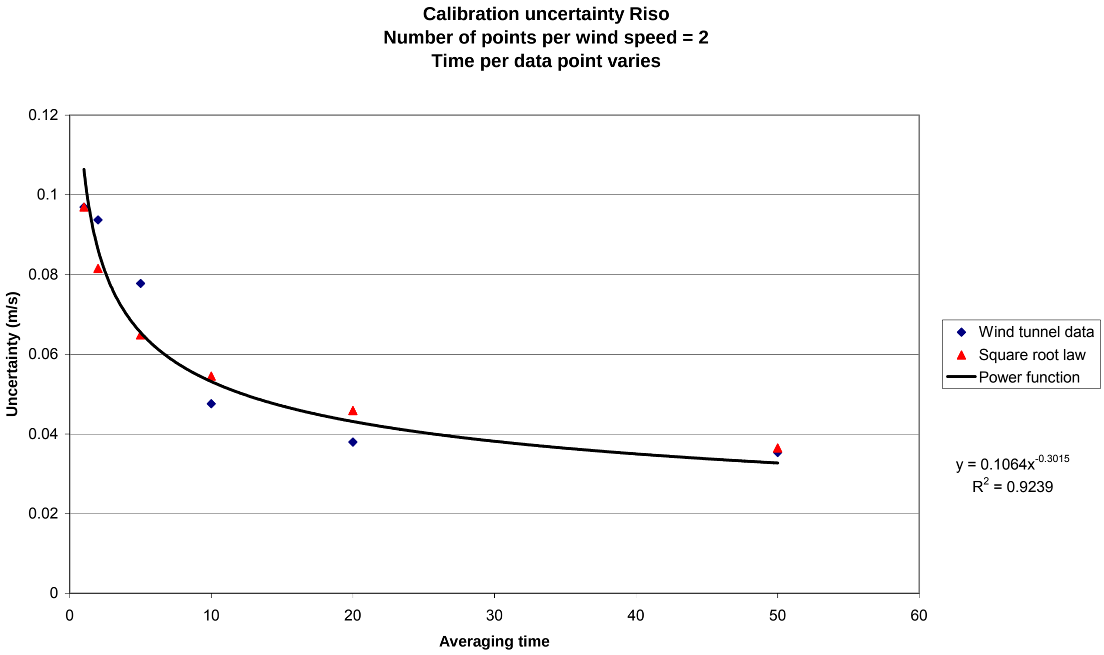

Figure 3-5 Fitting the relationship between the uncertainty and *M* using the square root law and a power function

For the Risø cup anemometer a relationship has been tried to find between *M* and the uncertainty, see Figure 3-5. A fit of the data with a power function leads to an empirical relationship between the uncertainty (y) and the time per data point (M):

$$y = 0.1064 * M^{-0.3015} \tag{19}$$

This is an empirical relationship that is not intuitive. The Guide to the expression of uncertainty in measurement [4] (GUM) gives the following relationship:

$$s^2(\bar{q}) = \frac{s^2(q_k)}{n} \tag{20}$$

This gives the relationship between the best estimate of the variance (*s*) of a quantity *q* that has been measured *n* times. In line with the previous equation a good fit is found when using:

$$\sigma_{\hat{y},M} \propto \frac{\sigma_{\hat{y},M=1}}{\sqrt[4]{M}} \tag{21}$$

This gives the relationship between the standard deviation of the Y–estimate and the time per data point (M). This function (21) fits as well as the power function and intuitively agrees to the GUM.

3.8 Response of cup anemometer in wind tunnel

The requirements of cup anemometer calibrations state a minimum sample frequency of 1Hz. Experiments using cup anemometers in wind tunnels have shown unstable behaviour of the cup anemometers themselves. In this section, we summarise some of the results of measurements in wind tunnels as performed by Frank Ormel in 2003. The measurements have been done in the German-Dutch DNW wind tunnel. The applied cup anemometers are summarised in Table 3-8. The cup anemometers are compared to measurements using a propeller anemometer. It is shown in Figure 3-6 and Figure 3-7 that the propeller anemometer has a very stable response, although it is able to measure fast fluctuations in wind speed. This shows that the wind speed of the tunnel is very stable. The fluctuating cup anemometer response is therefore the result of the cup anemometers themselves. This typical behaviour has been confirmed by measurements of FOI [9] in the FOI LT5 wind tunnel and the KTH-MTL tunnel that has turbulence levels lower than 0.025%.

Table 3-8. Overview of cup anemometers used in the analysis.

Brand	Type	ECN identification number	Reference
Mierij	018	DEST0005	Mierij 1
Mierij	019	DEWS0299	Mierij 2
Mierij	020	DEWS0418	Mierij 3
Lambrecht		DEWS0230	Lambrecht
NRG	Max40 dynamo	DEWS0366	NRG
Riso	P2546	DEWS0600	Riso
Gill	R3A sonic	DEWS0583	Gill

The measurements using the propeller anemometer have been confirmed in a separate measurement campaign. In these measurements, the propeller anemometer is compared to measurements with a Gill sonic anemometer. The responses of these anemometers are quite similar and show the same standard deviation around the mean. Compared to cup anemometers, the propeller and Gill sonic anemometer measure the relatively stable wind speed of the tunnel more accurate and it has been confirmed that the fluctuating response of cup anemometers is due to the cup anemometer itself.

The findings of these experiments support the findings in the previous section. The averaging time of wind speed measurements by cup anemometers in wind tunnels should be increased to reduce the cup anemometer calibration uncertainty. When the averaging time is increased, the fast fluctuations are averaged out and the standard deviation of the points in the regression of the cup anemometer calibration are reduced.

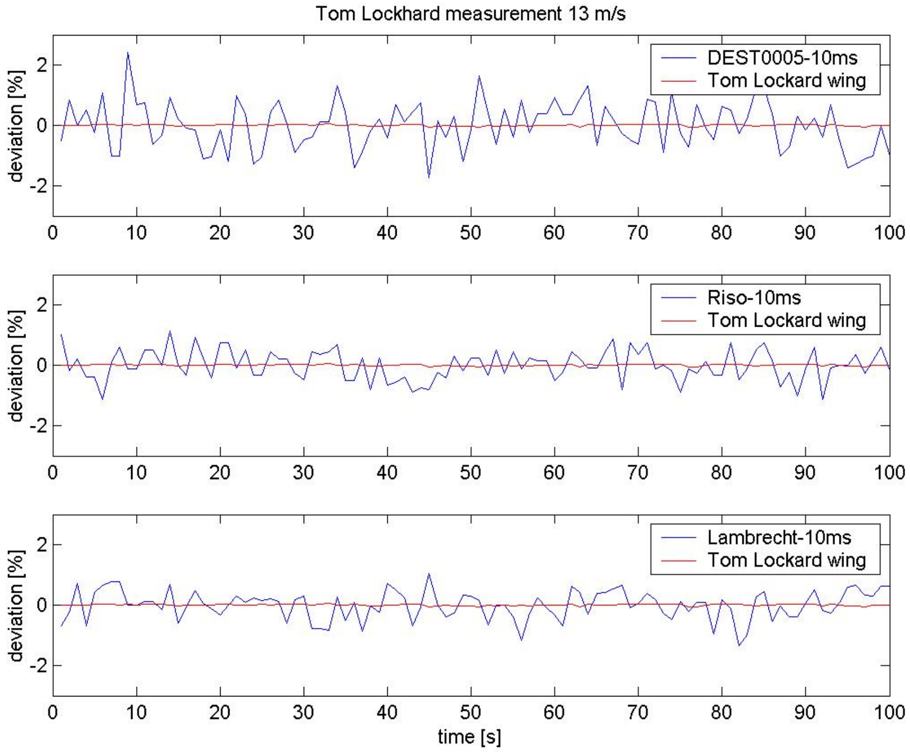

Figure 3-6. Shown is the comparison of wind speed measurements with a cup anemometer and a propeller anemometer. A Mierij (DEST0005), a Risø and a Lambrecht cup anemometer are compared to the propeller.

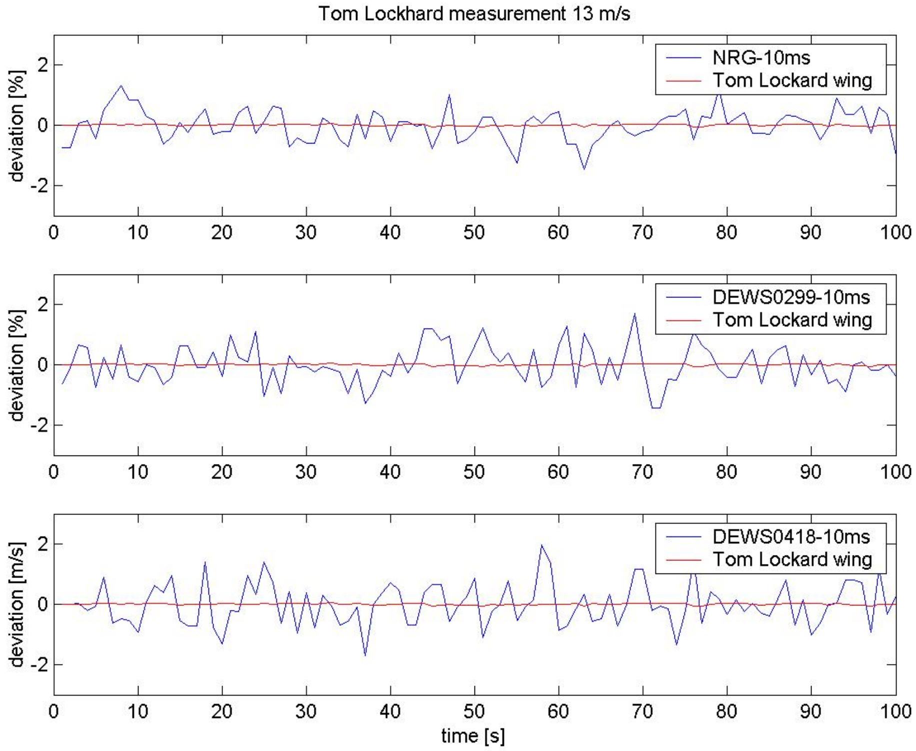

Figure 3-7. Shown is the comparison of wind speed measurements with a cup anemometer and a propeller anemometer. An NRG, and two Mierij (DEWS0299 and DEWS0418) cup anemometers are compared to the propeller.

3.9 Discussion

The variability of cup anemometers in wind tunnels leads to fluctuations in the wind speed measurements that originate from the unstable wakes behind the cups. Since the requirements on cup anemometer calibration require sample frequencies larger than 1Hz, these fluctuations are captured in the calibration procedure. Given the remarkable linearity of cup anemometers against wind speed, the regression through the averaged measurements of the calibration procedure leads to much smaller uncertainties. One could argue whether a minimum sample frequency of 1 sec would lead to higher type A uncertainties than longer averaging periods. The averaged values of the measurements using cup anemometers are much better than one would suspect from the variation in wind speed measurements during the 30-second period. This report will be followed by a request for information at MEASNET accredited wind tunnels, focussing on typical values for the type A and type B uncertainties of Risø cup anemometers and spread in measurements using the so-called golden reference cup anemometers.

For the ten Risø cup anemometers that are calibrated at the same day at DEWI, the type A uncertainty at 10m/s is determined using the derived type B uncertainty (see Chapter 2). In Figure 3-8 the total uncertainties (1σ) and the derived type A uncertainties are shown using the type B uncertainty of 0.03 m/s, as derived in Chapter 2. In the case of the DEWI wind tunnel, the type A uncertainty is of the order of the type B uncertainty or larger and has a large spread.

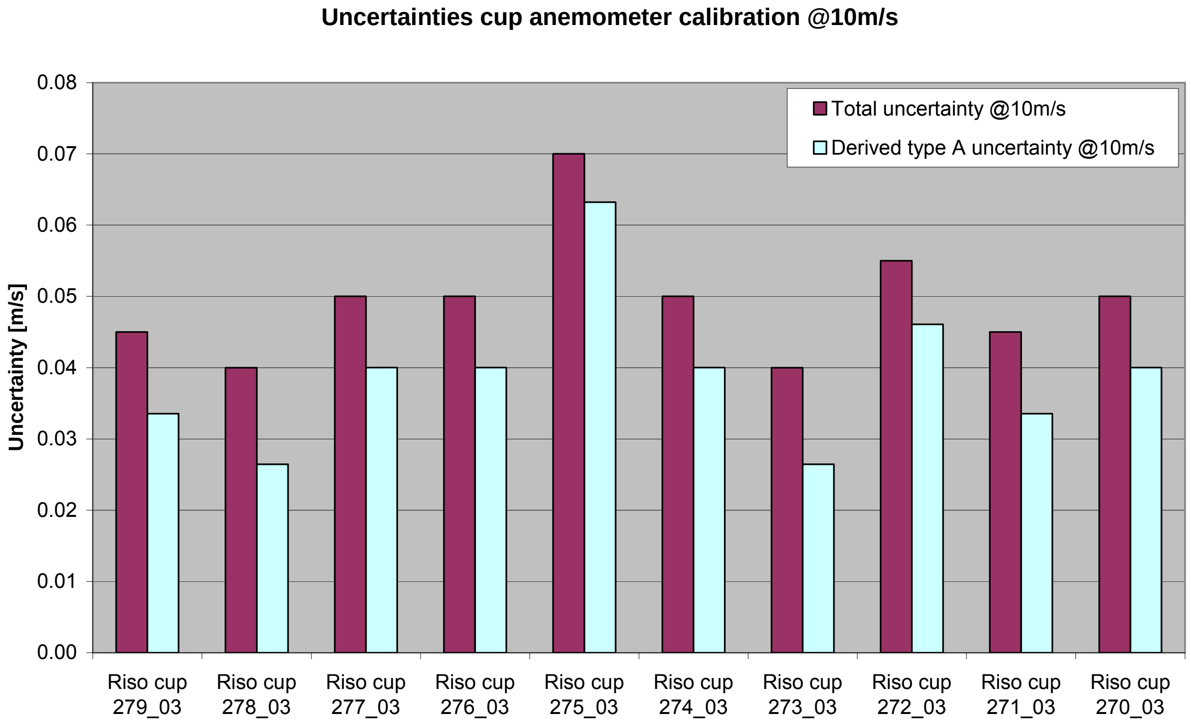

Figure 3-8. The total uncertainties and the derived type A uncertainties are shown for the ten Risø cup anemometers that are calibrated at the same day at DEWI. The type B uncertainty derived in Chapter 2 has been used (=0.03m/s).

4. Conclusions

ECN calibrates its cup-anemometers in the DEWI wind tunnel. For power performance measurements under MEASNET and IEC regulations uncertainty analyses are important. Since the calibration uncertainty is a large contribution to the uncertainty analysis in power performance measurements, ECN started the analyses of the reported uncertainties in the DEWI wind tunnel. Since DEWI does not report the type A and type B uncertainties separately, this poses a problem for the analysis. Both types of uncertainties are investigated separately.

A detailed analysis of the type B uncertainty has been performed. The analyses showed (minor) mistakes in the MEASNET document for cup-anemometer calibrations [1] and the proposal for the IEC document on power performance measurements [2]. This led to recommendations to the IEC, see section 2.4. The type A uncertainty of the cup anemometer calibration can be derived from the combined calibration uncertainty of the DEWI calibrations now the value for the type B uncertainty has been determined. ECN has calibrated ten anemometers at a single day and the spread in type A uncertainty is rather large, ranging from 0.026m/s to 0.063m/s.

The analysis of type A uncertainties starts with a description of the uncertainty of the regression. It has been shown that the measurements of cup anemometers include variations that originate from the cup itself. The resulting type A uncertainty, determined by these fluctuations, is larger than would be necessary based on the regression analysis. Although the cup anemometer fluctuates on small time scales, the averaged values show very good linearity against wind speed. It should be investigated whether other averaging schemes during cup anemometer calibrations could reduce the associated uncertainties.

5. References

[1] Measnet Cup Anemometer Calibration Uncertainty, Version 1 September 1997

[2] IEC 61400-121, Wind turbine generator systems- Part 12: Wind Turbine Power Performance testing

[3] Calibration Certificate 279-03, DEWI Wilhelmshaven, 27 March 2003

[4] *Guide to the expression of uncertainty in measurement*, corrected and reprinted 1995, International Organisation for Standardization, Geneva 91995).

[5] D. C. Montgomery, E. A. Peck, G. G. Vining, *Introduction to linear regression analysis*, third edition, John Wiley & sons, inc New York (2001).

[6] *MEASNET Power performance measurement procedure*, version 3, November 2000, Network of European Measuring Institutes, (2000).

[7] J. P. Molly et al, *Implementation of the Network of European Measuring Institutes, MEASNET*, subproject VI of project European Wind turbine Standards II, DEWI, Wilhelmshaven (1998).

[8] Jan-Åke Dahlberg, 'Some aspects of cup anemometer behaviour and its effect on calibration', presentation at IMTS 2003, FOI/FFA

Appendix A Calibration Report 279_3 from 27.03.2003

DEUTSCHER KALIBRIERDIENST DKD

Kalibrierlaboratorium für Strömungsgeschwindigkeit von Luft
Calibration laboratory for velocity of air flow
Akkreditiert durch die / *accredited by the*
Akkreditierungsstelle des DKD bei der
PHYSIKALISCH-TECHNISCHEN BUNDESANSTALT (PTB)

DEUTSCHES WINDENERGIE
INSTITUT
WILHELMSHAVEN

Deutscher Akkreditierungs Rat
DKD-K-28901

Kalibrierschein
Calibration Certificate

Kalibrierzeichen
Calibration label
DKD-K-
28901
279_03

Gegenstand *Object*	Cup Anemometer
Hersteller *Manufacturer*	Risoe DK-4000 Roskilde
Typ *Type*	P 2546 A
Fabrikat/Serien-Nr. *Serial number*	Body: 1271 Cup: -
Auftraggeber *Customer*	ECN NL-1755 LE Petten
Auftragsnummer *Order No.*	279_03
Anzahl der Seiten des Kalibrierscheines *Number of pages of the certificate*	3
Datum der Kalibrierung *Date of calibration*	27.03.03

Dieser Kalibrierschein dokumentiert die Rückführung auf nationale Normale zur Darstellung der Einheiten in Übereinstimmung mit dem Internationalen Einheitensystem (SI).
Der DKD ist Unterzeichner der multilateralen Übereinkommen der European co-operation for Accreditation (EA) und der International Laboratory Accreditation Cooperation (ILAC) zur gegenseitigen Anerkennung der Kalibrierscheine.
Für die Einhaltung einer angemessenen Frist zur Wiederholung der Kalibrierung ist der Benutzer verantwortlich.

This calibration certificate documents the traceability to national standards, which realize the units of measurement according to the International System of Units (SI).
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The user is obliged to have the object recalibrated at appropriate intervals.

Dieser Kalibrierschein darf nur vollständig und unverändert weiterverbreitet werden. Auszüge oder Änderungen bedürfen der Genehmigung sowohl der Akkreditierungsstelle des DKD als auch des ausstellenden Kalibrierlaboratoriums. Kalibrierscheine ohne Unterschrift und Stempel haben keine Gültigkeit.

This calibration certificate may not be reproduced other than in full exept with the permission of both the Accreditation Body of the DKD and the issuning laboratory. Calibration certificates without signature and seal are not valid.

Stempel *Seal*	Datum *Date*	Leiter des Kalibrierlaboratoriums *Head of the calibration laboratory*	Bearbeiter *Person in charge*
DKD-K- 28901	27.03.03	i.V. P. Busch i.V. Dipl.-Ing.(FH) P.Busche	R.Kluin

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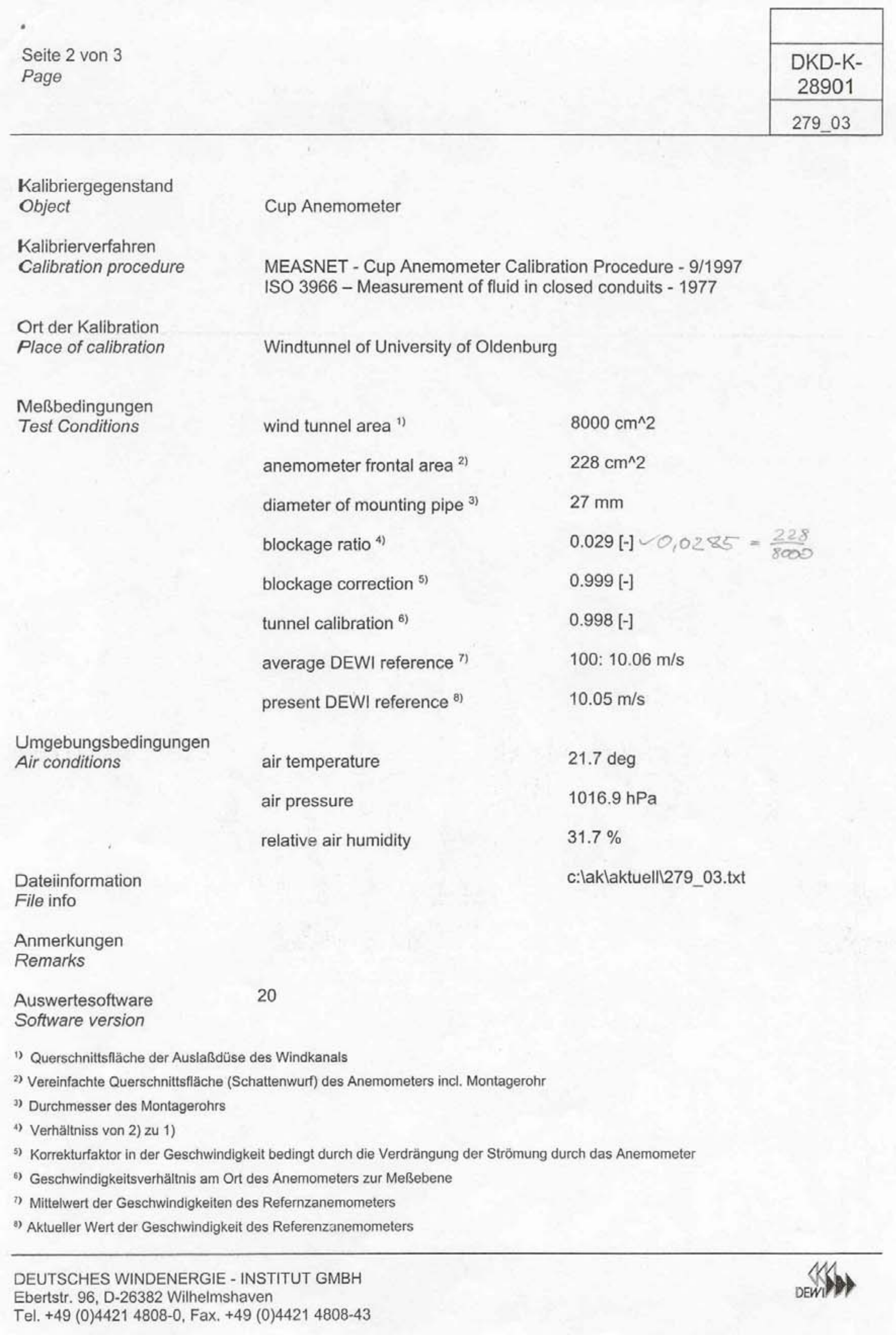


Kalibriergegenstand *Object*	Cup Anemometer	
Kalibrierverfahren *Calibration procedure*	MEASNET - Cup Anemometer Calibration Procedure - 9/1997 ISO 3966 – Measurement of fluid in closed conduits - 1977	
Ort der Kalibration *Place of calibration*	Windtunnel of University of Oldenburg	
Meßbedingungen *Test Conditions*	wind tunnel area [1]	8000 cm^2
	anemometer frontal area [2]	228 cm^2
	diameter of mounting pipe [3]	27 mm
	blockage ratio [4]	0.029 [-] ✓0,0285 = $\frac{228}{8000}$
	blockage correction [5]	0.999 [-]
	tunnel calibration [6]	0.998 [-]
	average DEWI reference [7]	100: 10.06 m/s
	present DEWI reference [8]	10.05 m/s
Umgebungsbedingungen *Air conditions*	air temperature	21.7 deg
	air pressure	1016.9 hPa
	relative air humidity	31.7 %
Dateiinformation *File* info		c:\ak\aktuell\279_03.txt
Anmerkungen *Remarks*		
Auswertesoftware *Software version*	20	

[1] Querschnittsfläche der Auslaßdüse des Windkanals

[2] Vereinfachte Querschnittsfläche (Schattenwurf) des Anemometers incl. Montagerohr

[3] Durchmesser des Montagerohrs

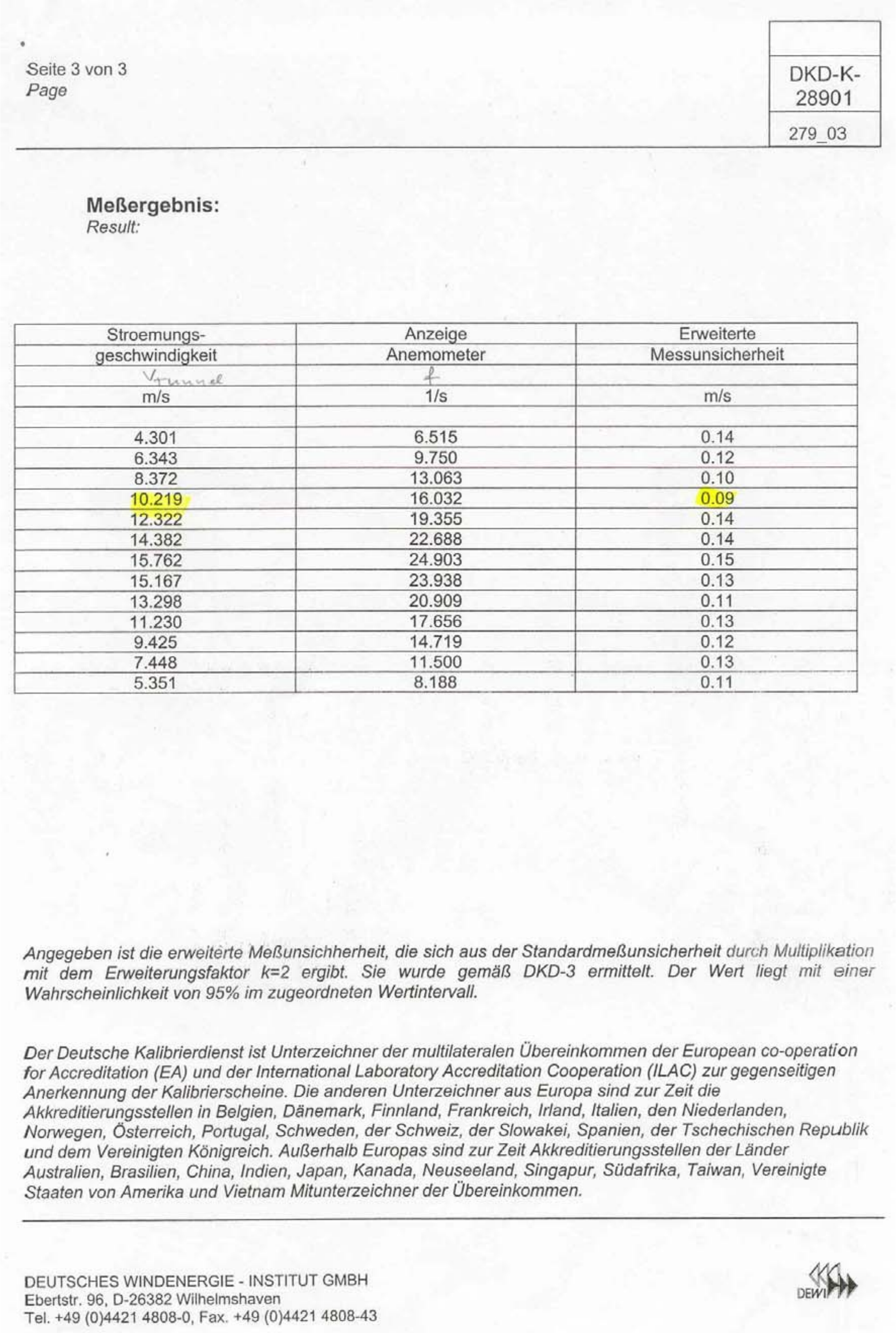
[4] Verhältniss von 2) zu 1)

[5] Korrekturfaktor in der Geschwindigkeit bedingt durch die Verdrängung der Strömung durch das Anemometer

[6] Geschwindigkeitsverhältnis am Ort des Anemometers zur Meßebene

[7] Mittelwert der Geschwindigkeiten des Refernzanemometers

[8] Aktueller Wert der Geschwindigkeit des Referenzanemometers



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Page

DKD-K-28901
279_03

Meßergebnis:
Result:

Stroemungs-geschwindigkeit	Anzeige Anemometer	Erweiterte Messunsicherheit
Vtunnel	f	
m/s	1/s	m/s
4.301	6.515	0.14
6.343	9.750	0.12
8.372	13.063	0.10
10.219	16.032	0.09
12.322	19.355	0.14
14.382	22.688	0.14
15.762	24.903	0.15
15.167	23.938	0.13
13.298	20.909	0.11
11.230	17.656	0.13
9.425	14.719	0.12
7.448	11.500	0.13
5.351	8.188	0.11

Angegeben ist die erweiterte Meßunsichherheit, die sich aus der Standardmeßunsicherheit durch Multiplikation mit dem Erweiterungsfaktor k=2 ergibt. Sie wurde gemäß DKD-3 ermittelt. Der Wert liegt mit einer Wahrscheinlichkeit von 95% im zugeordneten Wertintervall.

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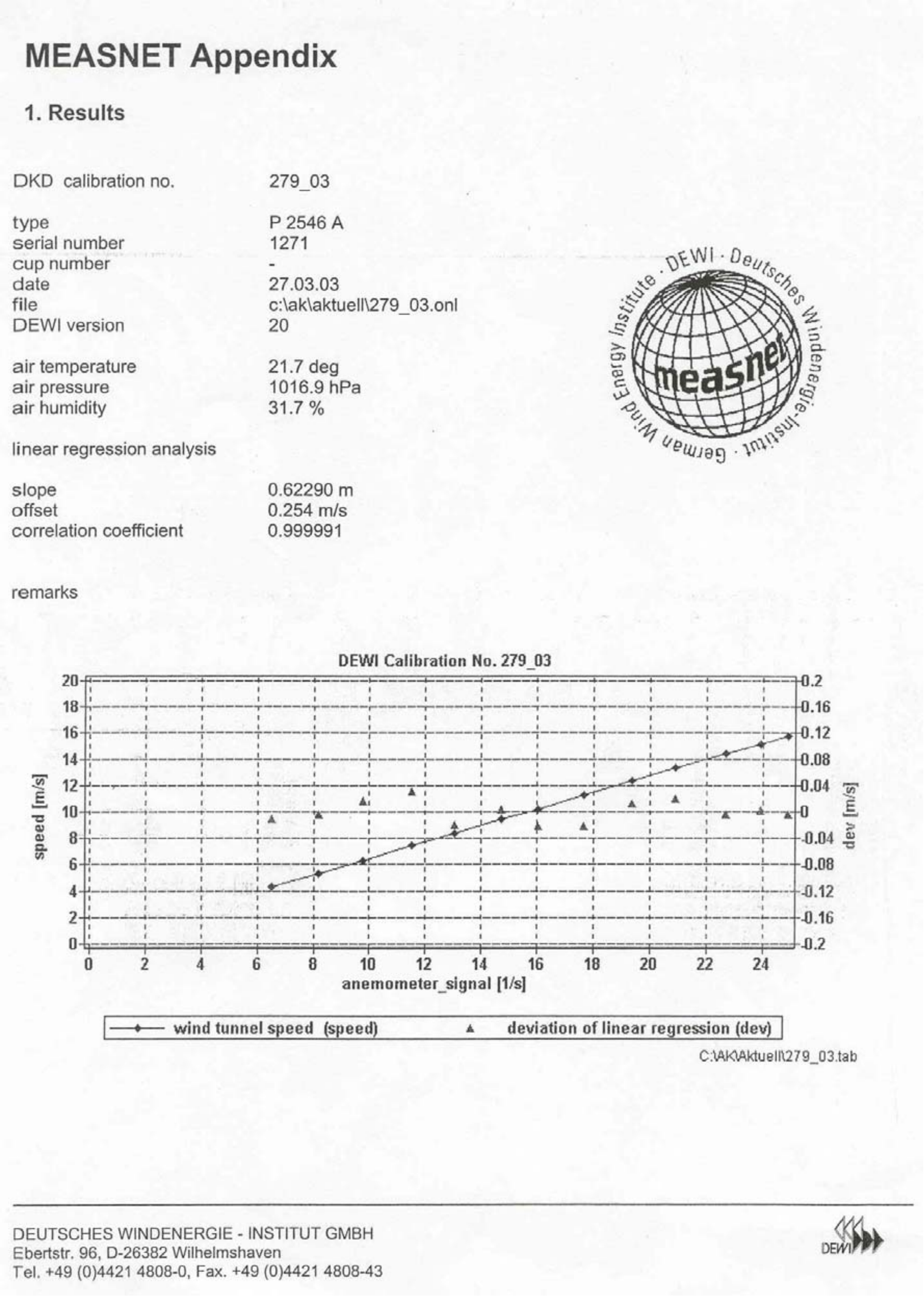

MEASNET Appendix

1. Results

DKD calibration no.	279_03
type	P 2546 A
serial number	1271
cup number	-
date	27.03.03
file	c:\ak\aktuell\279_03.onl
DEWI version	20
air temperature	21.7 deg
air pressure	1016.9 hPa
air humidity	31.7 %

linear regression analysis

slope	0.62290 m
offset	0.254 m/s
correlation coefficient	0.999991

remarks

DEWI Calibration No. 279_03

C:\AK\Aktuell\279_03.tab

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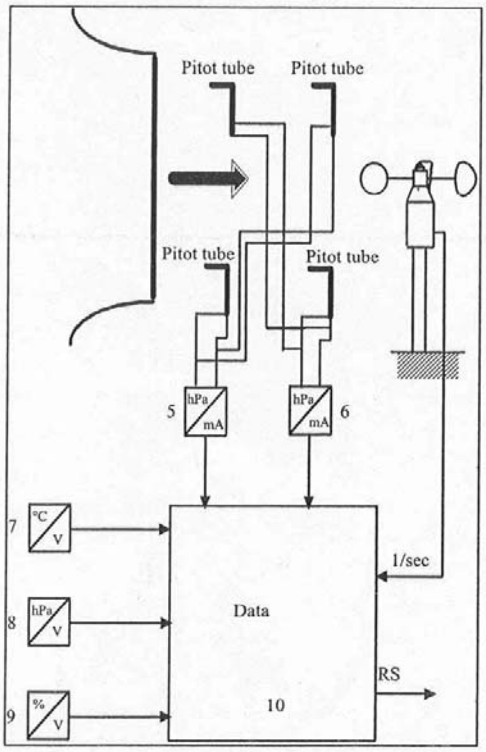
2 Instrumentation



Description of the data acquisition system

1 - 4 Pitot tube
Type: Airflow (ISO 3966)
Year: 1994
Calibration: No; ISO 3966 [1]

5 Pressure transducer
Type: ASHCROFT XLdp
Year: 1993
Calibration: 5/99

6 Pressure transducer
Type: ASHCROFT XLdp
Year: 1993
Calibration: recalibration DEWI

7 Thermometer
Type: Rotronic MP 300/340
Year: 1994
Calibration: checked with temperature standard

8 Barometer
Type: Vaisala PTA 427
Year: 1994
Calibration: checked with pressure standard

9 Humidity
Type: Rotronic MP 300/340
Year: 1994
Calibration: No

10 Data logger
Type: Ammonit V 492 B
Year: 1994
Calibration: checked with calibrated multi meter

Wind Tunnel: University of Oldenburg

Remark: Pressure standard is traceable calibrated by the German 'Eichamt' in 10/97
Temperature standard is traceable calibrated by the German 'Eichamt' in 4/94
The multi meter is traceable calibrated by the German 'DKD' in 5/99

3 Deviation to MEASNET Procedure

1. The time to get stable flow conditions between two steps in wind speed is approx. 30 seconds (it has been proved for this tunnel that 30 seconds are enough).

2. The humidity sensor is calibrated by the manufacturer.

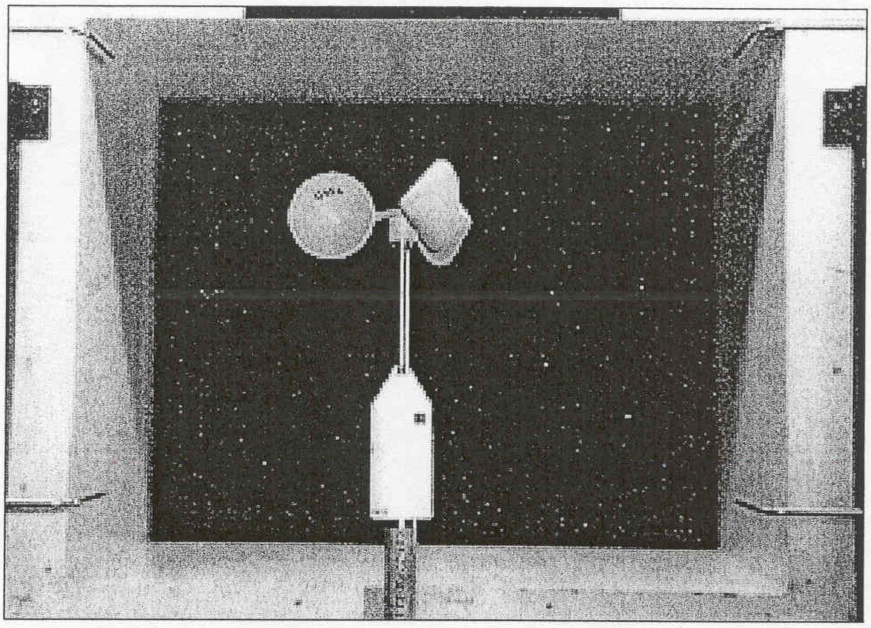


Photo showing the anemometer and the mounting system in the wind tunnel of the universtiy of Oldenburg.
The anemometer shown in the photo is not the calibrated one but identical with the calibrated anemometer.
Remark: The photo does not show the real proportions, it is distorted by the lens of the camera.

4 References

[1] MEASNET
Cup Anemometer Calibration Procedure
September 1997

[2] ISO 3966 1977
Measurement of fluid flow in closed conduits.

[3] D.Westermann, H. Klug, K. Junior 1999
DEWI Anemometer Calibration Procedure