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INTEGRAL BOUNDARY LAYER METHODS FOR WIND TURBINE AERODYNAMICS

A Literature Survey

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Abstract

The simulation of wind turbine rotor aerodynamics can be improved upon by increasing the physics content in the underlying approximations. The current effort is targeted at an aerodynamics model in which the flow field is decomposed into two domains. The inviscid external flow domain is modeled with a “panel method” type of flow solver and the viscous regions are modeled with an integral boundary layer model. For the simulation of separated flows the two models are coupled in strong interaction. The technology for the “panel method” flow solver is considered common knowledge and is not expected to raise serious problems during its development. In this report the possibilities and difficulties in the development of the integral boundary layer solver are investigated. The feasibility of the development of the rotor aerodynamics simulation code is shown through a discussion of successful integral boundary layer approaches based on the conservation form of the boundary layer equations and the impact of time-dependency and rotor blade rotation related terms on the system of equations. In addition a number of possible boundary layer velocity profiles is described. As a result the development of the targeted wind turbine rotor aerodynamics simulation code is recommended. Due to the vast amount of possible solution strategies it is advised to develop the boundary layer solver in close cooperation with experts in the field of computational fluid dynamics.

Acknowledgement This report is part of the project “preRotorFlow” in which the possibilities and risks areas in the development of an integral boundary layer solver for windturbine rotor applications are investigated.

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NOMENCLATURE

Roman symbols

C_τ	shear stress coefficient
c	distance in crossflow direction
D	viscous dissipation integral
H	boundary layer shape factor
L	length
n	distance in surface normal direction
q	velocity
Re	Reynolds number
r	distance (length of relative position vector), radius
St	Strouhal number
s	distance in streamwise direction, skin friction sign
t	time

Greek symbols

α	angle between local cell s and x coordinate directions
β	streamline angle w.r.t. external flow
δ	boundary layer thickness
δ_0^*	boundary layer displacement thickness
δ_0^{**}	boundary layer density thickness
η	dimensionless surface normal distance
κ	Von Kármán constant
ρ	mass density
τ	shear stress
$\theta_{()()}$	orientation dependent boundary layer momentum thickness
$\theta_{()()}^*$	orientation dependent boundary layer kinetic energy thickness

Tensors, matrices and vectors

\vec{f}	force vector
\mathbf{F}, \mathbf{G}	flux vectors
\mathbf{H}	source vector
\vec{n}	unit surface normal vector
\vec{r}	relative position vector
\mathbf{U}	solution vector
\vec{u}	velocity vector (also written as (u, v, w))
\vec{X}	displacement vector
\vec{x}	position vector (also written as (x, y, z))
∇	derivative operator pseudo-vector $(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$
$\vec{\Omega}$	angular velocity vector

Subscripts, superscripts and accents

$()_c$	value in crossflow direction
$()_e$	value at the boundary layer edge
$()_{eq}$	equilibrium value
$()_n$	value or derivative in surface normal direction
$()_{ref}$	reference value
$()_s$	value in streamwise flow direction
$()_w$	value at the wall
$()_x$	value in x-direction
$()_y$	value in y-direction
$\overline{()}$	averaged quantity
$()'$	fluctuating quantity

Acronyms

BC	Boundary Condition
BL	Boundary Layer
CFD	Computational Fluid Dynamics
FEM	Finite Element Method
FVM	Finite Volume Method
IBL	Integral Boundary Layer
RANS	Reynolds-Averaged Navier-Stokes
VII	Viscous-Inviscid Interaction
2D	Two-Dimensional
3D	Three-Dimensional

1 INTRODUCTION

1.1 Context

The current trend in wind turbine design is towards larger and larger rotor diameters and an accompanying increase in power output. Associated with this trend is a demand for more accurate and reliable wind turbine rotor aerodynamics simulation codes. The currently used rotor aerodynamics simulation codes can roughly be subdivided into two groups of models, each with its own merits and disadvantages:

1. In the first group of simulation codes available aerodynamic data tables for two dimensional (2D) airfoils, for example obtained by wind tunnel experiments, form the basis of the method. Correction formulas are introduced to take into account the effects of the physics that are left out. Among the correction formulas are those that take into account the influence of the rotor wake, the effects of yaw misalignment, dynamic inflow, dynamic stall and tip effects. The advantages of this group of codes are the very short simulation times and the low demand on user expertise making it possible to incorporate them into structural dynamics simulation codes. The disadvantages are the inherent inaccuracies and uncertainties in the computed results introduced by the correction formulas and the little detailed information computed by the methods. This group of simulation codes is used on a regular basis in wind turbine design practice.
2. The second group of simulation codes is based on a more fundamental description of the physics involved in time-dependent, three-dimensional, viscous flows around general configurations; the so called Reynolds averaged Navier-Stokes (RANS) equations. Only turbulence is approximated by a model. It should be noted that currently available turbulence models are open to improvements for wind turbine rotor applications. Advantages are the general applicability of the method, the extensive amount of information computed and the elimination of correction factors. Disadvantages are the great demand put on computer resources and the large effort required for input preparation and post processing. This group of rotor aerodynamics codes is currently used for isolated test cases in research environments where specialized expertise is available.

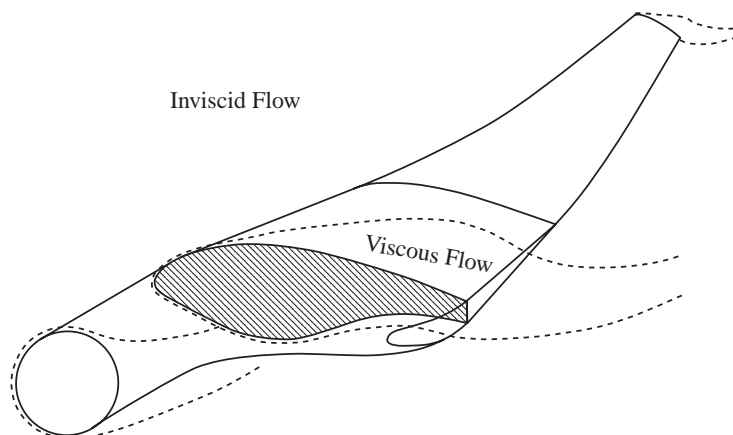


Figure 1: Flow domain decomposition into external inviscid flow and viscous boundary layer flow regions.

The goal of the current project is to investigate the technical possibilities and risk areas in the development of a new wind turbine rotor aerodynamics simulation code that requires little user expertise and computer power, but can compute in detail the unsteady aerodynamic characteristics of rotor blades. The simulation of separated flow and the coupling with structural dynamics simulation programs should be feasible.

This new rotor aerodynamics simulation program will be a combination of a “panel method” flow solver for the incompressible inviscid external flow and an integral boundary layer (IBL) solver for the three-dimensional (3D) viscous flow near the blade surface (see figure 1). The strong interaction between these two flow regimes in mildly separated flows will be accounted for by a so called viscous-inviscid interaction (VII) algorithm. The viability of this approach was confirmed positively by Veldman [40].

The technology for the “panel method” flow solver is considered common knowledge and is not expected to raise serious problems during its development. The feasibility of the development of the targeted rotor aerodynamics simulation code will be investigated through a discussion of existing successful IBL approaches and the effects caused by the introduction of time-dependency and rotor blade rotation related terms in the boundary layer equations.

1.2 Outline

In this project a large amount of literature on integral boundary layer theory was surveyed. It quickly became clear that the mathematics involved in the development of an IBL method capable of handling 3D separated flows about rotor blades, is of comparable complexity as those in the development of a RANS code. Many of the choices and problems encountered in the discretization of volume based RANS and Euler fluid dynamics codes transfer to the IBL equations also. A further obstacle was the large variation in notation schemes used in the various IBL articles.

Time constraints made it unfeasible to get, with the literature at hand, a complete roadmap or outline of the practical development of an IBL method for wind turbine rotor applications. As a result, in this report a concise survey is given of the available discretization and solution schemes for IBL methods that have proven themselves in practical computations of separated flows. Special attention is given to existing approaches for handling the effects introduced by blade rotation and unsteady flow features.

The current report makes significant use of the results from the research by Mughal reported in [20], [21] and the subsequent research by Nishida [24], [25], Milewski [19], Mughal [22] and Coenen [1].

The integral boundary layer equations and existing successful solution strategies are discussed in Chapter 2 together with the effects of introducing additional terms in the boundary layer equations to account for rotation and time dependency.

Some available velocity profile families for streamwise and crosswise laminar and turbulent flow are reported in Chapter 3.

In Chapter 4 some concluding remarks and recommendations regarding future developments are given.

2 THREE DIMENSIONAL BOUNDARY LAYER

2.1 Introduction

In this chapter existing solution methods for the 3D IBL equations are discussed. The effects of rotation and translation of the frame of reference on the system of equations are indicated together with the possibilities offered by the inclusion of time dependent terms. It should be mentioned that the IBL equations included in this chapter serve as illustration along with the text. No attempt is made to be complete in the definitions and describe the components in the boundary layer (BL) equations in detail. The interested reader is referred to the cited references.

A characteristic feature of a three-dimensional boundary layer is the existence of a cross-flow velocity component in viscous flow regions (see figure 1). The flow in the boundary layer region can be thought of to be composed of a velocity component in the direction of the external streamline and a crosswise component as is shown in figure 2. The principal mechanisms for the existence of such a crossflow in wind turbine rotor aerodynamics are the transverse pressure gradients and the forces due to the rotation of the frame of reference.

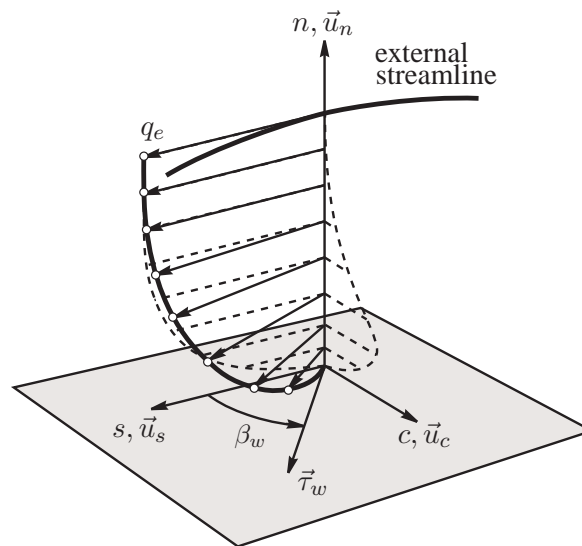


Figure 2: Streamwise \vec{u}_s and crossflow \vec{u}_c velocity components.

Notice that in figure 2 a uni-directional crossflow velocity profile is sketched. For a flow with changing direction in transverse pressure gradient the resulting crossflow profiles are bi-directional.

2.2 3D Integral Boundary Layer Formulation

As is shown in numerous textbooks on fluid dynamics (for example [27],[41]) the integral form of the boundary layer equations are obtained from the differential form by integration with respect to the surface normal direction n (see figure 2). It should be noted that the integral form of the BL equations, commonly referred to as the “integral boundary layer equations”, are differential equations also. For two-dimensional airfoil flow problems this approach was shown to give accurate results even for mildly separated flows (see [6], [7], [44]).

The choice for an integral method formulation over a differential field method for the solution of the 3D boundary layer equations is based on the following observations:

1. The combination of an IBL method formulation, a panel method solver for the external flow and a viscous-inviscid interaction (VII) algorithm is more frequently encountered.
2. The IBL formulation can be shown to be hyperbolic in nature which facilitates the design of stable spatial discretization schemes. Integral methods tend to be more robust and are better suited for viscous-inviscid interaction.
3. The integral method formulation only requires a surface grid; possibly the same grid used by the solver for the inviscid external flow. Field method BL solvers require a separate volume grid around the domain of interest. Associated disadvantages are the increase in required user expertise for grid generation and the increase in number of unknowns to be solved for.
4. There are more opportunities to tune the IBL method to measurements because of the direct dependence on BL closure relationships. Field method BL solvers can be tuned only indirectly through the turbulence model.

Prior to the solution method introduced in [20],[21], the IBL equations were mostly written in a finite difference curvi-linear formulation for non-orthogonal grids. The resulting equations are analytically rather involved and contain a vast amount of metric coefficients and geodesic curvature terms. Examples of this approach can be found in [2], [15], [23], [28], [30], [31] and [42].

Mughal, 1992

In references [20] and [21] Mughal expressed the steady IBL equations in conservation form in simple (local) Cartesian coordinate systems and solved the system using a cell-centered finite-volume method (FVM). This approach circumvented the explicit analytical introduction of the grid stretching and curvature terms in the BL equations. In this work only turbulent boundary layer computations were considered with external flow velocity components supposed to be known in advance. For a detailed description of the intrinsics of FVM discretization options the reader is referred to standard CFD textbooks like [8], [10], [11] and [43]. For a local

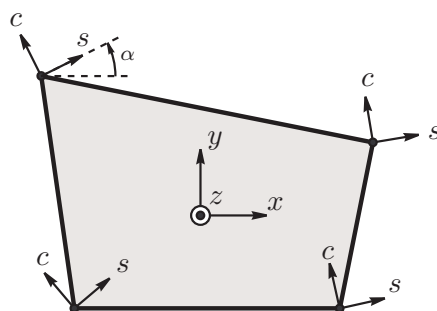


Figure 3: Gridcell Cartesian (x, y) and nodal streamline (s, c) coordinate systems.

Cartesian coordinate system (x, y) tangent to the surface of a grid cell (see figure 3) the IBL

equations read:

$$\frac{\partial}{\partial x}(\rho_e q_e^2 \theta_{xx}) + \frac{\partial}{\partial y}(\rho_e q_e^2 \theta_{xy}) + \rho_e q_e \delta_x^* \frac{\partial u_e}{\partial x} + \rho_e q_e \delta_y^* \frac{\partial u_e}{\partial y} = \tau_{w_x}, \quad (1)$$

$$\frac{\partial}{\partial x}(\rho_e q_e^2 \theta_{yx}) + \frac{\partial}{\partial y}(\rho_e q_e^2 \theta_{yy}) + \rho_e q_e \delta_x^* \frac{\partial v_e}{\partial x} + \rho_e q_e \delta_y^* \frac{\partial v_e}{\partial y} = \tau_{w_y}, \quad (2)$$

$$\frac{\partial}{\partial x}(\rho_e q_e^3 \theta_x^*) + \frac{\partial}{\partial y}(\rho_e q_e^3 \theta_y^*) + \rho_e q_e \delta_x^{**} \frac{\partial q_e^2}{\partial x} + \rho_e q_e \delta_y^{**} \frac{\partial q_e^2}{\partial y} = 2D. \quad (3)$$

The first two equations involve the x- and y-momentum relations and are 3D extensions of the well-known Von Kármán equation in two dimensions. The third equation used by Mughal is a relation involving kinetic energy dissipation. This hyperbolic system of equations can be written in a generic conservation form as:

$$\frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{H}. \quad (4)$$

With these three (non-linear) IBL equations it is possible to solve for three primary unknowns which in [20], [21] were selected to be momentum thickness θ_{ss} , wall streamline angle β_w (see figure 2) and displacement thickness δ_s^* . In order to solve the system all other parameters in the equations must be connected to these primary unknowns by some closure relationships to make the problem determinate. To be able to build upon established empirical closure relations for laminar and turbulent flow, which are based on streamwise and crossflow velocity profiles, it is necessary to work in a streamwise coordinate system.

The relation between the (u, v) velocity components in the local Cartesian (x, y) coordinate system (see figure 3) and their counterparts in the nodal streamwise coordinate system are

$$\begin{aligned} u &= u_s \cos \alpha - u_c \sin \alpha, \\ v &= u_s \sin \alpha + u_c \cos \alpha. \end{aligned} \quad (5)$$

where α is the angle between the x and s directions. With these relations it is possible to express boundary layer integral parameters in the (x, y) coordinate system like θ_{xx} and δ_x^{**} as functions of their values in streamline (s, c) coordinates.

The central-differencing scheme employed by Mughal [20], [21] permitted the occurrence of unwanted odd-even decoupling. The encountered numerical instabilities in the solution of the system were damped by introducing variable value weighing at each of the four nodes in the evaluation of the source terms \mathbf{H} (see equation (4)) of a cell. This effectively introduced numerical diffusion terms into the system of equations.

For the solution of the non-linear system of equations a Newton-Raphson scheme was selected. The boundary layer computations are started with initial conditions for the primary variables along the first row of cells where characteristics of the hyperbolic equations enter the computational domain. Recall that for hyperbolic systems a disturbance in a point P (see figure 4) is felt throughout its domain of influence whereas only disturbances in the domain of dependence can influence properties at P itself. In reference [4] Cousteix and Houdeville showed that the characteristic lines (figure 4) lie within the limits defined by the wall streamline β_w and the external flow streamline.

For situations where all or part of the initial conditions along a domain boundary were not known in advance, zero-curvature Neumann boundary conditions (BC's) were found to perform better than simple Dirichlet or Neumann BC's. With zero-curvature Neumann BC's, the values from two adjacent interior nodes are simply extrapolated.

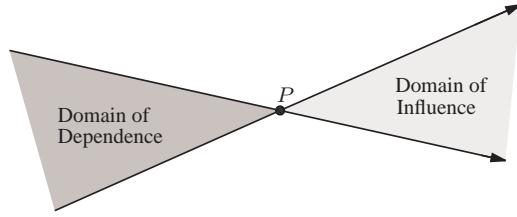


Figure 4: Hyperbolic equation domains of dependence and influence.

In his conclusions Mughal [20] recommends to replace the ad hoc source-term weighing approach by an upwind based discretization scheme. Further suggestions made are, the inclusion of the method in a viscous-inviscid interaction code, the addition of a more accurate model for non-equilibrium boundary layers and the extension of the method to laminar flows with the associated need for laminar-turbulent transition modeling. Very short computer CPU times of 15 seconds are reported in reference [22] for a swept wing test case with 112 chordwise and 22 spanwise grid cells. However, the results exhibit some spanwise oscillations.

Nishida, 1996

Nishida [24], [25] extended the original system of equations used by Mughal with an extra turbulent shear stress lag equation as derived by Green, Weeks and Brooman [9]:

$$\frac{\delta}{C_\tau} \frac{\partial C_\tau}{\partial \xi} = K_c \left(C_{\tau_{eq}}^{\frac{1}{2}} - C_\tau^{\frac{1}{2}} \right). \quad (6)$$

The shear stress direction ξ was approximated and set to be the chordwise direction. The four selected independent boundary layer variables were the shear stress coefficient factor $C_\tau^{1/2}$, the momentum thickness θ_{ss} and the two displacement thicknesses δ_s^* and δ_c^* . The system of equations was discretized using a weighed residual Petrov-Galerkin finite element method (FEM). In this approach the upwind-biased weight function, different from the shape function, provided for a stable discretization scheme. For a description of FEM discretization solutions used in CFD codes the reader is referred to reference [10] and [11]. The IBL equations were coupled in full simultaneous interaction with a full-potential solver for the external flow. A Newton-Raphson method was applied for the combined solution of the non-linear equations for the external flow and the boundary layer region.

Boundary layer computations are started with initial conditions specified for the primary variables along the attachment line. Zero-curvature Neumann BC's applied in a test case where *not all* initial conditions along a domain boundary were known in advance, were shown to give anomalous results. Nishida states that the correct BC's should specify all incoming characteristic variables. However, this was considered too difficult to implement and a modified BC was constructed in which one of the primary variables was allowed to float, Nishida chose the crossflow displacement thickness δ_c^* , while all others were explicitly specified.

An additional set of BL parameter closure relationships was included to enable the simulation of laminar boundary layers. Laminar-turbulent transition was assumed to occur at pre-determined chordwise locations. The effect of boundary layer displacement thickness on the external flow was accounted for by a surface normal transpiration velocity.

Impressive results for separated transonic flow about a swept wing were shown in reference [24]. However, some difficulties with the robustness of the code were reported that could be

attributed to the specific handling of the kinetic energy equation (3). The fully simultaneous coupling scheme for the viscous and inviscid flow regions was found to put large demands upon computer CPU time and memory allocation.

Milewski, 1997

Milewski's work in reference [19] was based on Nishida's formulation with the essential difference being the handling of the external flow by a first order accurate panel method instead of a full-potential code. An explicit numerical dissipation term was added to the x- and y-momentum IBL equations (1) and (2) to prevent spanwise oscillations in the solution that occurred in cases with coarse spanwise grid resolution. For a swept wing test case good agreement with experiment was found for the overall lift force, even in 3D separated flow. Computed pressure distributions for a circular duct were compared with experimental data and, apart from some discrepancies at the trailing edge, found to be in good agreement. Drag forces were generally underpredicted.

In his recommendations, Milewski mentions the large computational requirements that result from the assembly and inversion of the Jacobian matrix used in the Newton-Raphson scheme. Another suggestion made is to examine in more detail the finite trailing edge loading that was believed to be caused by the algorithm used to compute the external velocities at the trailing edge nodes.

Mughal, 1998

A more fundamental study of the mathematical aspects of 3D integral boundary layer approximations was reported in the PhD. thesis of Mughal [22]. The derivation of all relevant IBL equations was based on the differential form of the boundary layer equations for conservation of mass and x- and y-momentum:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0, \quad (7)$$

$$\rho \frac{\partial u}{\partial t} + \rho \vec{u} \cdot \vec{\nabla} u + \frac{\partial p}{\partial x} = \frac{\partial \tau_x}{\partial z}, \quad \tau_x = \mu \frac{\partial u}{\partial z} - \overline{\rho u' w'}, \quad (8)$$

$$\rho \frac{\partial v}{\partial t} + \rho \vec{u} \cdot \vec{\nabla} v + \frac{\partial p}{\partial y} = \frac{\partial \tau_y}{\partial z}, \quad \tau_y = \mu \frac{\partial v}{\partial z} - \overline{\rho v' w'}. \quad (9)$$

The time dependent version of any set of IBL equations was written in a generic conservation form (see also equation (4)) as:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{H}. \quad (10)$$

In this equation \mathbf{U} is the solution vector, \mathbf{F} and \mathbf{G} are flux vectors and \mathbf{H} is a source vector. It is mentioned here that all results shown by Mughal in reference [22] were actually based on the steady form of the BL equations.

A number of streamwise and crossflow velocity profile families for laminar and turbulent flow were considered. Normally these velocity profiles are used to derive some explicit closure relationships between boundary layer parameters. However, Mughal computed the closure relations on the fly from the assumed set of BL velocity profiles. This gives greater flexibility for the choice of streamwise and crossflow velocity profile families. A drawback of this approach is the loss of direct control of the closure functions. Based on the nature of the unsteady differential form of the BL equations, a justification for the *requirement* for hyperbolicity of the

complete resulting IBL system was given (i.e. BL equations plus closure relations). It should be mentioned that only for some simple closure functions the hyperbolic character of the IBL system can be proven. In practice, hyperbolicity of the system is *assumed*. A method to enforce hyperbolicity by constraining the closure relationships was devised but discarded for actual implementation. Several strategies for the solution of the system of steady and unsteady IBL equations were suggested. The Petrov-Galerkin FEM method from Nishida (references [24] and [25]) was enhanced with a more sophisticated upwinding based weight function. In order to be able to compute bi-directional crossflows (see figure 5) the system of momentum equations and total kinetic energy equation was extended with a mixed velocity component kinetic energy equation.

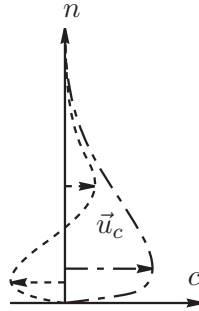


Figure 5: Uni-directional and bi-directional crossflow velocity profiles \vec{u}_c .

A demonstration computer program was constructed, employing a fully simultaneous coupling of the steady 3D IBL equation model and a first order accurate panel method for the external flow. The method, flexible in the choice of applied velocity profiles, was demonstrated to be able to compute 3D separated flows. For the theory of panel methods, also known as boundary element methods or linear potential flow solvers, the interested reader is referred to references [12], [13] and [14].

In the conclusions of reference [22], Mughal recommends to refine and implement the scheme to enforce hyperbolicity of the combined system of IBL equations and closure functions. Further suggestions made are the implementation of laminar-turbulent transition criteria and the replacement of the CPU-intensive direct matrix inversion method by an iterative solver in conjunction with a sparse matrix approximation for the full residual Jacobian matrix. In addition, to simplify the data structure for the BL parameters, it is proposed to integrate the differential BL equations themselves numerically on the fly. The rotation of BL thicknesses can then be implemented by just decomposing the velocity vectors in streamwise and crossflow direction.

Coenen, 2000

The main topic of the work by Coenen [1] was the interaction scheme for the coupling of the external flow model and an IBL method for the viscous flow region. In the 3D IBL method the kinetic energy integral equation (3) was replaced by an entrainment equation:

$$\frac{1}{q_e} \frac{\partial}{\partial x} (u_e \delta - q_e \delta_x^*) + \frac{1}{q_e} \frac{\partial}{\partial y} (v_e \delta - q_e \delta_y^*) = C_E. \tag{11}$$

For the solution of this system of equations the momentum thickness θ_{ss} , the shape factor H (where $H = \delta_s^* / \theta_{ss}$), and the limiting wall streamline angle β_w were selected to be the

independent variables. The hyperbolic nature of the IBL equations was taken into account by the use of upwind discretizations for the derivative terms in the cell-vertex FVM. Along the attachment line initial conditions are specified for the primary variables. Symmetry BC's were applied at the wing root section and for the tip section the variables were determined by extrapolation of values at interior nodes.

For the laminar part of the boundary layer a very simple 2D integral method was used. The position of natural transition to turbulent flow was predicted with a simple empirical criterion and the effect of a possible laminar separation bubble on starting conditions for the turbulent boundary layer was taken into account. The IBL equations were coupled, with the quasi-simultaneous interaction algorithm by Veldman [38], to a first order accurate panel method for the external flow. The coupled system of boundary layer, external flow and interaction equations was solved using a Newton-Raphson method. For some 3D wing aerodynamics test cases the pressure distributions and boundary layer parameters were found to be in reasonable agreement with experimental data. Some discrepancies were noticed in the trailing edge region that were attributed to the inaccurate modeling of the Kutta condition in the panel method. For an overview of viscous-inviscid interaction techniques the interested reader is referred to reference [17].

2.3 Rotation and Time Dependency

The momentum equations so far are formulated for an inertial coordinate system. For a *steadily* rotating frame of reference two force vector terms have to be added to \mathbf{H} in the right hand side of equation (10). The first is the Coriolis force term

$$\rho \vec{f}_1 = -2\rho(\vec{\Omega} \times \vec{u}), \quad (12)$$

and the second is a centrifugal force term:

$$\rho \vec{f}_2 = -\rho \vec{\Omega} \times (\vec{\Omega} \times \vec{r}). \quad (13)$$

When incompressible flow is assumed and the rotation associated velocity component $\vec{u} = \vec{\Omega} \times \vec{r}$ is incorporated in the pressure formula, the centrifugal force term does not have to appear explicitly in the momentum equations (see [30]).

For an arbitrarily translating and rotating frame of reference two more force vectors are introduced in the momentum equations. Due to the time dependency of the translation velocity $\partial \vec{X} / \partial t$, a force term is introduced:

$$\rho \vec{f}_3 = -\rho \frac{\partial^2 \vec{X}}{\partial t^2}, \quad (14)$$

and the time dependency of the angular velocity gives rise to another force vector:

$$\rho \vec{f}_4 = -\rho \frac{\partial \vec{\Omega}}{\partial t} \times \vec{r}. \quad (15)$$

In compressible flow all four force terms will appear in the source term \mathbf{H} of equation (10) and will merely add inhomogeneities to the problem. No additional discretization difficulties are expected in the implementation of these terms.

Wind turbine rotor blades generally operate in low frequency periodic flows with small characteristic Strouhal numbers

$$St = L_{ref} / (t_{ref} q_{ref}).$$

As a result, the flow can be considered to be quasi-steady: at any point in time the solution behaves as the corresponding steady solution with the instantaneous outer flow (see [27]). This means that periodicity effects on the assumed set of BL velocity profiles can be neglected.

In flutter conditions with small amplitude, high frequency oscillations, the flow cannot be considered quasi-steady and unsteady flow effects should be reflected in the assumed set of BL velocity profiles. A possible approach is to combine the basis set of velocity profiles with a velocity profile family typical for oscillating boundaries. In figure 6 such a profile family is shown: Stokes' steady state solution (see [27]) for BL flow near an in-plane oscillating wall with velocity $u_w = \sin T$.

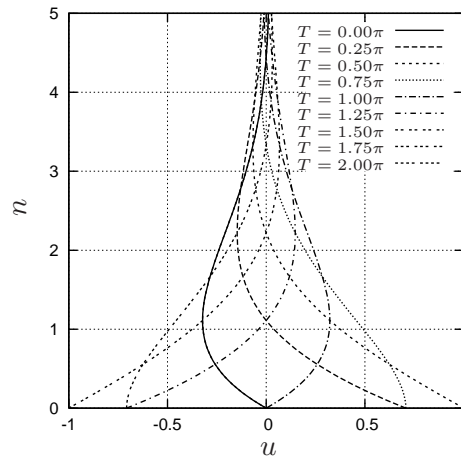


Figure 6: Stokes' steady state solution for flow near an in-plane oscillating plate.

A crucial step towards solving the set of time-dependent IBL equations (see the system of equations (10)), is to recognize that the spatially discretized problem represents a non-linear system of ordinary differential equations. As was mentioned by Mughal [22], the solution of this unsteady IBL system can be accomplished by integrating the equations in time. For the discretization a multitude of options are available with differences in accuracy, stability, robustness, efficiency and complexity. Many of the schemes can be found in textbooks like [8], [10], [11] and [43]. A discussion of these numerical algorithms is outside the scope of this project.

Swafford [33] and Swafford and Whitfield [35] derived the unsteady form of the momentum and kinetic energy IBL equations for compressible flow in a non-orthogonal curvilinear coordinate system. The unsteady IBL equations were used to obtain solutions for steady flow problems. Two- and four-stage explicit Runge-Kutta time-stepping schemes were used for integration in time. Local time steps were used to accelerate convergence to steady state. For the spatial discretization an upwind scheme was found to give the best results.

The differential form of the unsteady IBL equations was used by Van der Wees and Van Muijden [42] in strong interaction with a full-potential flow solver. They stated that an important advantage of using the unsteady boundary layer equations for solving steady boundary layer flow is the robustness of such an approach. For the time integration scheme a fully implicit backward Euler method was selected. In later reports the code was demonstrated for test cases with mildly separated flow. For the VII scheme the steady form of the quasi-simultaneous interaction algorithm was used.

The unsteady version of the quasi-simultaneous VII scheme was found by Coenen [1] to lead

to a better conditioned interaction problem than its steady counterpart. This conclusion was positively confirmed by Veldman [40].

3 CLOSURE RELATIONS

3.1 Introduction

As was already mentioned in Chapter 2 the number of boundary layer integral parameters exceeds the number of equations available (see equation (10)). It is only possible to solve for as many primary unknowns as there are equations. In order to make the problem determinate all other BL parameters have to be expressed directly, or indirectly via a chain of closure relations, as functions of these primary unknowns. These closure formulas can be constructed from experimental BL data, from numerically obtained velocity profiles or through an assumed set of velocity profiles.

Almost all existing IBL equation solvers use à priori determined explicit algebraic formulas to relate the BL parameters to one another. In the thesis of Mughal [22], the closure relations were computed on the fly from an assumed set of BL velocity profiles. The advantage of this approach is the great flexibility in the choice of streamwise and crossflow velocity profile families. A drawback could be the resulting increase in work and possibly the loss of direct control of the closure relationships.

Generally it is assumed that the streamwise velocity profiles in 3D boundary layer flow resemble the 2D velocity profiles and as a consequence enables the utilization of established empirical closure relations for 2D laminar and turbulent flow.

In this Chapter some of the BL velocity profiles available from literature for streamwise and crosswise laminar and turbulent flow are introduced.

3.2 Streamwise Flow

Laminar Flows

One of the first methods that used a family of velocity profiles was one due to Pohlhausen. For two dimensional flow the velocity profile family was assumed to be a function of a single parameter Λ :

$$\frac{u_s}{q_e} = F(\eta) + \Lambda G(\eta).$$

For the two functions $F(\eta)$ and $G(\eta)$ Pohlhausen selected quartic polynomials as function of the dimensionless normal direction $\eta = z/\delta$. The same idea can be extended to a multi-parameter velocity profile family of which an example is the two-parameter velocity profile by Wieghardt (see Rosenhead [26]) that is constructed from three basis functions and reads

$$\frac{u_s}{q_e} = F_1(\eta) + aF_2(\eta) + bF_3(\eta).$$

The three basis functions and two constructed BL velocity profiles are shown in figure 7. Of course, the extra parameter introduces another variable in the system of equations that has to be solved for.

In reference [22] an algebraic one-parameter function is fitted to the velocity profiles obtained from the numerical solution of the Falkner-Skan self-similar BL equation. From this *approximation* of the Falkner-Skan velocity profiles the closure relations can be obtained. Another one-parameter approximation of the Falkner-Skan velocity profiles is by Stock and can be found in reference [32].

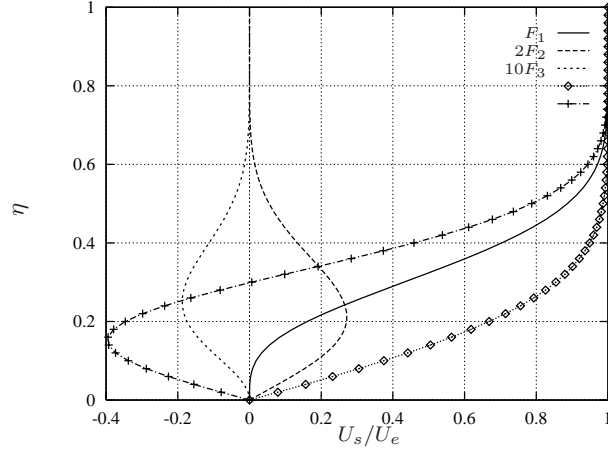


Figure 7: Wieghardt's streamwise velocity functions [26].

Of course the closure relationships could also have been constructed from the numerical results of the Falkner-Skan equation directly. In his thesis Drela [6] obtained a number of one-parameter curve fits that way.

Turbulent Flows

For turbulent flows exact numerical solutions to the BL equations are not possible. Velocity profiles for turbulent boundary layers therefore have been constructed with the help of experimental data. Multiple layers can be distinguished in a turbulent velocity profile and a one-parameter velocity profile family is inadequate to describe all turbulent boundary layers.

Noticeable two-parameter models for complete turbulent velocity profiles, able to represent separated flow, are due to Swafford [34] and Cross [5] which respectively take the form

$$\frac{u_s}{q_e} = \frac{s}{0.09u_e^+} \arctan(0.09y^+) + \left(1 - \frac{s\pi}{0.18u_e^+}\right) \sqrt{\tanh(a\eta^b)},$$

and

$$\frac{u_s}{q_e} = \frac{u_\tau \cos \beta_w}{\kappa} \left(\frac{1}{2} \ln(R_\delta u_\tau \eta)^2 + A \right) + B_s \sin^{\chi_s} \left(\frac{\pi}{2} \eta \right).$$

As was mentioned in the introduction of this chapter, except for Mughal [22], the velocity profile models are almost exclusively used to obtain algebraic closure relations. Closure relations that were derived from Swafford's model were used by Drela [6], Mughal [20], Nishida [24] and Milewski [19].

3.3 Crossflow

Laminar Flows

As in Pohlhausen's approximation of streamwise laminar profiles it is possible to construct one- or multi-parameter velocity profiles for crosswise flow. In reference [32], Stock constructed a two-parameter crossflow velocity profile family allowing for bidirectional crossflows. The specific form of the two basis functions G_1 and G_2 , shown in figure 8, was established with the help of results from a 3D differential BL solver. In the same figure some resulting crossflow velocity profiles $u_c = cG_1(\eta) + dG_2(\eta)$ are shown.

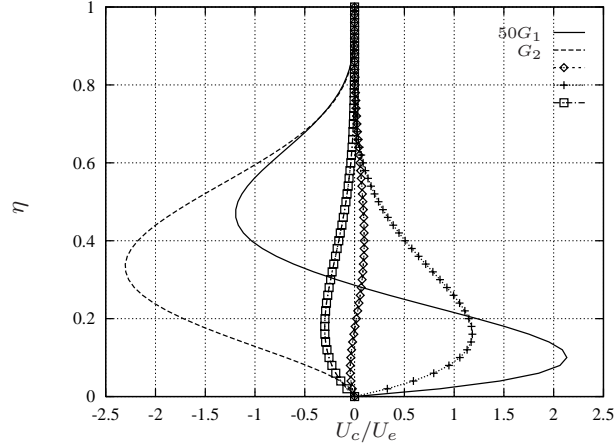


Figure 8: Stock's crossflow velocity functions [32].

A two-parameter model taken from reference [22] uses the streamwise velocity profile in its definition:

$$\frac{u_c}{q_e} = \left(1 - \frac{u_s}{q_e}\right)(c\eta + d\eta^2).$$

To solve for the second unknown parameter an extra equation is required. In reference [22] Mughal examined a system of equations and obtained results comparable with those from a differential type of BL solver for a laminar flow problem with the crosswise pressure gradient alternating along the streamlines.

Turbulent Flows

A common approach in modeling crossflow velocity profiles is to base them on the scaling of the streamwise velocity profile. Two frequently used models are due to Johnston and Mager. The model of Johnston, used by Mughal [20], Nishida [24] and Swafford [34], reads:

$$\frac{u_c}{q_e} = \begin{cases} \frac{u_s}{q_e} \tan \beta_w & \eta \leq \eta^*, \\ c\left(1 - \frac{u_s}{q_e}\right) & \eta > \eta^*. \end{cases}$$

Mager's model was used by Coenen in reference [1]. The model uses a simple parabolic scaling function and is expressed as

$$\frac{u_c}{q_e} = \frac{u_s}{q_e}(1 - \eta)^2 \tan \beta_w.$$

In reference [36] Tai suggested a modification of Mager's function that prevents the unrealistic increase in velocity magnitude in the combined velocity profile at large crossflow angles.

A model for the complete crossflow velocity profile was defined in reference [5] by Cross. Its use was recommended by Coenen [1] over Mager's model which was established for fairly small limiting crossflow angles β_w .

$$\frac{u_c}{q_e} = \frac{u_\tau \sin \beta_w}{\kappa} \left(\frac{1}{2} \ln(R_\delta u_\tau \eta)^2 + A \right) + B_c \sin^{\chi_c} \left(\frac{\pi}{2} \eta \right).$$

4 CONCLUSIONS AND RECOMMENDATIONS

In this report the feasibility of a rotor aerodynamics simulation code composed of a “panel method” solver for the external flow and an integral boundary layer method for the viscous flow regions is shown through a discussion of existing successful integral boundary layer approaches and the impact of time-dependency and rotor blade rotation related terms on the system of equations. As a result, the development of the targeted wind turbine rotor aerodynamics simulation code is recommended.

Some detailed conclusions and recommendations are given below.

4.1 Conclusions

A concise survey is given of the available theories and solution approaches for integral boundary layer methods that have proven themselves in practical computations of separated flows. (see [1], [19], [20], [21], [22], [24], [25], [42]).

The viability of an aerodynamic simulation model employing the unsteady quasi-simultaneous coupling of an IBL method for the viscous flow and a panel method for the inviscid external flow was confirmed positively by Veldman [40].

The conservation form of the integral boundary layer equations was first employed by Mughal in reference [20] and [21]. The finite volume method that was used for the solution of the equations circumvents the explicit introduction of awkward metric coefficients and geodesic curvature terms.

From the options to suppress instabilities and oscillations in the solution of the integral boundary layer equations the upwind discretization schemes are preferred over central schemes with explicit numerical damping terms.

Time-dependent terms in the integral boundary layer equations *and* in the quasi-simultaneous interaction scheme both enhance the robustness of the flow solver.

The fully simultaneous coupling scheme for the interaction between viscous and inviscid flow regions is expensive with regard to the allocation of computer resources.

For a steadily rotating frame of reference, Coriolis force and centrifugal force vectors must be added to the source term of the momentum equations. In case of an arbitrarily translating and rotating frame of reference two more force vectors should be included in addition.

Both Coenen [1] and Milewski [19] employed a panel method for the external flow about wing geometries and reported difficulties in the interaction with the boundary layer solver at the trailing edge. The problem was attributed to the pressure difference across the trailing edge as computed by the panel method.

Most of the existing integral boundary layer solvers use *à priori* determined explicit algebraic closure relations to render the system of equations determinate. An exception is the approach taken by Mughal [22] where the closure relations were computed on the fly from an assumed set of boundary layer velocity profiles.

In reference [22] a method was devised to enforce hyperbolicity of the system of boundary layer equations by constraining the closure relationships. It was recommended to refine the scheme and implement it in the integral boundary layer solver.

Laminar-turbulent transition modeling is a topic that received very little attention in the reviewed reports.

For the temporal and spatial discretization of hyperbolic unsteady integral boundary layer equa-

tions with finite volume or finite element methods a vast amount of schemes is available each with its own characteristics regarding accuracy, stability, robustness, efficiency and complexity. This vastness of options poses a problem for the selection of the “right” discretization schemes.

4.2 Recommendations

For the development of an integral boundary layer solver, to be coupled in strong interaction with a panel method for the external flow, some recommendations can be given:

- Employ a Finite Volume Method or a Finite Element Method to solve the conservation form of the time-dependent integral boundary layer equations.
- The vast amount of available solution strategies for the time-dependent integral boundary layer equations strongly suggests to do code development in close cooperation with experts in the field of computational fluid dynamics.
- Separated flow about wind turbine rotor blades exhibit large crossflow components. This suggests the use of an advanced model for the turbulent crossflow velocity profile family like the model by Cross [5].
- In order to avoid problems with the viscous-inviscid interaction scheme at the trailing edge, the implementation of a zero pressure difference Kutta condition in the panel method is recommended.
- If a coupling of the integral boundary layer solver with a compressible flow solver is anticipated, the compressible form of the boundary layer equations should be used.
- Laminar-turbulent transition modeling is a topic that should be part of the development of an integral boundary layer solver.
- Mughal [22] integrated the boundary layer velocity profiles on the fly, avoiding the need for explicit boundary layer closure functions. In addition, a recommendation was to integrate the differential equations over the boundary layer thickness in the course of the simulation. Both ideas are worth investigating.

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Title	INTEGRAL BOUNDARY LAYER METHODS FOR WIND TURBINE AERODYNAMICS		
Subtitle	A Literature Survey		
Author(s)	A. van Garrel		
Principal(s)	Novem BV.		
ECN project number	7.4188		
Principal(s) project number	2020-02-13-20-008		
Programme(s)	BSE DEN 2002		
Abstract	<p>The simulation of wind turbine rotor aerodynamics can be improved upon by increasing the physics content in the underlying approximations. The current effort is targeted at an aerodynamics model in which the flow field is decomposed into two domains. The inviscid external flow domain is modeled with a “panel method” type of flow solver and the viscous regions are modeled with an integral boundary layer model. For the simulation of separated flows the two models are coupled in strong interaction. The technology for the “panel method” flow solver is considered common knowledge and is not expected to raise serious problems during its development. In this report the possibilities and difficulties in the development of the integral boundary layer solver are investigated. The feasibility of the development of the rotor aerodynamics simulation code is shown through a discussion of successful integral boundary layer approaches based on the conservation form of the boundary layer equations and the impact of time-dependency and rotor blade rotation related terms on the system of equations. In addition a number of possible boundary layer velocity profiles is described. As a result the development of the targeted wind turbine rotor aerodynamics simulation code is recommended. Due to the vast amount of possible solution strategies it is advised to develop the boundary layer solver in close cooperation with experts in the field of computational fluid dynamics.</p>		
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