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ENDOW: Validation and improvement of ECN's wake model

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Abstract

This report describes the tasks performed by ECN within the EU 5th Framework project 'EfficieNt Development of Offshore WindFarms', (ENDOW). In this project ECN cooperated with the following partners:

- RISØ National Laboratory, Dept. of Wind Energy and Atmospheric Physics (Dk), Coordinator;
- Uppsala University, Dept. of Earth Sciences, Meteorology (S);
- Garrad Hassan and Partners (GH), (UK);
- Robert Gordon University (RGU), School of Mechanical and Offshore Engineering (UK);
- University of Oldenburg (OU), Dept. Of Energy and Semiconductor Research EHF, Faculty of Physics (FRG);
- Seas Distribution (Dk);
- Techwise A/S, (Dk);
- NEG Micon Project (Dk);
- ECOFYS, energy and environment (NL)

The main aim of the project was to validate, evaluate, enhance and interface wake and boundary-layer models for offshore utilisations.

The present report, describes the tasks which are performed by ECN within the project. These tasks mainly refer to the validation and development of ECN's wake model WAKEFARM. Some modifications have been implemented into this model, which led to a considerable improvement in results.

The model has been validated with different wake measurements:

- Measurements within the off-shore Vindeby wind farm. This includes measurements of meteorological data from conventional fixed masts as well as measurements which are taken with a mobile SODAR system mounted on a boat;
- Measurements within the Alsvik Wind farm;
- Measurements which are taken in a model wind farm, which is placed in a wind tunnel.

In addition, a comparison with calculational results from other partners in the project offered much insight into the accuracy of the WAKEFARM model.

Keywords

Wind Farm Design

Aerodynamic wake effects

SUMMARY

Within Europe many off-shore wind energy projects are planned and it is expected that several thousand megawatts will be installed in the first decade of the millennium.

In order to estimate the power production of such off-shore windfarms it is necessary to estimate the wake effects, i.e. the velocity decrease experienced by a wind turbine which is placed downstream of another wind turbine. Indications exist that these wake effects in off-shore conditions differ considerably from the wake effects on land. In particular it is expected that the wakes in off-shore conditions will be propagated over a larger distances. The likely result is that in order to optimise power output, offshore wind farms will require larger distances between rows than is common in design of onshore wind farms.

In order to gain insight into these effects, the EU 5th framework project 'EfficieNt Development of Offshore WindFarms', (ENDOW) was carried out. This project started on March 1 2000 and ended on March 1 2003. In this project ECN cooperated with the following partners:

- RISØ National Laboratory, Dept. of Wind Energy and Atmospheric Physics (Dk), Coordinator;
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The main aim of the project is to validate, evaluate, enhance and interface wake and boundary-layer models for offshore utilisations. This results in a significant advance in the state of the art in both wake and marine boundary layer models leading to improved prediction of wind speed and turbulence profiles within large offshore wind farms.

In the present report the activities are described which are performed by ECN within the ENDOW project and which are related to the validation and improvement of the WAKEFARM program. Note that another task of ECN was to carry out SODAR measurements. These activities are described in [15].

The WAKEFARM program models the wake flow downstream of a turbine, using the basic flow field parameters and some wind turbine properties as input. Output properties from the model are among others the wake wind speeds and the turbulence intensities in the wake.

Model calculations have been compared with:

- Measurements within the off-shore Vindeby wind farm. This includes measurements of meteorological data from conventional fixed masts as well as measurements which are taken with a mobile SODAR system mounted on a

boat. ;
- Measurements within the Alsvik Wind farm;
- Measurements which are taken in a model wind farm, which is placed in a wind tunnel.

In addition a comparison with calculational results from other partners in the project offered much insight into the accuracy of the WAKEFARM model. This holds in particular for the comparison with the results from Robert Gordon University. The difference between ECN's and RGU's model lies mainly in the fact that RGU uses a full elliptic approach, where the ECN model is parabolised. The parabolisation yields an efficient calculational procedure but on the other hand the near wake cannot be modelled in great detail. In fact the near wake is modelled through an empirical initial near wake profile, where RGU's method models the near wake in a physical way. The comparison with RGU's results led to the insight that ECN's model could be improved by changing the original initial near wake model, which was 'hat-shaped', into a Gaussian profile.

The validation performed in ENDOW showed that the model with the Gaussian profile yields, generally speaking, results which compare much better to the measurements. Nevertheless the modified near wake model still overpredicts the turbulence intensities, but to a smaller extent than previously. A further improvement in the prediction of turbulence intensity was possible by assuming the turbulence generated by the wake to be of a more isotropic kind.

Some improvements in ECN's model are still expected to be possible in the future. Indications for possible future improvements are mainly found from the SODAR measurements:
- The SODAR measurements, indicated that a 'double dip' near wake profile, is more realistic than the Gaussian profile. The double dip is a result of the load distribution along the blade.
- The SODAR measurements indicated that the near wake profile depends on the ambient conditions (turbulence level, stability). Taking into account such dependancy is expected to improve the agreement.

Furthermore, the intialisation of more quantities than the wake speeds alone (i.e. the turbulent kinetic energy and the dissipation) will improve the agreement.

CONTENTS

SYMBOLS

Quantity	Description	Unit
a	Axial induction factor in rotor plane	-
$C_{D.ax}$	Thrust coefficient	-
D	Rotor diameter	m
D_{exp}	Expanded rotor diameter	m
F_{ax}	Axial force	-
h	Height	m
h_{hub} or h_t	hub height	m
I(h)	Turbulence intensity at height h	%
k(h)	Turbulent kinetic energy at height h	m^2/s^2
k	von Karman constant	[-]
L or MOL	Monin-Obukhov length scale	m
R	Rotor radius	m
RGU	Robert Gordon University	
x	Coordinate in wind direction	m
y	Coordinate in horizontal plane, perpendical to wind direction	m
z or h	Coordinate in vertical direction	m
x_{nw}	near wake length (=2.25 D in WAKEFARM)	m
t	temperature	K
t.s.r.	tip speed ratio	-
U(h)	Free stream wind speed at height h	m/s
U_{hub}	Free stream wind speed at hub height	m/s
u^*	friction velocity	m/s
u_{def}	Velocity defect (relative to free stream)	m/s
z_0	Roughness height	m
ϵ	Dissipation rate	$[m^2/s^3]$
λ	Tip speed ratio	[-]
Ω	Rotor speed	[rad/s]
σ	Standard deviation wind speed or standard deviation in 'Gaussian' wake defect, i.e. a measure for the wake width	[m/s] or [-]
θ	Pitch angle	[deg]
ξ	h/L	[-]

1,2 (subscript)	first or second turbine in a row	
add (subscript)	added	
w (subscript)	wake	
∞ (subscript)	free stream	

1. INTRODUCTION

Within Europe many off-shore wind energy projects are planned and it is expected that several thousand megawatts will be installed in the first decade of the millennium. While experience gained through the demonstration projects currently operating is valuable, a major uncertainty in estimating power production and dynamic loads on the wind turbines, lies in the prediction of the dynamic links between the atmosphere and wind turbines. Due to lower turbulence offshore, wake effects (velocity decrease, turbulence increase downstream of a wind turbine rotor) will be propagated over larger distances downstream than is the case over land. The likely result is that in order to optimise power output, offshore wind farms will require larger distances between rows than is common in design of onshore wind farms.

In order to gain insight into these effects, the EU 5th framework project 'EfficieNt Development of Offshore WindFarms', (ENDOW) was carried out. This project started on March 1 2000 and ended on March 1 2003. In this project ECN cooperated with the following partners:

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The main aim of the project is to validate, evaluate, enhance and interface wake and boundary-layer models for utilisation offshore. This will result in a significant advance in the state of the art in both wake and marine boundary layer models leading to improved prediction of wind speed and turbulence profiles within large offshore wind farms.

This report describes the ECN activities which have been performed within the ENDOW project. All described activities are related to the validation and improvement of the WAKEFARM program. Note that another task of ECN was to carry out SODAR measurements. These activities are described in [15]. The WAKEFARM program models the wake flow downstream of a turbine, using the basic flow field parameters and some wind turbine properties as input. Output properties from the model are among others the wake wind speeds and the turbulence intensities in the wake.

The WAKEFARM model is based on the UPMWAKE model, which has been developed by the Universidad Polytecnica de Madrid. Although some modifications have been made to the UPMWAKE program, see chapter 2, these modifications are relatively minor and the basic modelling can be considered similar for both programs.

The UPMWAKE/WAKEFARM model is a 3D Parabolized Navier Stokes Code

using a k-ϵ turbulence model which accounts for the turbulent processes in the far wake. The parabolisation yields an efficient calculational procedure, which requires the near wake to be initialised with an (empirical) velocity profile. A true physical modelling of the near wake would require a full elliptic, time consuming approach.
In chapter 2 some of these modelling aspects are discussed in more detail.

Within the ENDOW project, the WAKEFARM program could be validated against many measurement cases. Also much insight was gained from a comparison of ECN's calculational results with the calculational results from Robert Gordon University, RGU. The difference between ECN's and RGU's model lies mainly in the fact that RGU uses a full elliptic approach. As described above, the elliptic approach makes it possible to model the near wake in much more detail, and it avoids the necessity of an empirical initial near wake profile.

The comparison with RGU's results led to the insight that ECN's model could be improved by changing the initial near wake model. In addition the isotropy of the turbulence has been considered as an option to improve the agreement between calculated and measured turbulence intensities in the wake. These modifications of the WAKEFARM program are described in more detail in chapter 3. The effect of these modifications is validated in chapter 4. Thereto a comparison is made between calculational results and:

- Wind tunnel measurements, made by Garrad and Hassan [1];
- Full scale measurements from the Vindeby off-shore wind farm. A very interesting validation was offered by SODAR measurements which were taken in this farm, see section 7. Thereto the SODAR measurement system was mounted on a boat. This made it very easy to perform measurements on different locations in the wake;
- Full scale measurements from the Alsvik wind farm (supplied by FFA);
- RGU's calculations (not described in this report, but they can be found in different ENDOW reports).

Although most activities were related to single wake conditions, multiple wake conditions have been considered as well. The modelling of multiple wakes is described in chapter 5. Calculations in multiple wake situations have been compared with measurements made in the wind tunnel and in the Vindeby wind farm, see chapter 6. Multiple wake results have also been compared with results from RGU's model.

Finally in chapter 8, some numerical aspects (i.e. convergence) of the WAKEFARM program are discussed.

2. DESCRIPTION OF WAKEFARM PROGRAM

Figure 2.1 gives a schematic view of the wake modelling in the WAKEFARM program. As described in chapter 1, the WAKEFARM program is largely similar to the UPMWAKE program.

The free stream wind field (axial wind speed and turbulence intensity as function of height) is modelled in accordance with the Panofsky-Dutton [2] model, see section 2.1. The wake is then divided in a near wake (with length $x_{nw} = 2.25D$) and a 'far wake'. The turbulent processes in the far wake are modelled with a k-ϵ turbulence model (see section 2.3), assuming an initial near wake velocity profile at the near wake length, $x = x_{nw}$. In the original version of the WAKEFARM program, this initial velocity profile is calculated with the inviscid momentum theory from the undisturbed wind speed and the axial force coefficient $C_{D.ax}$ of the wind turbine (see section 2.2). In the modified version of WAKEFARM, see section 3, an empirical initial velocity profile is used.

Starting from the initial velocity profile, the WAKEFARM/UPMWAKE program calculates a number of quantities in the far wake, among others the wake profile (i.e the mean wind speeds in 3 directions) and the turbulence intensities in the wake. These quantities are calculated at a discrete number of grid points in the wake. The grid size in flow (axial) direction is taken to be 0.25D. The grid size in lateral and vertical direction is taken to be $D_{exp}/7$ with D_{exp} the expanded rotor diameter (Explained in section 2.2).

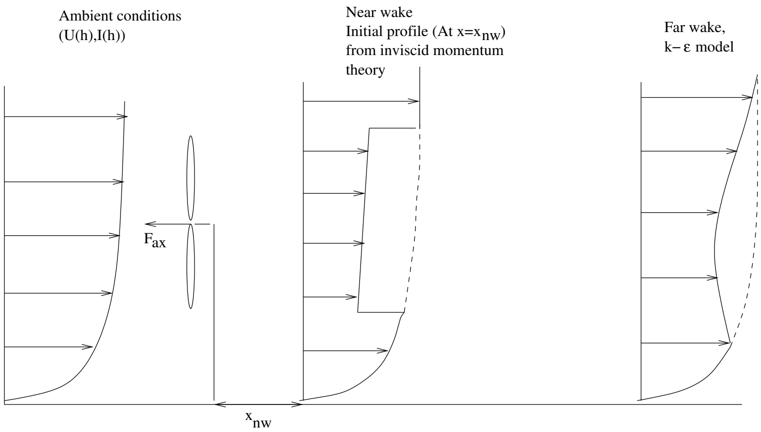

Figure 2.1 *Wake modelling in the WAKEFARM program (The figure only shows wind speed profiles)*

Hence different versions of the UPMWAKE/WAKEFARM program exist. In the sequel they are usually denoted by:

- UPMWAKE, i.e. the basic version supplied by the Universidad Polytecnica de Madrid;
- 'Original', i.e. the original WAKEFARM version;
- 'Near wake', i.e. a version which is a modification of the original WAKEFARM program and which is developed within the present project.

The main difference between the UPMWAKE program and the original version of the WAKEFARM program lies in the value which is assumed for x_{nw}: The UPMWAKE model considers x_{nw} to be 0.0 and WAKEFARM considers $x_{nw} > 0$, see also section 2.2. The main difference between the original WAKEFARM program and the modified version of the WAKEFARM program, lies in the near wake model, see section 3. In addition to the program versions described above, there is a so-called 'isotropic' version. In this version the anisotropy of wake turbulence is treated in a special way, see section 3.2.

2.1 Free stream modelling

In the WAKEFARM program, the free stream wind model is based on Panofsky-Dutton, [2]. This model is valid for the lower 10% of the atmospheric boundary layer. Although the thickness of this boundary layer depends on many parameters, the validity of the model generally extends to a height of at least 100 m.

The free stream wind speed as function of height is calculated from the friction velocity (u^*), the roughness height (z_0) and the Monin-Obukhov length scale (L):

$$U(h) = 2.5u^*[\ln(\frac{h}{z_0}) - \Psi_m(\frac{h}{L})] \tag{2.1}$$

The Monin-Obukhov length scale is a measure for the stability of the surface layer of the atmosphere. For $L > 0$ the surface layer is stable: The earth surface cools the air, which limits the vertical motion and the turbulent mixing. For $L < 0$ the surface layer is instable: The heating of the earth surface promotes the vertical motion and turbulent mixing in the atmospheric boundary layer. For L = ∞ the surface layer is neutral. The function Ψ_m is found from:

$$\Psi_m = \int_0^\xi [1 - \Phi_m(\xi)]\frac{d\xi}{\xi} \tag{2.2}$$

with ξ = h/L.

$$\Phi_m = (1 - 16\xi)^{-0.25} \qquad (L < 0) \tag{2.3}$$

$$\Phi_m = 1 + 5\xi \qquad (L > 0) \tag{2.4}$$

For stable conditions ($L > 0$), equation 2.2 then reduces to:

$$\Psi_m = -5\frac{h}{L} \tag{2.5}$$

For instable conditions ($L < 0$), equation 2.2 becomes more complicated:

$$\Psi_m = \ln[\frac{(1+\gamma^2)(1+\gamma)^2}{8}] - \mathrm{atan}(\gamma) + \pi/2 \tag{2.6}$$

with $\gamma = 1/\Phi_m = (1 - 16\xi)^{+0.25}$

It should be noted that the above mentioned equations are only valid for $h >> z_0$, which is usually the case for wind turbine applications.

The ambient turbulence intensity follows from the turbulent kinetic energy (k):

$$I(h) = 1.026\frac{\sqrt{k}(h)}{U(h)} \tag{2.7}$$

The expression for the turbulent kinetic energy k in the ambient flow is given in i.e. [3]:

$$k(h) = \frac{u^{*2}}{\sqrt{C_\mu}} \cdot f_k(\xi) \tag{2.8}$$

In this equation it is assumed that C_μ = 0.033.

For stable atmosphere ($L > 0$), the function f_k, which is implemented in WAKEFARM, is given by:

$$f_k(\xi) = [\frac{(1. + 2.5\xi^{0.6})^{1.5}}{\Phi_m}]^{0.5} \tag{2.9}$$

For unstable atmosphere ($L < 0$), the function f_k is given by:

$$f_k(\xi) = [\frac{1. - \xi}{\Phi_m}]^{0.5} \tag{2.10}$$

For neutral atmosphere, it holds that f_k=1. Hence for neutral atmosphere, the ambient turbulence intensity is given by:

$$I(h) = 2.4\frac{u^*}{U(h)} \tag{2.11}$$

Note that the derivation of the turbulence intensity from the turbulent kinetic energy requires assumptions on the anisotropy of turbulence. These are described in section 3.2.

2.2 Near wake modelling

According to (inviscid) momentum theory the velocity deficit in the rotorplane is:

$$u_{def}(x = 0) = (0.5 - 0.5\sqrt{1 - C_{D.ax}}) \cdot U_\infty \tag{2.12}$$

with $C_{D.ax}$ the axial force coefficient on the whole rotor. The value for u_{def} is constant along the rotor plane.
According to the same inviscid momentum theory, this velocity deficit is doubled, at the limit state, infinitely downstream of the turbine:

$$u_{def}(x = \infty) = (1.0 - 1.0\sqrt{(1 - C_{D.ax})}) \cdot U_\infty = 2u_{def}(x = 0) \tag{2.13}$$

Hence the actual wake velocity decreases in streamwise direction and as a result (mass conservation), the wake diameter expands from the rotor diameter at x=0 to the so-called expanded diameter (D_{exp}) at x=∞. The continuity equation together with the equations 2.12 and 2.13 yield the following value for D_{exp}:

$$D_{exp}(x = \infty) = D\sqrt{(1 - a)/(1 - 2a)} \tag{2.14}$$

with a, the axial induction factor in the rotor plane:

$$a = u_{def}(x = 0)/U_\infty = (0.5 - 0.5\sqrt{1 - C_{D.ax}}) \tag{2.15}$$

It is stressed that it is the hypothetical inviscid assumption which yields the doubled velocity deficit at infinity. Obviously turbulent mixing makes the velocities at infinity recover to the free stream values.

A simple cylindrical vortex model can be used to gain insight into the wake expansion rate. Such model is to some extent comparable to the momentum theory. The cylindrical wake model yields the following relation for the velocity deficit as function of the downstream distance to the rotor (Note that this downstream distance is made non-dimensional with the rotor radius):

$$u_{def}(x)/u_{def}(x=\infty) = 0.5 + 0.5 \cdot f/[(f^2+1.)^{0.5}] \quad (2.16)$$
$$f = x/R$$

The relation is shown graphically in figure 2.2: It is seen that the wake induced velocity in the rotorplane is half the value at infinity, similar to the considerations given above. Within ≈ 3R the infinite value has almost been reached, although it is expected that in reality the wake development takes place with a slightly slower rate, due to the expansion of the wake. The figure clearly shows that the assumption made in the UPMWAKE program (x_{nw} = 0.0) is somewhat irrealistic: The wake definitely needs some distance before it is fully expanded.

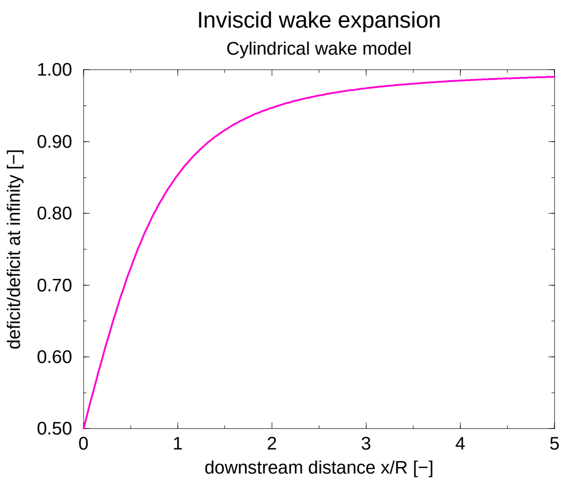

Figure 2.2 *Wake deficit (inviscid)*

The wake development as sketched in figure 2.2, i.e. the transition from the velocities given by equation 2.12 to the values from equation 2.13 cannot be modelled in the WAKEFARM program. This is due to the parabolised nature of the solution procedure. Such parabolisation implies that only upstream influences are modelled, where the modelling of the above mentioned inviscid wake effects also require feedback influences, i.e. an elliptic approach. For this reason an initial velocity profile is fixed at a certain distance downstream of the rotor ($x_{nw} > 0$) and no wake modelling takes place upstream of this position. Actually all upstream wake effects are assumed to be included in this initial velocity profile. This initial profile is then a boundary coundition for the far wake model (section 2.3), which models the wake downstream of x_{nw}.

The initial velocity defect which is imposed in the original WAKEFARM program is found from equation 2.13 and is assumed to be constant over the expanded diameter, where the expanded diameter is found from equation 2.14. Outside the expanded diameter the velocity is equal to its free stream value. The resulting near wake profile is then (almost) 'hat' shaped, with sharp discontinuities at the edges of the wake, see figure 2.1.

The value for x_{nw} has been chosen to be 2.25 D. The choice for this value was based on previous validation cases, but nevertheless its value should be considered as rather arbitrary.

Hence, the WAKEFARM model assumes that for $x < 2.25D$ no turbulent mixing takes place and the wake develops according to an inviscid model. On the other hand, for $x > 2.25D$, the inviscid wake expansion is ignored and only turbulent mixing is modelled. These assumptions can be justified by realising that according to figure 2.2, the inviscid wake expansion is almost completed at x=2.25D. So the ignorance of the inviscid wake expansion for distances x>2.25D is expected to have limited effects. On the other hand, turbulent mixing is expected to be limited for distances x<2.25D: In [4] it is explained that the shear stress profile develops relatively slowly in the near wake. This slow development leads to a limited mixing of momentum from the ambient flow to the wake. Therefore turbulent mixing hardly has had any chance for distances x<2.25D and the start of the turbulence modelling at x = 2.25D does not necessarily neglect a substantial part of the turbulent mixing process.

2.3 Far wake modelling

As described in section 2.2, the far wake is initialised with a near wake profile at $x = x_{nw}$. Using this near wake profile as boundary condition, the turbulent mixing is modelled with a k-ϵ turbulence model. The model is described in more detail in [5] and comprises a set of 7 equations:

- Continuity equation;
- 3 Momentum equation in x, y and z direction;
- Energy equation (for adiabatic temperature);
- Equation for turbulent kinetic energy;
- Equation for dissipation rate of turbulence

These equations solve the added values (i.e. the disturbances from the basic flow values) of the following 7 unknowns:

- Velocities in three directions;
- Pressure;
- Adiabatic temperature;
- Turbulent kinetic energy k;
- Dissipation rate of turbulent kinetic energy ϵ

The model is parabolised by neglecting the streamwise diffusion and the pressure gradient in streamwise direction. In this way all elliptic terms have disappeared.

The turbulent stresses are modelled using a turbulent kinematic viscosity and closure is then needed for the turbulent kinematic viscosity, turbulent diffusivity of heat, turbulent diffusivity of k, and turbulent diffusivity of ϵ.

The turbulent intensities can be derived from the turbulent kinetic energy assuming some kind of anisotropy, see section 3.2.

A detailed description of the equations, as well as the numerical approach, can be found in [6], see also section 8.

3. MODIFICATION OF WAKEFARM PROGRAM

3.1 Near wake profile

As described in section 2.2, the near wake is initialised with a wind speed profile at x = 2.25D. This initial profile was 'hat-shaped' with a constant deficit given by equation 2.13. From calculations performed by RGU and wind tunnel measurements made at distances of approximately 2.25D downstream of the rotor, indications were found that the wake profile is better approximated by an axisymmetric Gaussian shaped profile. Hence the following expression is adapted:

$$u_{def}(y,z) = 1.3 \cdot (1.0 - 1.0\sqrt{1 - C_{D.ax}}) \cdot U_\infty \cdot (e^{-0.5(y/r\sigma_y)^2} e^{-0.5((z-h_t)/r\sigma_z)^2}) \tag{3.1}$$

The resulting profile is Gaussian with a maximum deficit at hub height: $u_{def,max} = 2.6aU_\infty$ (with a the axial induction factor in the rotor plane, see equation 2.15). The values for σ_y and σ_z were tuned to wind tunnel measurements: σ_y = σ_z = D_{exp}/(2D), where D_{exp} is given by equation 2.14.

Figure 3.1 shows an example of the Gaussian near wake profile in a 2-dimensional environment (u_{def} is only a function of y). Furthermore it is assumed that D_{exp}/D = 1.5 and hence σ_y = σ_z = 0.75. It is seen that the maximum deficit is found at the rotor centre. At y/R = D_{exp}/D = 2σ, the velocity has almost reached its ambient value. Actually, only 5% of the 'integrated' velocity defect is found at $\sqrt{y^2 + (z-h_t)^2} > D_{exp}$/D. Hence the main velocity defect appears within the expanded rotor diameter, which was also the case in the original modelling: In the original model (see section 2.2), 100% of the velocity defect is concentrated within the expanded diameter.

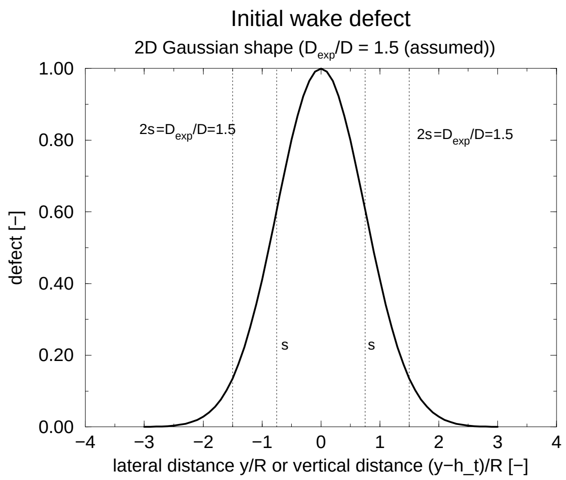

Figure 3.1 *Example of Gaussian near wake profile (2-Dimensional; D_{exp}/D = 1.5 (assumed)*

3.2 Relation between turbulent kinetic energy and turbulence intensity

3.2.1 Definition of wake turbulence intensity

Before entering the discussion on the wake turbulence intensities, it is important to realise that many definitions of such turbulence intensity can be found in literature. The differences in definition often lead to misunderstandings and confusions and therefore some attention needs to be paid to these definitions.

The usual definition of turbulence intensity is given by the standard deviation of the wind speed fluctuations (in a particular direction) normalised with the resultant mean velocity, which in the present report is assumed to be in x-direction. This common definition of turbulence intensity is used for the free stream flow, i.e. the turbulence intensity in x-direction is given by:

$$I_{x,\infty}(z) = \sigma_{x,\infty}/U_\infty(z) \tag{3.2}$$

Note that $I_{x,\infty}$ and U_∞ are assumed to be a function of height (z) only.

Now, at first sight it would be most consistent to define the wake turbulence intensity by the standard deviation of the wake wind speed fluctuations normalised with the local mean wake wind speed. Hence, a higher wake turbulence intensity is not necessarily a result of a higher standard deviation, but it may also be caused by a lower mean wake velocity. Furthermore,it should be kept in mind that there is a large variation of mean wake velocities in y- and z direction. As such, normalisation on the local mean wake wind speed yields a large variation of the turbulence intensity over the wake. These effects often obscured the physical understanding of the meaning of the wake turbulence intensity.

Therefore, instead of normalising with the local wake wind speed, the wake turbulence intensity is commonly defined as the standard deviation of the wind speed fluctuations at position (y,z) (in a particular direction) normalised with the free stream wind speed at height z, i.e.

$$I_{w,x}(y,z) = \sigma_{w,x}(y,z)/U_\infty(z) \tag{3.3}$$

As such, the comparison between $I_{w,x}$ and $I_{x,\infty}$ offers direct insight into the increase in the standard deviation of the wind speed fluctuations. Furthermore it offers the possibility to define an added turbulence intensity at position (y,z) which is a measure for the increase in standard deviation:

$$I_{add,x}(y,z) = \sqrt{I_{w,x}(y,z)^2 - I_{x,\infty}(z)^2} = 1/U_\infty \sqrt{\sigma_{w,x}(y,z)^2 - \sigma_{x,\infty}(z)^2} \tag{3.4}$$

In the WAKEFARM program the definitions according to equations 3.3 and 3.4 are used.

Other definitions of the wake turbulence intensity can also be found in the literature. In particular a normalisation based on the free stream wind speed at hub height, instead of a normalisation on the free stream wind speed at height z, is a frequently used alternative.

3.2.2 Derivation of turbulence intensities from turbulent kinetic energy

It should be realised that a k-ϵ model does not produce a value for the turbulence intensity. Instead, the turbulence intensity should be derived from the turbulent kinetic energy k. The basic relation between the turbulent kinetic energy and the standard deviation of wind fluctuations in x, y, and z direction (σ_x, σ_y or σ_z) is given by:

$$k = 0.5\sigma_x^2 + 0.5\sigma_y^2 + 0.5\sigma_z^2 \tag{3.5}$$

Now the standard deviations (i.e. the turbulence intensities) can only be derived from the turbulent kinetic energy when assumptions are made on the anisotropy.

For the basic atmosphere, the following relations between the standard deviation of the wind fluctuations and the turbulence friction velocity u^* are available, see i.e. [7]:

$$\sigma_{x,\infty} = 2.4u_\infty^* \tag{3.6}$$

$$\sigma_{y,\infty} = 1.9u_\infty^* \tag{3.7}$$

$$\sigma_{z,\infty} = 1.25u_\infty^* \tag{3.8}$$

Substitution of the equations 3.6, 3.7 and 3.8 into equation 3.5 yields:

$$k_\infty = 5.47u_\infty^{*2} \tag{3.9}$$

Since u_∞^* = $\sigma_{x,\infty}$/2.4 (equation 3.6) this yields:

$$k_\infty = 5.47/(2.4)^2\sigma_{x,\infty}^2 = 0.95\sigma_{x,\infty}^2 \tag{3.10}$$

As a result, the turbulence intensity in x-direction is

$$I_{x,\infty} = \sigma_{x,\infty}/u_\infty = 1.026\sqrt{k_\infty}/u_\infty \tag{3.11}$$

Using the definition of the added turbulence, (equation 3.4),

$$I_{add,x}(y,z) = \sqrt{I_{w,x}(y,z)^2 - I_{x,\infty}(z)^2} \tag{3.12}$$

it is found that:

$$I_{w,x}(y,z) = \sigma_{w,x}(y,z)/U_\infty(z) = \sqrt{I_{add}(y,z)^2 + I_{x,\infty}(z)^2} \tag{3.13}$$

In the original modelling, it was assumed that the anisotropy in the wake is similar to the anisotropy in the free stream ($k_w = 0.95\sigma_{w,x}^2$). Hence:

$$I_{w,x}(y,z) = 1.026\sqrt{k_w(y,z)}/U_\infty(z) \tag{3.14}$$

Substitution of equations 3.14 and 3.11 into 3.4 yields:

$$I_{add,x}(y,z) = 1.026\sqrt{k_{add}(y,z)}/U_\infty(z) \tag{3.15}$$

with:

$$k_{add}(y,z) = k_w(y,z) - k_\infty(z) \tag{3.16}$$

It is noted that k_{add} is straightforwardly available from the WAKEFARM program, see section 2.3.

The basic underlying assumption for equation 3.15 is given by an anisotropy in the wake, which is supposed to be similar to the anisotropy in the free stream (and which is given by the equations 3.6 to 3.8).

Now, due to the fact that the original WAKEFARM model often appeared to overestimate the turbulence intensity in x-direction, the ENDOW project group suggested to assume fully isotropic turbulence in the wake. This decreases the turbulence intensity in x-direction and as such it would improve the agreement with measurements. However, such assumption obviously increases the turbulence in the other 2 directions. Since measurements of turbulence intensity in y- and z-direction are usually not available, the assumption of isotropic turbulence in the wake could not be confirmed by experiments. Some doubt on the validity of this assumption is found from physical arguments, i.e. the presence of the ground is expected to yield different turbulence levels in z-direction in the wake as well.

Nevertheless, some validations have been made by assuming that $\sigma_x = \sigma_y = \sigma_z$, instead of using the relations 3.6 to 3.8.

This implies that $k = 1.5\sigma_x^2$ and $\sigma_x = \sigma_y = \sigma_z = 0.82\sqrt{k}$.

The isotropic turbulence is only assumed for the added turbulence in order to avoid discontinuities at the edges of the wake. Then equation 3.15 is replaced by:

$$I_{add,x}(y,z) = c_{add}\sqrt{k_{add}(y,z)}/U_\infty(z) \tag{3.17}$$

where c_{add} accounts for the wake anisotropy (0.82 in case of isotropic turbulence and 1.026 in case of 'free stream anisotropy').

Then according to equation 3.13 the following equation for the wake turbulence intensity (again normalised with the free stream wind speed) is found:

$$I_{w,x}(y,z) = 1/U_\infty(z)\sqrt{c_{add}^2 k_{add}(y,z) + 1.026^2 k_\infty(z)} \tag{3.18}$$

with c_{add} is 0.82, since isotropic added wake turbulence is assumed. Hence the value for the wake turbulence approaches the free stream value (equation 3.11) at the edge of the wake, where k_{add} approaches zero.

4. SINGLE WAKE: VALIDATION

Calculational results obtained from different model versions of the WAKEFARM program have been compared with wind tunnel measurements performed by Garrad and Hassan and with measurements performed in the Vindeby wind farm. Furthermore a comparison is made with measurements which are performed by FFA in the Alsvik wind farm.

4.1 Single wake: Validation with wind tunnel measurements

4.1.1 Single wake: Description of wind tunnel measurements

A comparison is made with measurements which have been performed in the boundary layer wind tunnel of Marchwood Engineering Laboratories. These measurement are reported by Hassan, [1]. Very detailed wake measurements have been performed on a wind farm which consists of turbines with a rotor diameter of 0.27 m and a hub height of 0.3 m. Single wake measurements are performed for three different tip speed ratios: λ = 2.9, λ = 4.0 and λ = 5.1. The corresponding (measured) axial force coefficients are:
$C_{D.ax}$ = 0.62; $C_{D.ax}$ = 0.79, resp. $C_{D.ax}$ = 0.85

Wake traverses have been made in lateral and vertical direction at 2.0D, 2.5D, 5D and 7.5D behind the model turbine. The wake velocities are processed to mean wake profiles and to turbulence intensities.

The incoming wind profile can be approximated by:

$$U_\infty(z) = \frac{u^*}{k} \cdot \ln(\frac{z}{z_0}) \tag{4.1}$$

The value of the incoming tunnel speed at hub height (z= 0.3 m) turned out be approximately 4.1 m/s. The velocity at z= 0.15 m turned out to be approximately of 3.7 m/s and the velocity at z = 0.45 turned out to be approximately 4.3 m/s.

The incoming turbulence intensity is approximately 9.2 % at hub height (z = 0.3 m), approximately 7.1 % at z = 0.45 m and approximately 14.2% at z = 0.15 m.

4.1.2 Single wake: Simulations of wind tunnel measurements

Measurements of wake profiles and turbulence intensities have been compared with calculational results from 3 WAKEFARM program versions:

- The original version with a 'hat shaped' near wake profile and wake anisotropy which is assumed to be similar to the ambient anisotropy (see section 2);
- A modified version with the 'Gaussian shaped' near wake profile (section 3.1), but the wake anisotropy is still assumed to be similar to the ambient anisotropy;
- A modified version with the 'Gaussian shaped' near wake profile, and the added wake turbulence is assumed to be isotropic, see equation 3.18;

The input for the WAKEFARM program basically consists of the diameter, the hub height and the thrust coefficient. These quanties are straigthforwardly known, see section 4.1.1. Furthermore the incoming basic tunnel flow should be prescribed.

Wind tunnel: Basic flow
It is reminded that the incoming wind flow needs to be estimated from a roughness height, a Monin-Obukhov length scale and a friction velocity. Hence there are only three degrees of freedom, which makes it possible to tune only 3 discrete points in the wind and/or the turbulence intensity profile. The best fit was found from z_0 = 7.56 10^{-6} m, L = 2.4 m and u^* = 0.1444 m/s, see ([8]). In this way the incoming wind speed profile was fitted well. The resulting wind speed at z = 0.15 m is approximately 3.7 m/s (measured: 3.7 m/s), the resulting wind speed at z = 0.3 m is approximately 4.05 m/s (measured: 4.1 m/s) and the wind speed at z = 0.45 m is approximately 4.3 m/s (measured 4.3 m/s). The turbulence intensity at hub height is also fitted well. The actual value of the turbulence intensity at hub height is 9.3 % (measured: 9.2%) However the turbulence intensities at other heights deviate: At z = 0.15 m the actual value turns out to be 10.2 % (instead of the measured value of 14.2%) and at z = 0.45 m the turbulence intensity is 8.7% (instead of the measured value of 7.1%).

4.1.3 Single wake: Validation with wind tunnel measurements: Comparison between calculational and measured results

In the figures 4.1 to 4.6, the calculational results from the different model versions are compared with the measurements. The results which are identified with 'original' denote the calculational results from the original WAKEFARM program. The results which are identified with 'nearwake' are the calculational results in which the Gaussian near wake profile is implemented, and the results which are identified with 'isotropic' are the results in which the added wake turbulence is assumed to be isotropic. The latter modification is obviously only relevant for the turbulence intensities (figures 4.4 to 4.6) and not for the wake profiles (figures 4.1 to 4.3). Turbulence intensities are only presented for x = 5 D and x = 7.5 D.

The following observations can be made:

- Wake profiles:
  - It can be observed that the modified initial near wake profile improves the agreement between calculated and measured wake profiles at x = 2.5D considerably. This is not surprising since the modified near wake profile has more or less been fitted on the present measurements;
  - It is somewhat surprising to see that the better estimate of the initial near wake profile leads to a poorer agreement between the calculated and measured wake profiles at x = 5.0 D (in particular for λ = 2.9 and 4.0). On the other hand, the agreement between the calculated and measured wake profiles at x = 7.5 D is considerably improved again for λ = 4.0 and 5.1!
- Turbulence intensities:
  - In most cases, the modification of the initial near wake profile improves the agreement between calculated and measured turbulence intensity profiles. A further improvement is found by the assumption of isotropic added turbulence. The exception is at x = 2.5D. Although the turbulence intensities from all model versions, show an underestimation at this position, the underestimation is much more in the results from the modified models. This is a consequence of the assumed initialisation at x=2.25D. Hence the production of wake turbulence is started at x=2.25D. A position of x=2.5D is a too short distance behind x=2.25D to create a significant amount of turbulence, but nevertheless the very strong shear which results from the initial 'hat-shaped' profile yields so much turbulence that a reasonable, though misleading, agree-

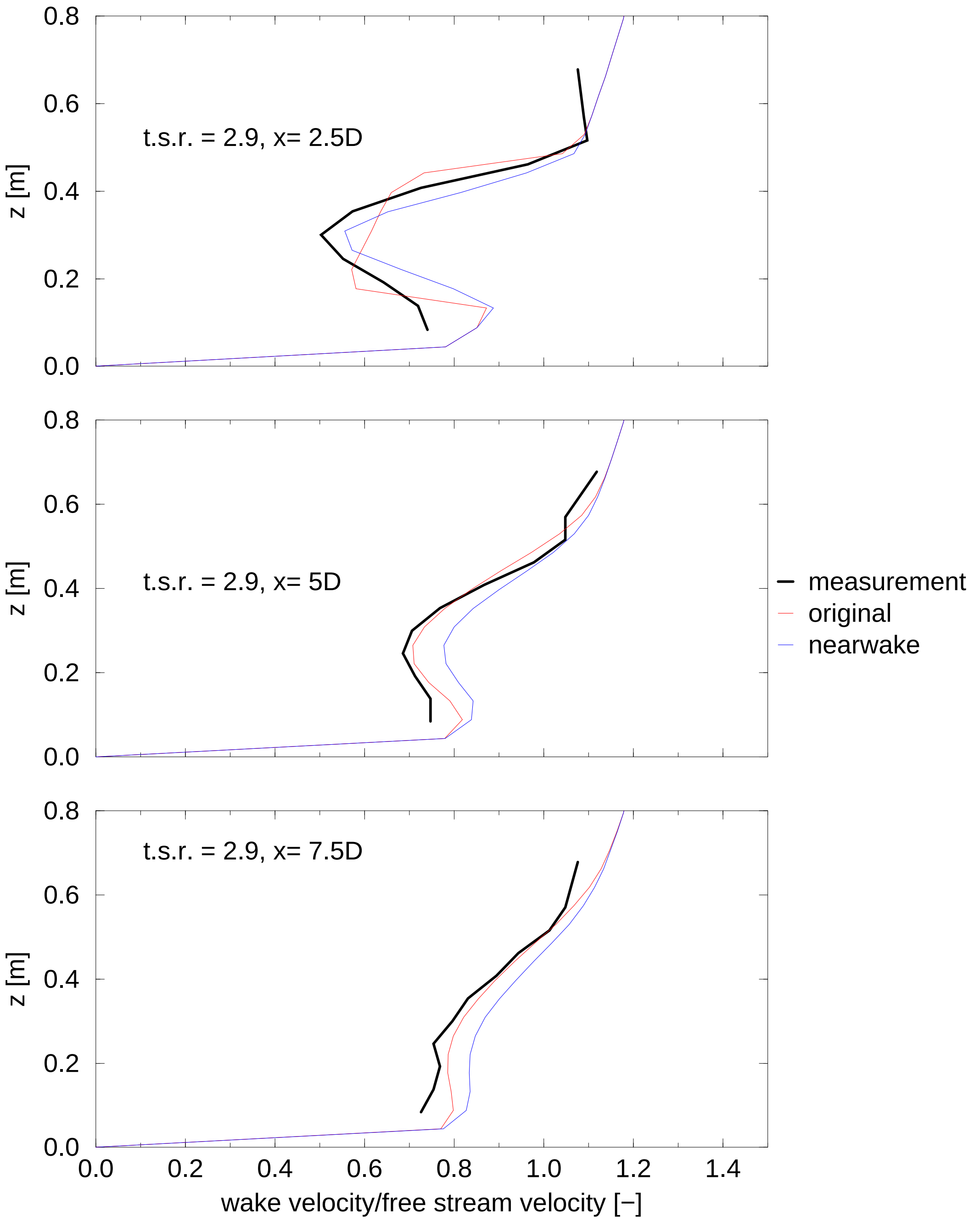

Figure 4.1 *Wind tunnel: Single wake: Measured and calculated wake profiles at λ = 2.9 (y=0)*

ment with the measured values is found: At distances further downstream, this large production of turbulence leads to a considerable overestimation of the turbulence intensity.
Generally speaking the best agreement is found from the 'isotropic model'. However it is again emphasised that the assumption of isotropic added turbulence improves the agreement between calculated and measured turbulence intensities in x-direction, but it is unknown how the calculated and measured turbulence intensities in the other two directions compare.

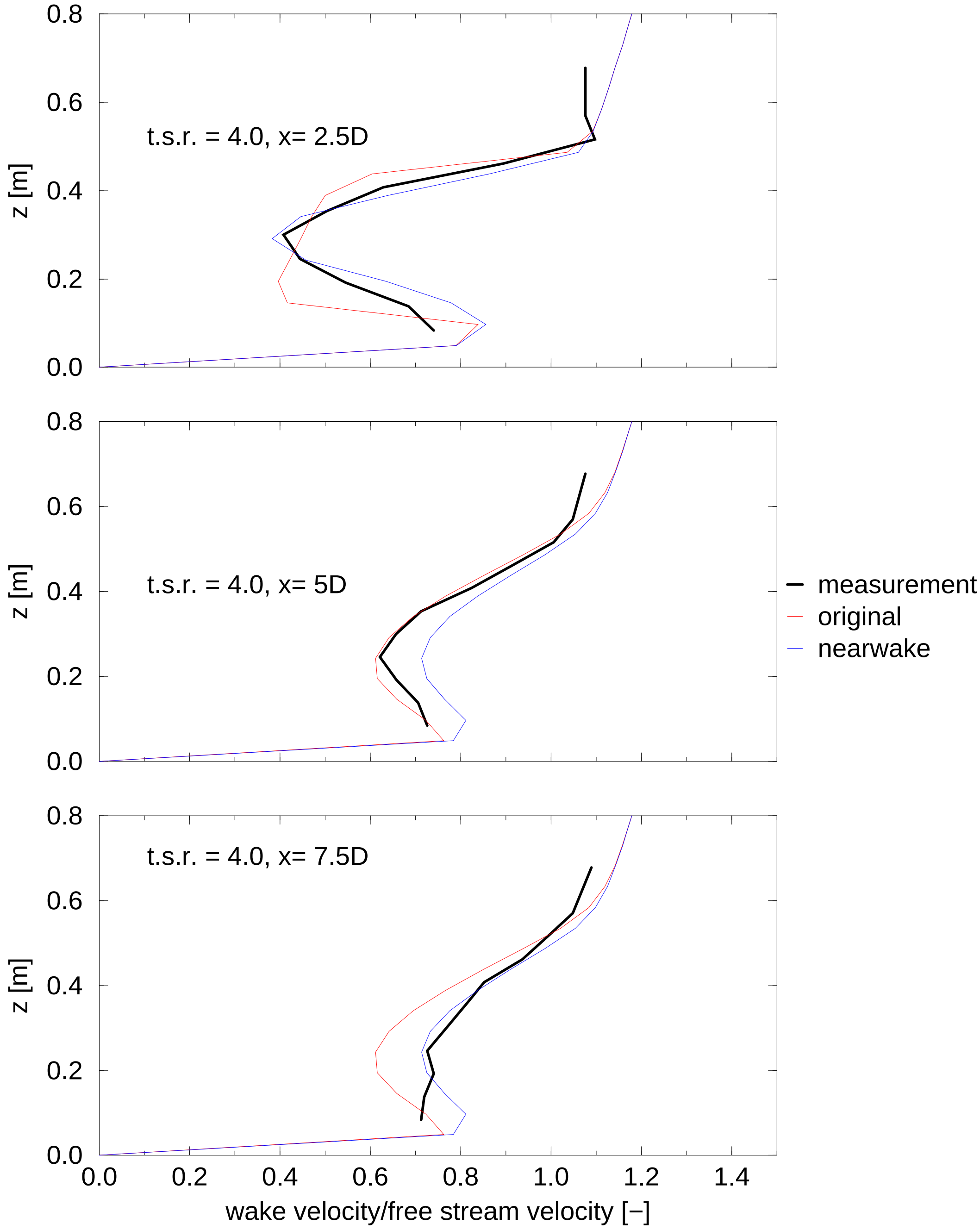

Figure 4.2 *Wind tunnel: Single wake: Measured and calculated wake profiles at* λ *= 4.0 (y=0)*

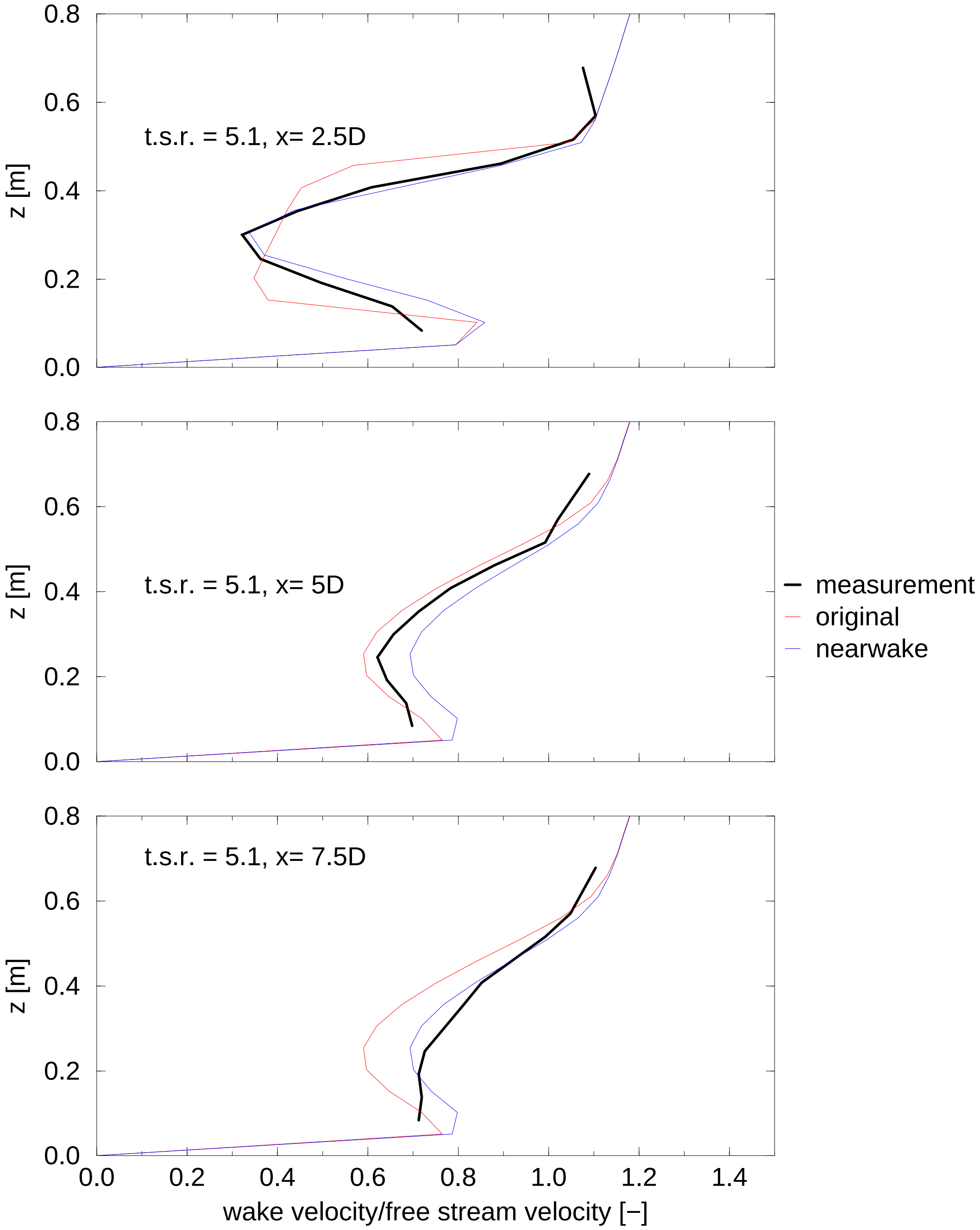

Figure 4.3 *Wind tunnel: Single wake: Measured and calculated wake profiles at λ = 5.1 (y=0)*

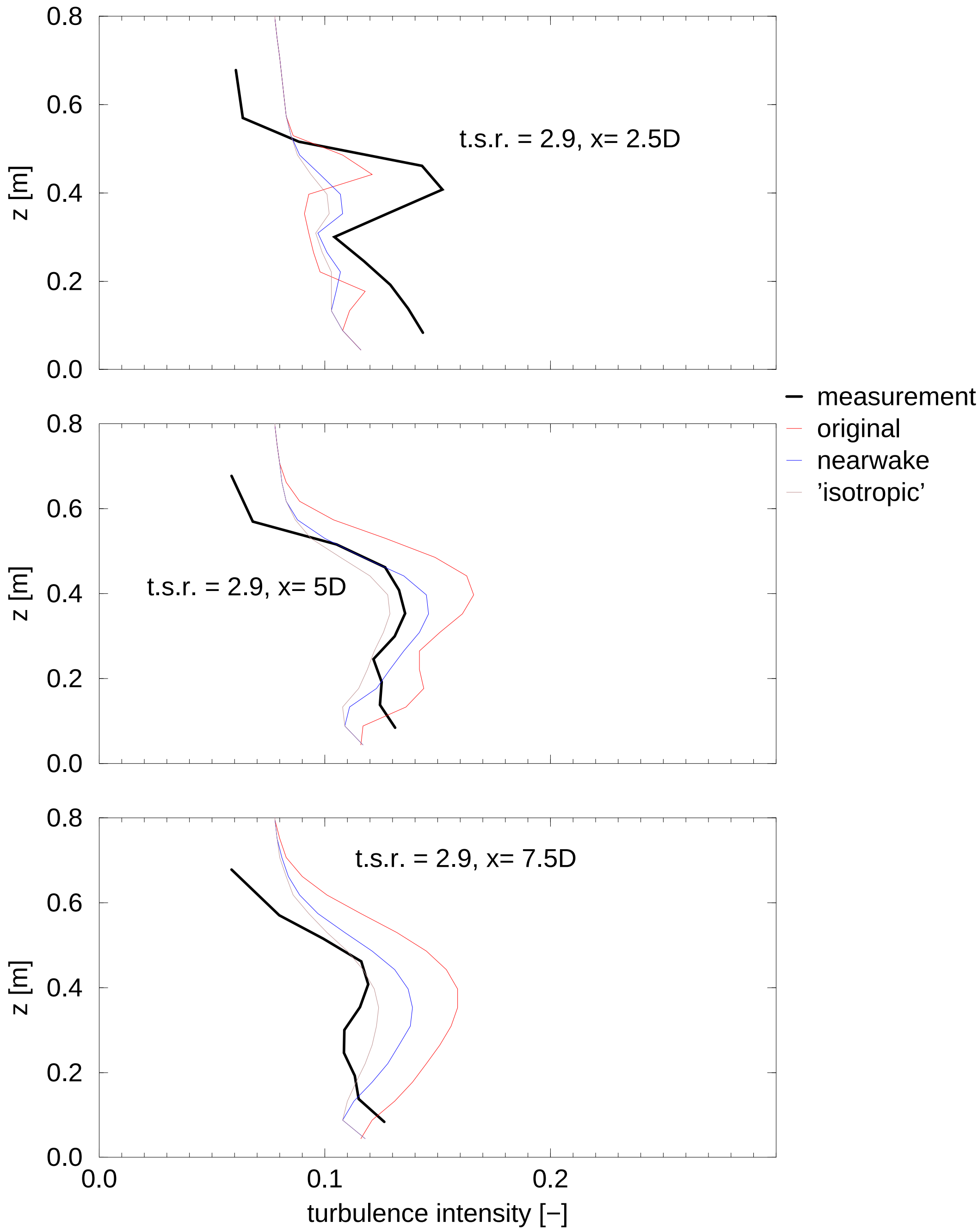

Figure 4.4 *Wind tunnel: Single wake: Measured and calculated turbulence intensity profiles at* λ *= 2.9 (y=0)*

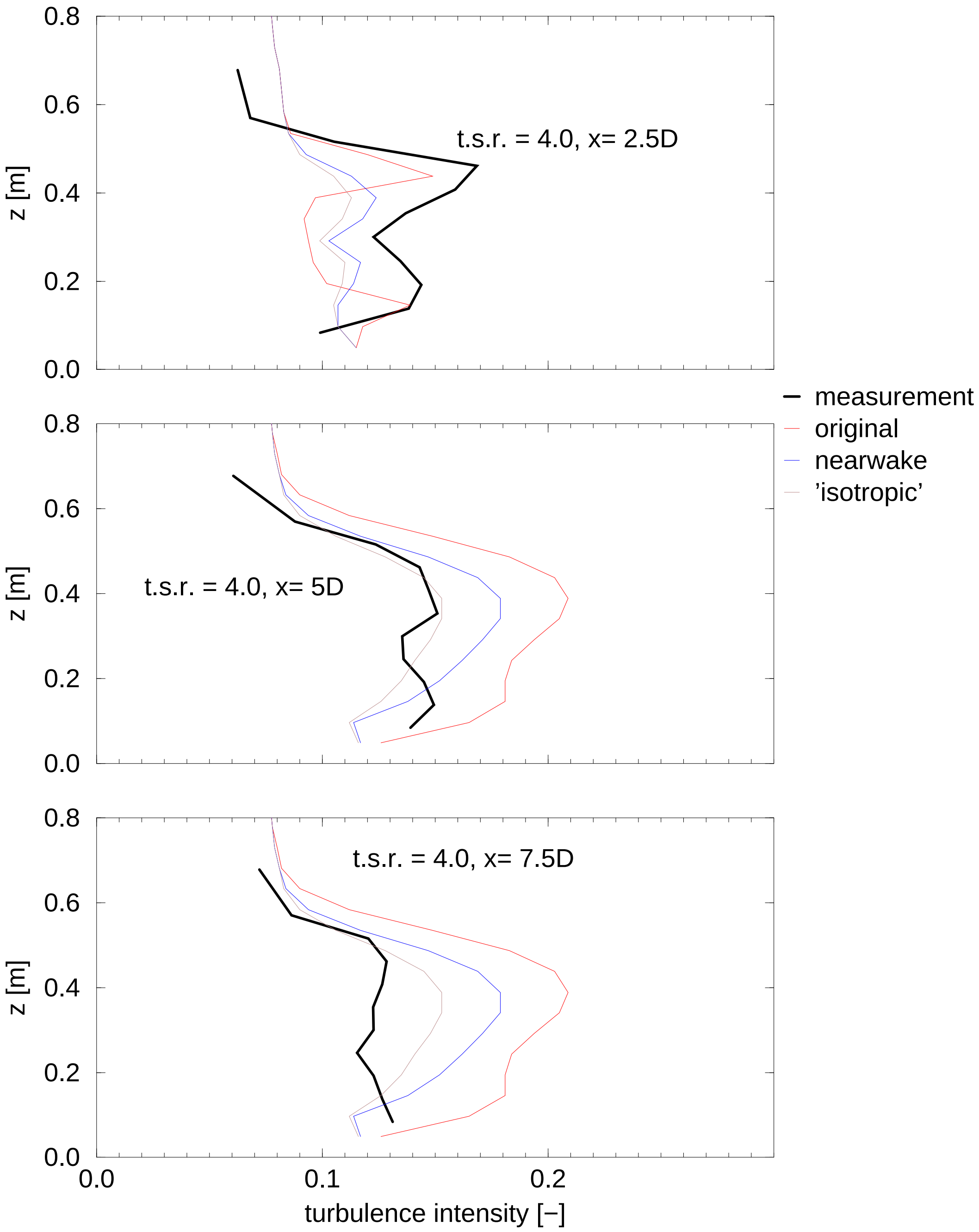

Figure 4.5 *Wind tunnel: Single wake: Measured and calculated turbulence intensity profiles at λ = 4.0 (y=0)*

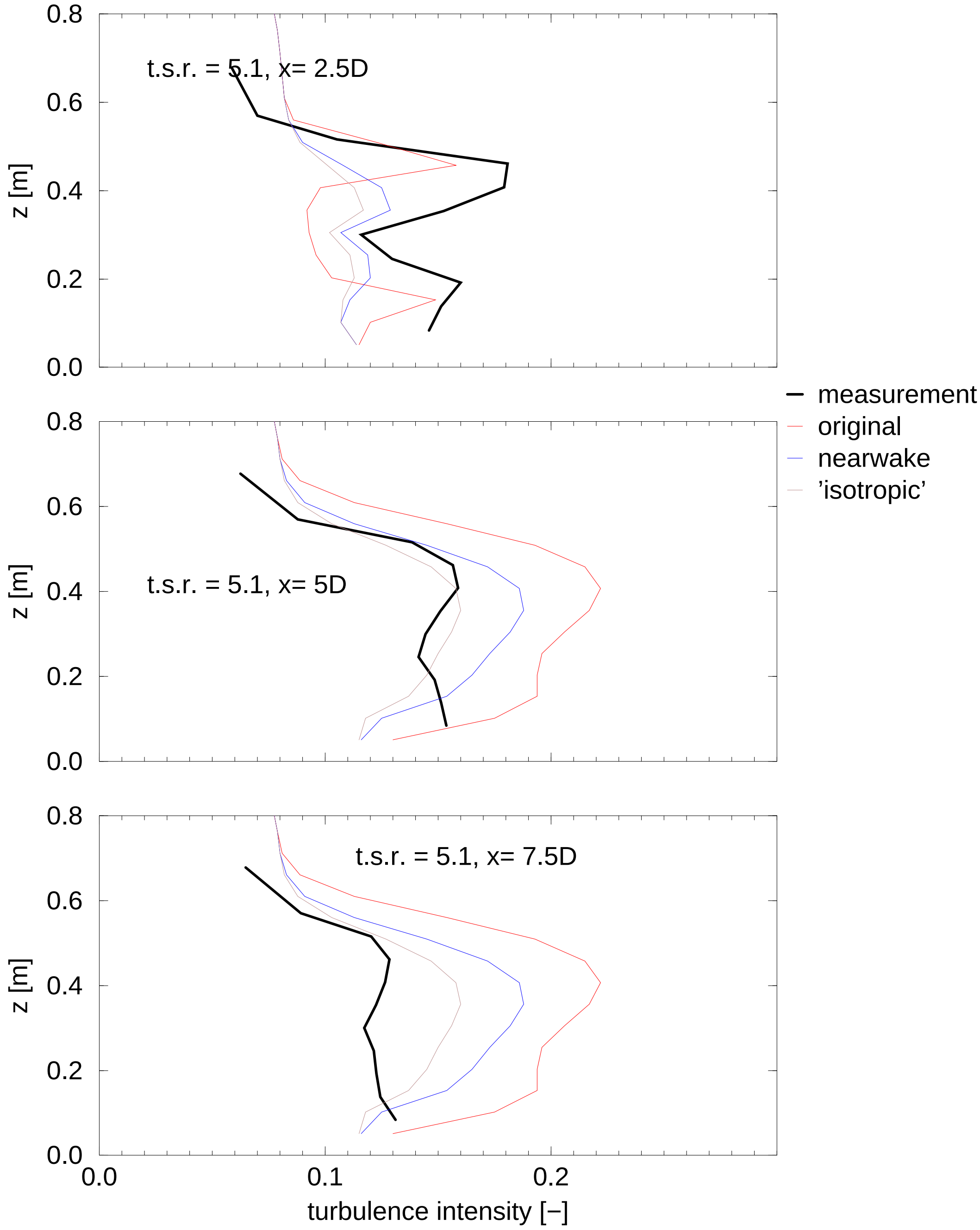

Figure 4.6 *Wind tunnel: Single wake: Measured and calculated turbulence intensity profiles at λ = 5.1 (y=0)*

4.2 Single wake: Validation with Vindeby measurements

4.2.1 Single wake: Description of Vindeby wind farm and measurements

The Vindeby wind farm is located in the Baltic See, some 2 km from the north-west coast of the Danish Island of Lolland. The wind farm consists of 11 wind turbines. The turbines (450 kW Bonus) are stall controlled and fixed speed. The rotor diameter is 35 m and the hub height is 38 m. The thrust curve has been calculated by BONUS and supplied within the ENDOW project. The 11 turbines are arranged in two rows, i.e. 1 row of 5 wind turbines and a second row of 6 wind turbines, see figure 4.7.

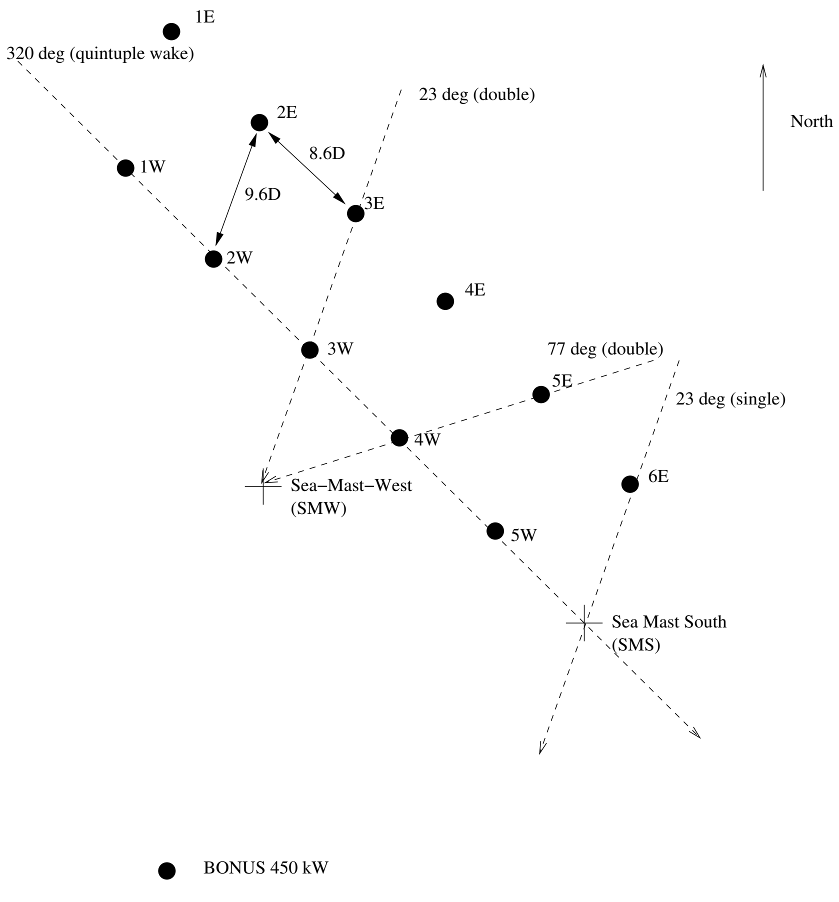

Figure 4.7 *Vindeby: Layout of wind farm, location of meteorological masts and wind directions with wake interferences*

The spacing between the turbines in a row is 300 m (8.6 D). The spacing between the two rows is also 8.6 D. However, due to the fact that the turbines in the row are shifted, this leads to a distance of 9.6 D between adjacent turbines from different rows. Three meteorological masts have been erected close to the wind farm. One mast (not indicated in figure 4.7) is located nearly 2 km (i.e. some 40 D) south of

the most southerly turbine in the array (turbine 5W). Then there are two off-shore masts: The so-called SMW mast at 8.6 D from turbines 3W and 4W, and the so-called SMS mast at 8.6 D from turbine 5W.
The table below shows the heights at which anemometers are placed above the ground or the mean sea level:

LM	SMW	SMS
-	48	48
46	-	-
-	43	43
38	38	38
-	29	29
20	20	20
-	15	15
7	7	7

Table 4.1 *Heights of anemometers on meteorological masts in Vindeby wind farm*

A more detailed description of the site and the meteorological monitoring is given in [9]. A description of the turbines and the wake interference is given in [10].

Within the ENDOW project, RISØ supplied a number of measurements which have been simulated with the wake models, see [11]. The single wake measurements, which are discussed in the present chapter have been performed for a wind direction of approximately 23 degrees, where the SMS mast is in the wake of turbine 6E (see figure 4.7). Thereto 1 minute averaged data, which were collected during a period of 2 years, and which were taken in a wind direction interval from 18 to 28 degrees (wake mast) were selected. At this wind direction, free stream atmospheric data could be obtained from the LM mast.

The data were binned on ambient wind speed, turbulence intensity and stability. Wind speed bins of 4-6 m/s, 6-9 m/s, 9-11 m/s and above 11m/s have been used. The turbulence intensity bins are: 5-7%, 7-9%, 9-11% and above 11%. The atmospheric stability bins are: $|L| > 1000$m, $0 < L < 1000$ m and $-1000 < L < 0$ m.

It should be realised that for some bins, insufficient data points were available. In the present study, sufficient data were available for the following bins:

stability	wind speed [m/s]	turbulence intensity [%]
neutral	5	6
neutral	5	8
neutral	7.5	6
neutral	7.5	8
neutral	10	6
stable	5	6
unstable	5	6
unstable	5	8
unstable	7.5	6
unstable	7.5	8

Table 4.2 *Centre bin values of free stream wind speed and turbulence intensity, for which sufficient measurements were available*

4.2.2 Single wake: Simulation of Vindeby measurements

The measurements, which are described in the previous section have been simulated by WAKEFARM. Again, calculational results from 3 WAKEFARM program versions have been supplied, see section 4.1.2

As explained in section 4.1.2, the input for the WAKEFARM program basically consists of the diameter, the hub height and the thrust coefficient. These quantities are straigthforwardly known, see section 4.2.1. Furthermore the incoming basic flow should be prescribed.

Vindeby wind farm: Basic flow

The basic flow has been modelled by tuning values for the friction velocity and roughness height to the centre bin values of hub height velocity and turbulence intensity from table 4.2. The Monin-Obukhov length scale has been fixed to +200 m for stable conditions and -200 m for unstable conditions. It must be noted that the basic flow field is approximated in this way because only two parameters hub height velocity and hub height turbulence intensity) have been fitted. The model then produces a wind speed profile and turbulence profile, which is not necessarily the same as the actual measured profile, although the hub values are similar. Furthermore the rather rough bins may lead to some deviations. In order to get an indication on the differences in the free stream modelling, the figures below will also show the comparison between modelled and actual basic flow.

Usually the results of WAKEFARM calculations are considered for only one single wind direction. Therefore the calculational results in section 4.2.3 assume the ’centre wake wind direction’ to occur for 100% of the time.

However, it should be realised that the measurement results which are used in the present validation, are taken in a relatively wide wind direction interval from 18 to 28 degrees. Therefore, at a later stage, RISØ supplied the wind direction frequency distribution within this wind direction bin, based on an interval of 1 degree. Calculational results have then been produced, which are ’weighted’ with this frequency distribution, see the results which are presented in chapter 4.2.4.

4.2.3 Single wake: Validation with Vindeby measurements: Comparison between calculational and measured results

In the figures 4.8 to 4.11 the calculated velocity profiles are compared with the measured profiles. The figures 4.12 to 4.15 show the turbulence intensities. Similar to the results presented in section 4.1.3, calculational results from three different model versions are presented. The results which are identified with 'original' denote the calculational results from the original WAKEFARM program. The results which are identified with 'nearwake' are the calculational results in which the Gaussian near wake profile is implemented, and the results which are identified with 'isotropic' are the results in which the added wake turbulence is assumed to be isotropic. The latter effect is obviously only relevant for the turbulence intensities

The following observations can be made:

- Some important differences can be observed between the calculated and measured free stream profiles of wind and turbulence. As explained above, the free stream input data for the WAKEFARM program consist of roughness length, friction velocity and Monin-Obukhov length scale. The latter one is prescribed, the other two make it possible to tune two parameters, i.e. the wind speed and turbulence intensity at hub height (h=38 m). As such, the modelled wind speed and turbulence intensity at hub height should coincide with the measurement data, but the profiles of free stream wind speed and turbulence intensity are not tuned.
  It should be noted that the comparison is obscured by doubt on the reliability of the free stream measurement reading at h=20 m, which has a large optical effect on the shape of the free stream velocity profile. Nevertheless there are important differences between the assumed and the actual free stream wind profile. In particular the actual wind speed at h= 7 m is often much larger than the modelled wind speed.
  Some important differences between modelled and actual free stream turbulence profile can be found as well: In some cases the modelled free stream turbulence profile follows the actual free stream profile quite closely. This is in particular true for 1) neutral atmosphere and U = 5 m/s, 2) neutral atmosphere, U = 7.5 m/s and I = 8%, and 3) unstable atmosphere, U = 5 m/s and I = 8%. In most other cases the actual gradient dI/dz is much smaller than the modelled gradient, i.e. the actual gradient is much more negative than the modelled gradient, for unknown reasons.
- Wake profiles:
  - It can be observed that the modified initial near wake profile leads to a better agreement between calculations and measurements;
  - The calculations generally show an overprediction of the wake effects, in particular at the lower part of the rotor plane. This may partly be caused by the underpredicted free stream wind speed at h = 7 m, which is expected to yield a too low wake velocity as well. It may also be caused by the underprediction of free stream turbulence in the lower part of the rotor plane.
  - At some conditions, i.e. at neutral atmosphere, U = 10 m/s and I= 6% and at unstable atmosphere, U = 7.5 m/s and I = 8%, it can be seen that the wake velocity is larger than the free stream velocity, where obviously the opposite would be expected. These are conditions where the wake effects are expected to be rather limited, but this cannot explain the larger wake velocity. The discrepancy is most likely a result of the spatial difference between the wake and free stream measurement points.

  - The only result which is available for stable conditions, shows a very poor agreement between calculations and measurements, even though the results may be obscured by measurement errors.
- Turbulence profiles in the wake:
  - It can be observed that all versions of the model overestimate the turbulence intensities in the wake. However the modified models improve the agreement between calculations and measurement.
- Some further improvement can be expected when the variation in wind direction is taken into account. The calculations assume a constant wind direction, such that the meteorological mast is exposed to maximum wake effects. In reality the variation in wind direction and the wake meandering are expected to 'smoothen' the wake effects. This is explained in section 4.2.4.

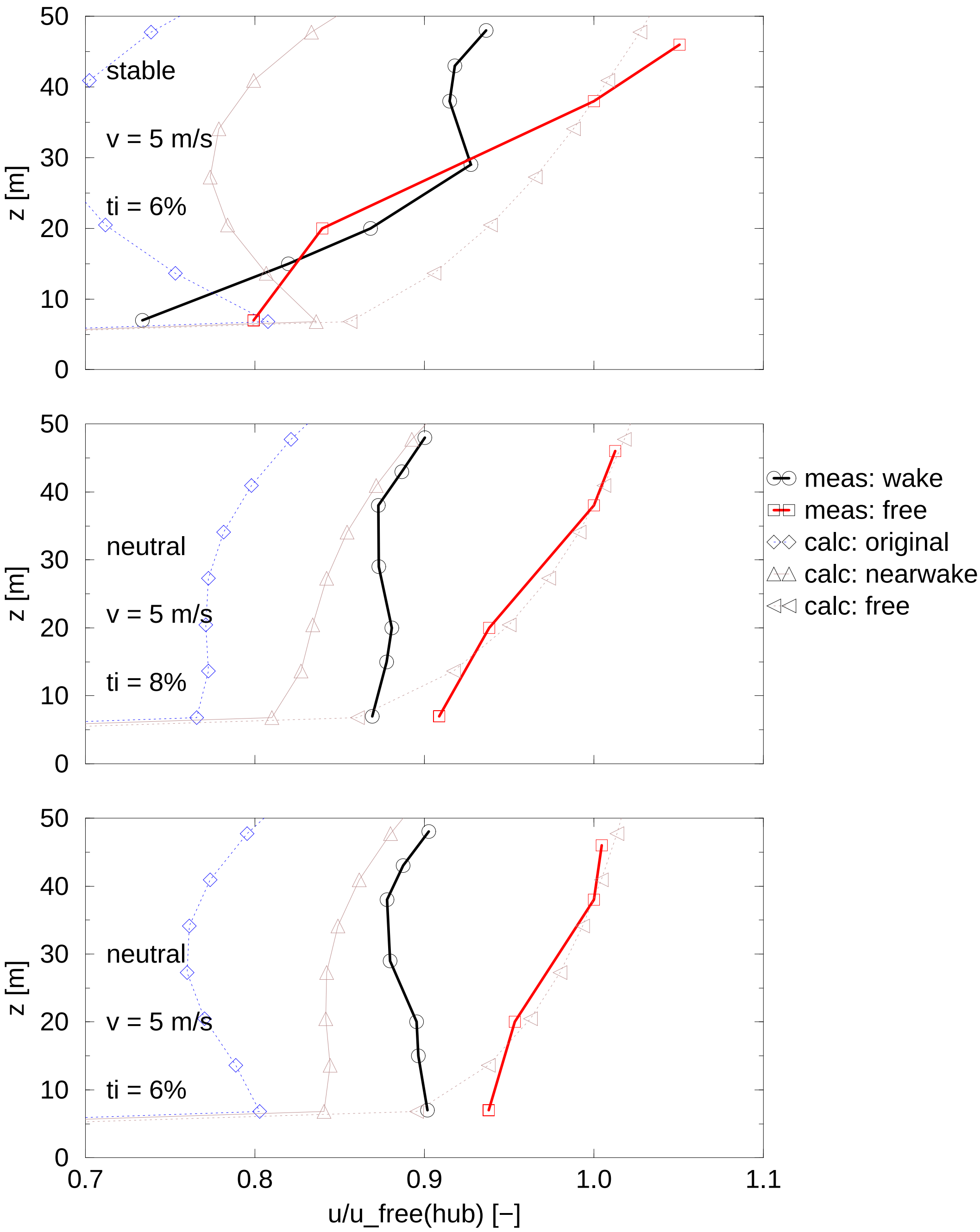

Figure 4.8 *Vindeby: Single wake: Measured and calculated non-dimensional velocity profiles at neutral and stable conditions*

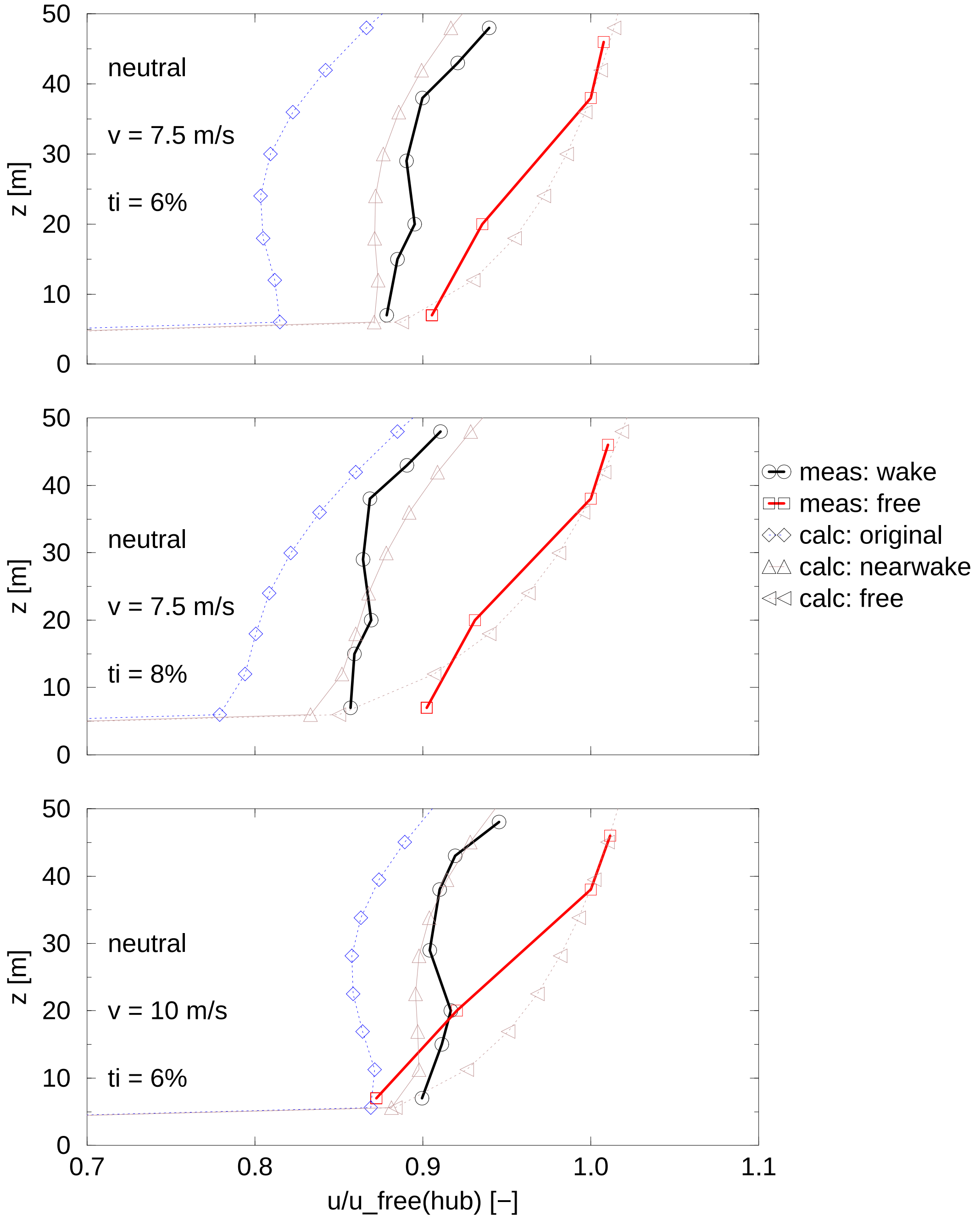

Figure 4.9 *Vindeby: Single wake: Measured and calculated non-dimensional velocity profiles at neutral conditions*

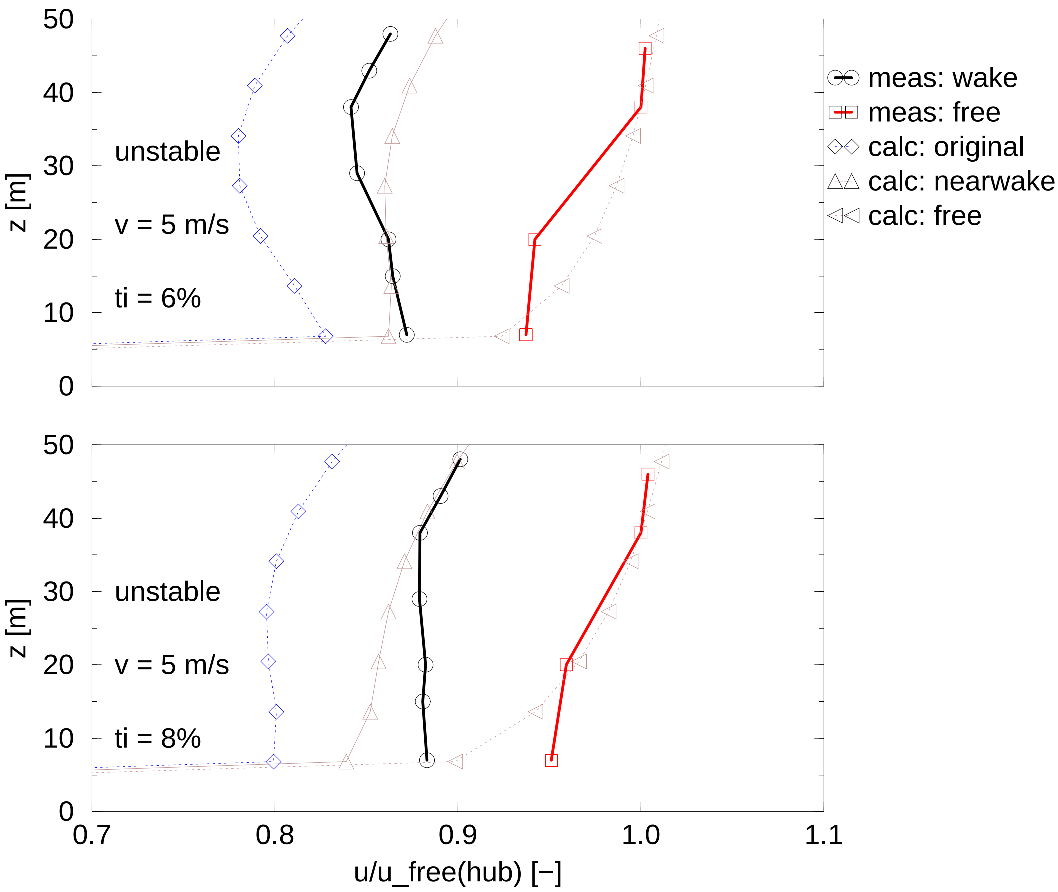

Figure 4.10 *Vindeby: Single wake: Measured and calculated non-dimensional velocity profiles at unstable conditions*

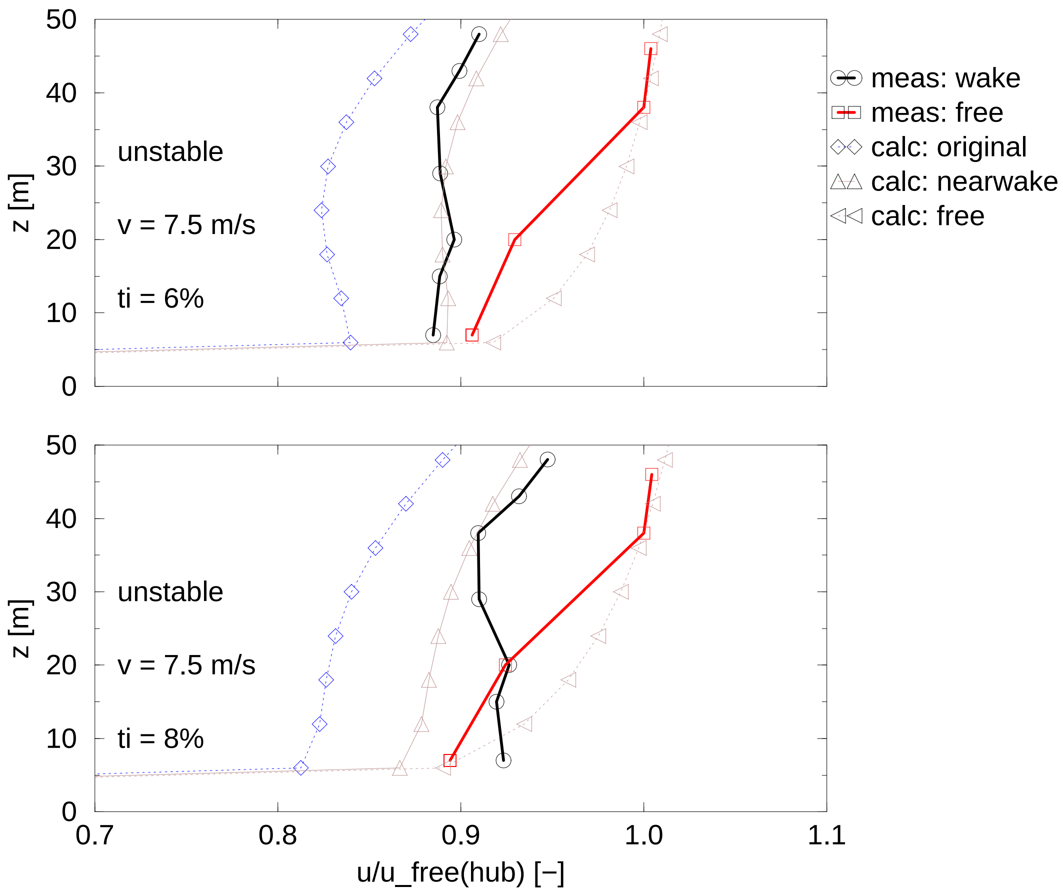

Figure 4.11 *Vindeby: Single wake: Measured and calculated non-dimensional velocity profiles at unstable conditions*

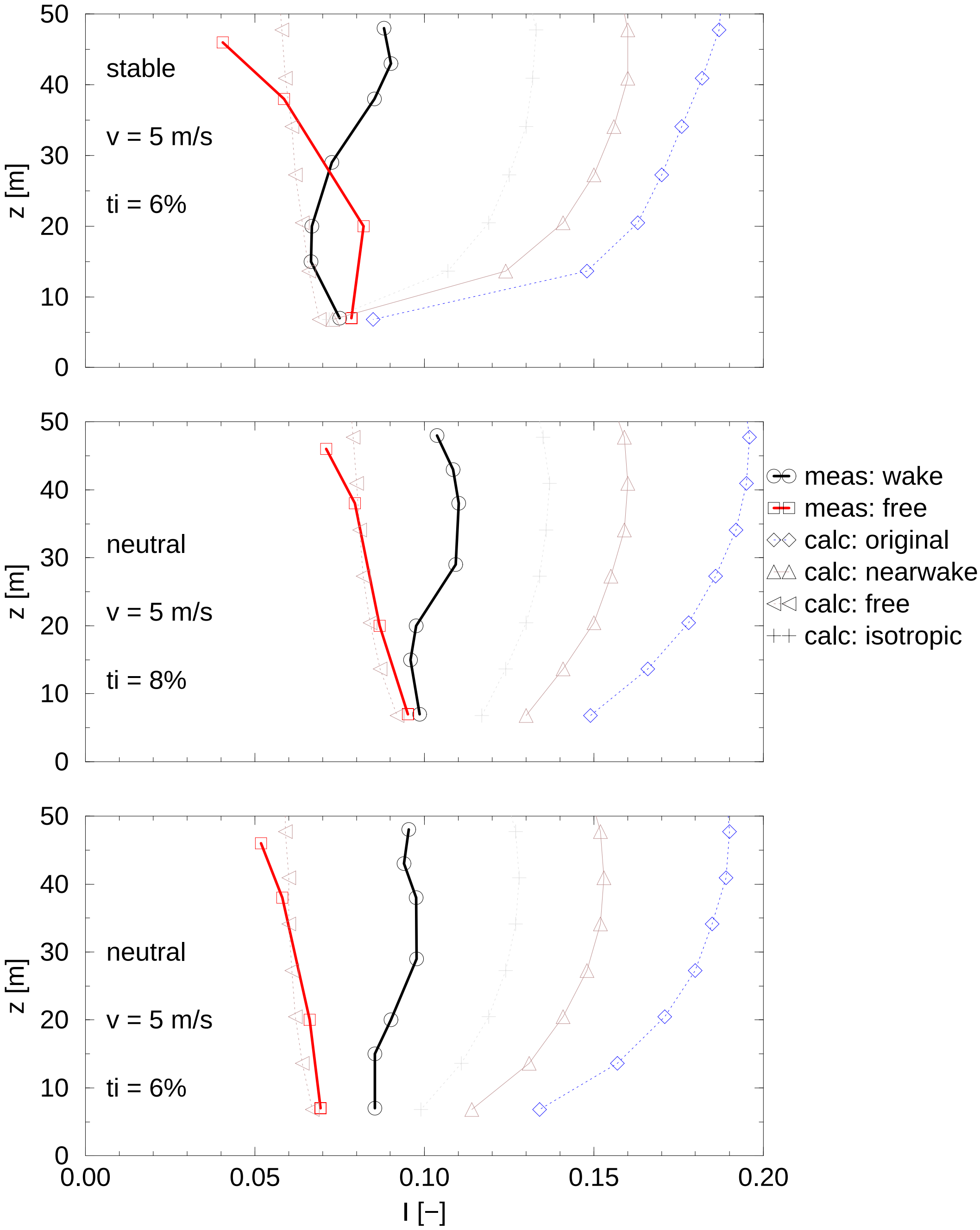

Figure 4.12 *Vindeby: Single wake: Measured and calculated turbulence intensities at neutral and stable conditions*

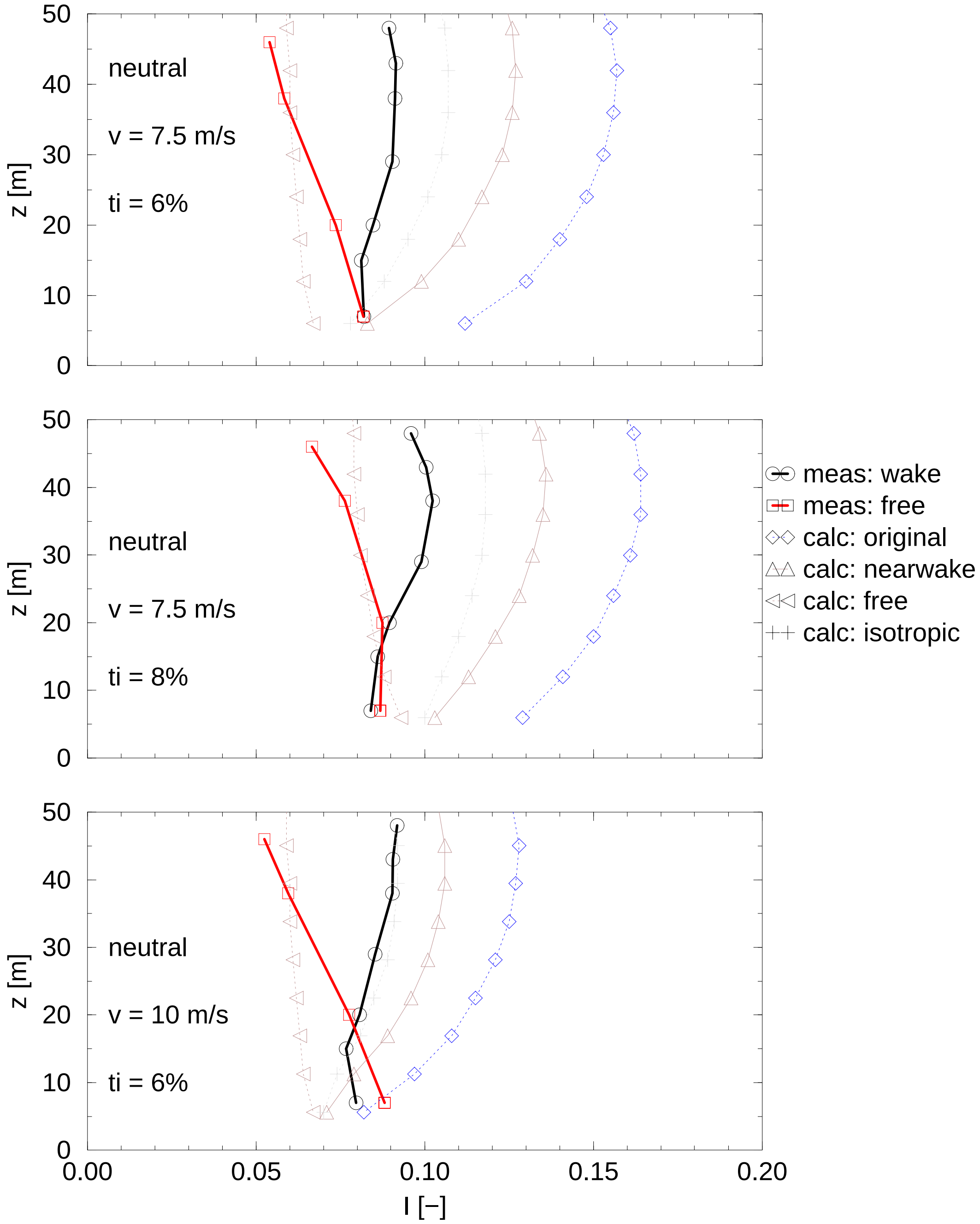

Figure 4.13 *Vindeby: Single wake: Measured and calculated turbulence intensities at neutral conditions*

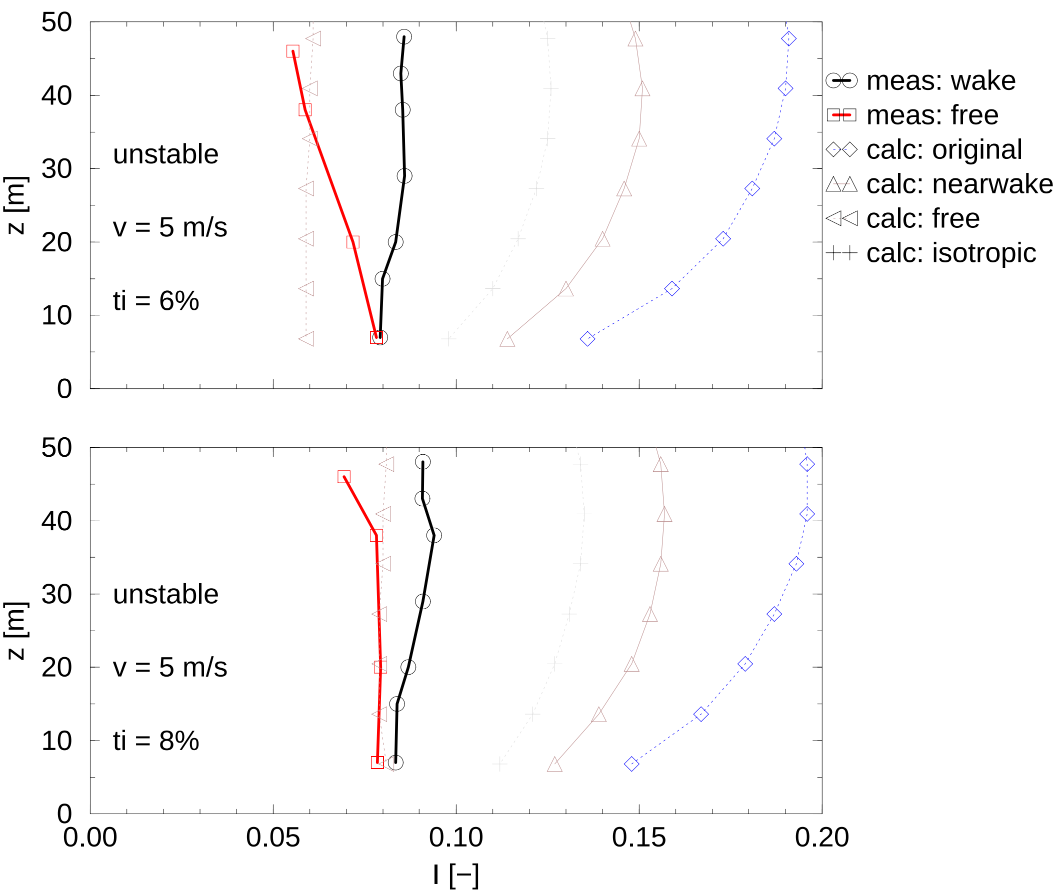

Figure 4.14 *Vindeby: Single wake: Measured and calculated turbulence intensities at unstable conditions*

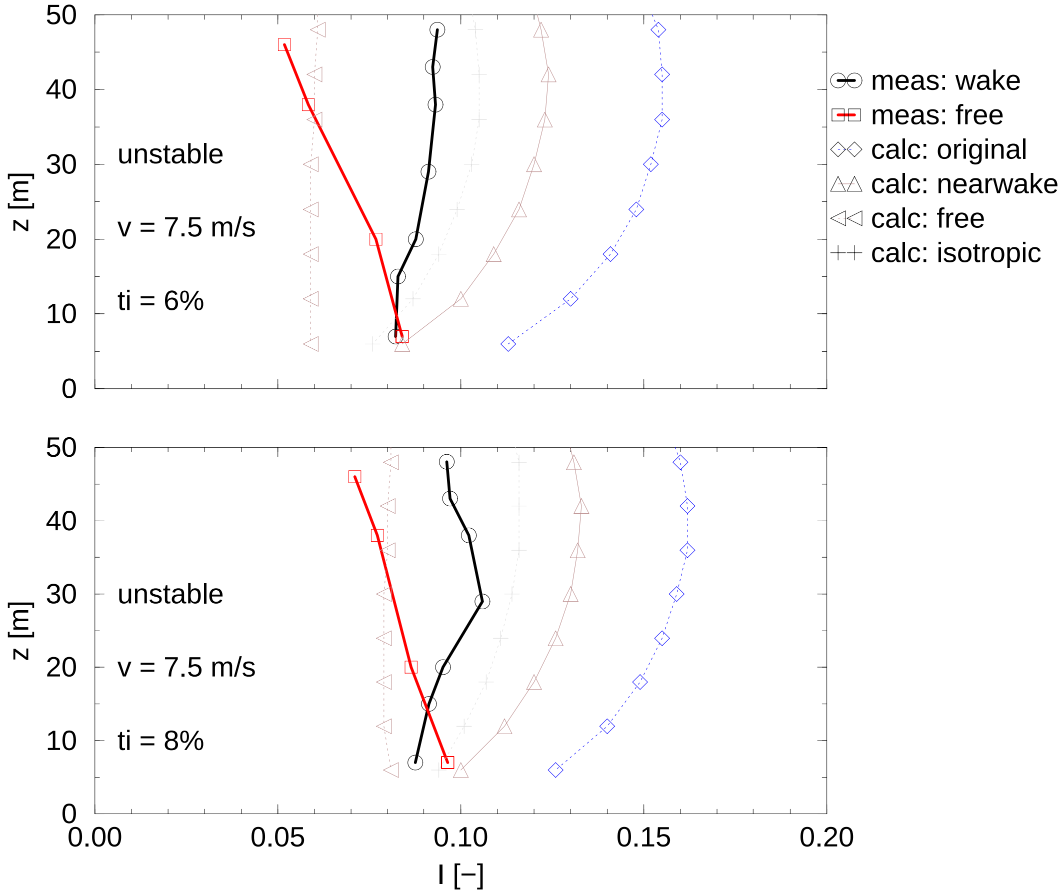

Figure 4.15 *Vindeby: Single wake: Measured and calculated turbulence intensities at unstable conditions*

4.2.4 Single wake: Validation with Vindeby measurements taking into account the wind direction fluctuations

As explained in section 4.2.1, the measurement results which are used in the present validation, are taken in a relatively wide wind direction interval from 18 to 28 degrees. However, all calculations which have been described in the previous sections, were produced under the assumption of a constant wind direction, along the wind turbine connection line. In order to get an indication for the effect of the wind direction fluctuations on the results, RISØ supplied a refined frequency distribution within the 18 to 28 degrees wind direction bin, based on a 1 degree resolution.

Then WAKEFARM results at different wind directions are 'weighted' using this measured frequency distribution.

The results are shown in the figures 4.16 to 4.23. Most of the comments which have been given in chapter 4.2.3 remain. The main conclusion comes from the comparison of the present results with the 'unweighted' results from the figures 4.8 to 4.15. The present results show, as expected, less pronounced wake effects. Due to the fact that the former results generally showed an overprediction of the wake effects, this implies that the agreement between calculations and measurements has improved.

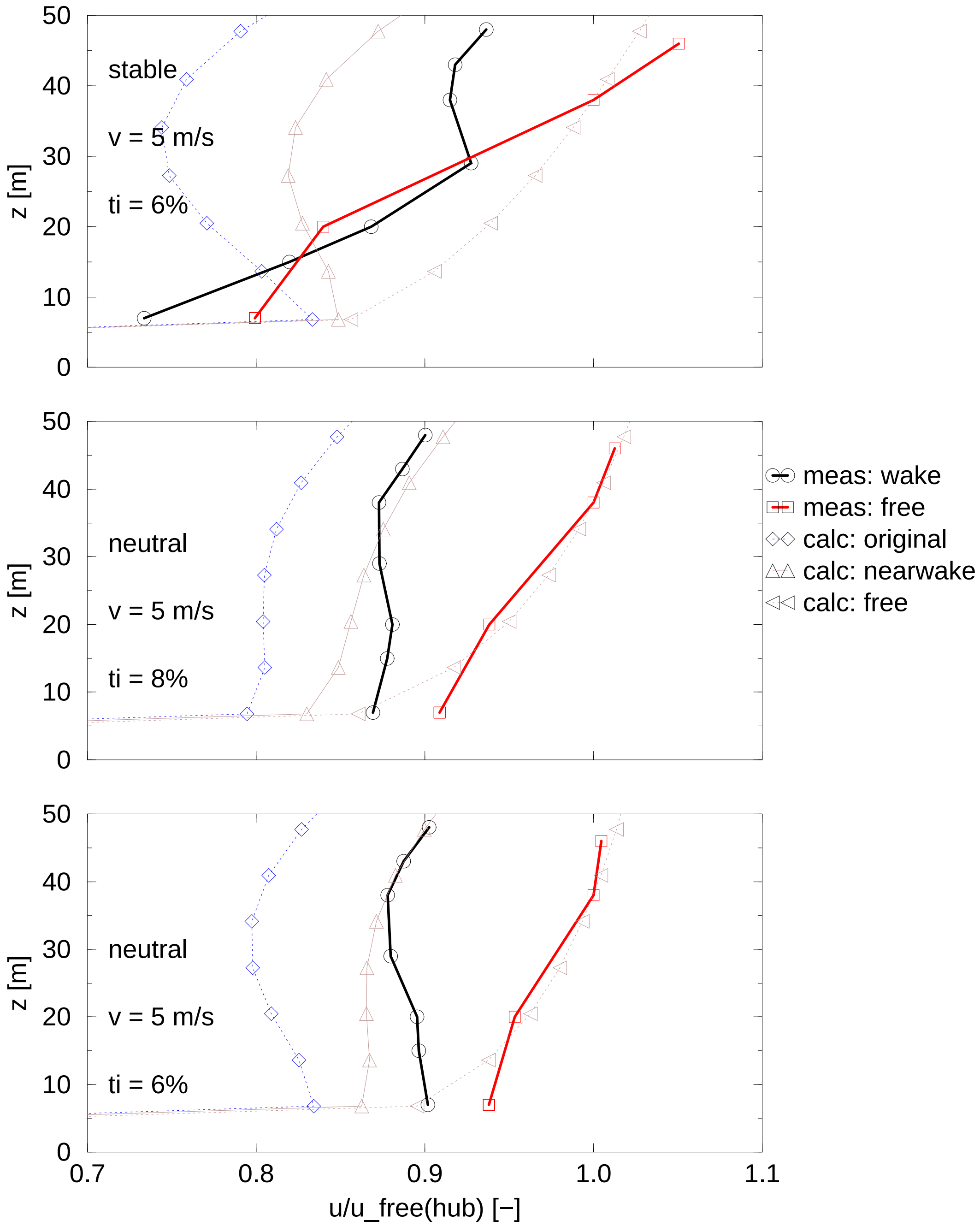

Figure 4.16 *Vindeby: Single wake: Measured and calculated non-dimensional velocity profiles at neutral and stable conditions; Wind direction fluctuations are taken into account*

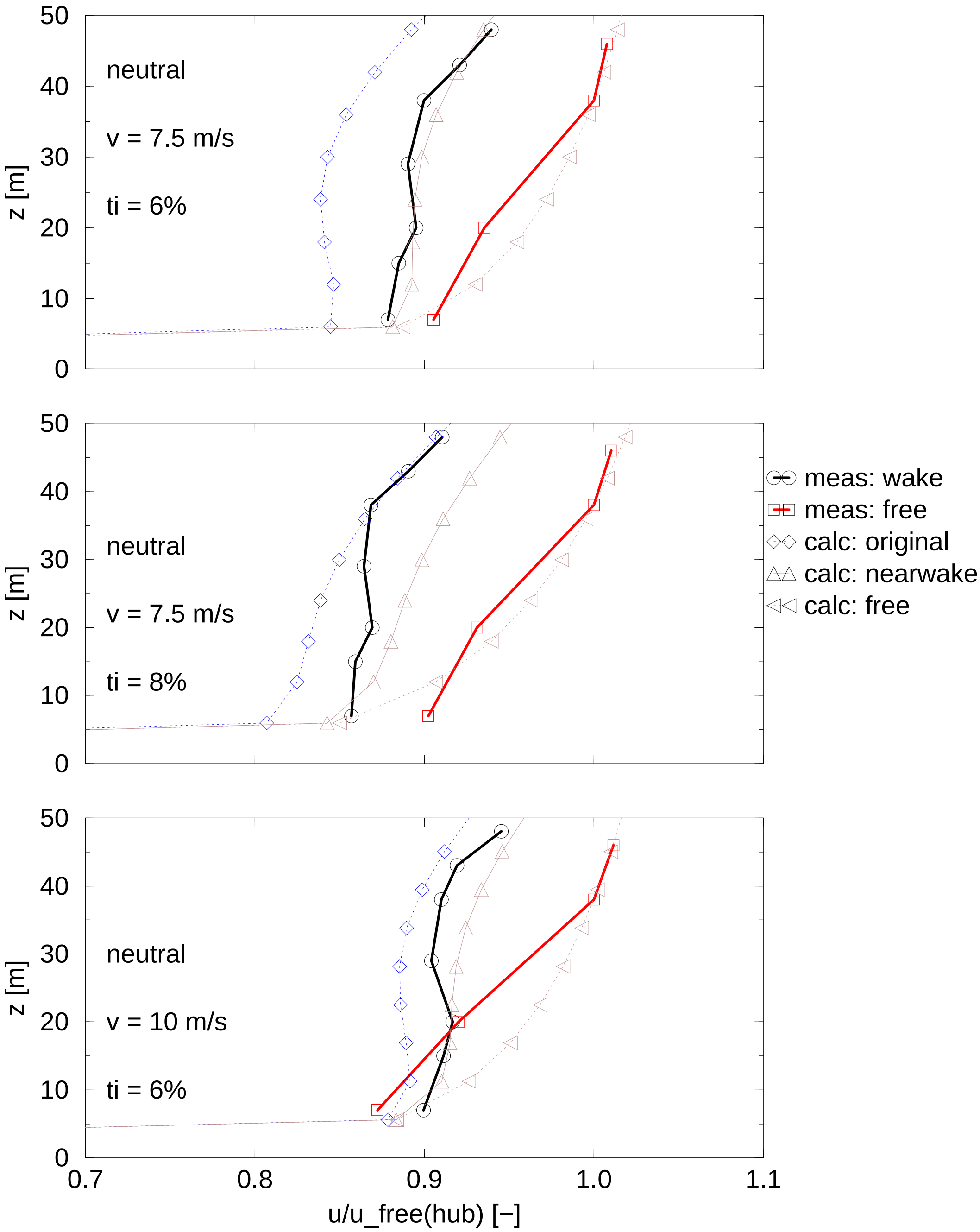

Figure 4.17 *Vindeby: Single wake: Measured and calculated non-dimensional velocity profiles at neutral conditions; Wind direction fluctuations are taken into account*

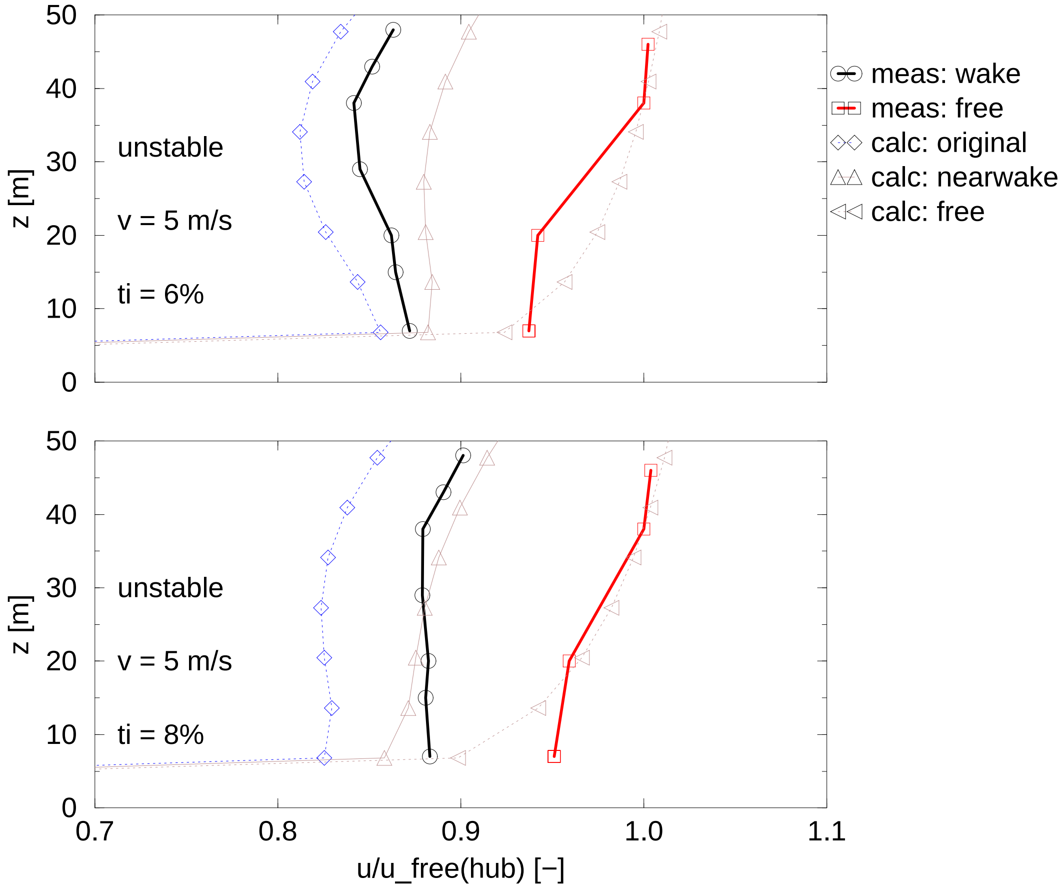

Figure 4.18 *Vindeby: Single wake: Measured and calculated non-dimensional velocity profiles at unstable conditions; Wind direction fluctuations are taken into account*

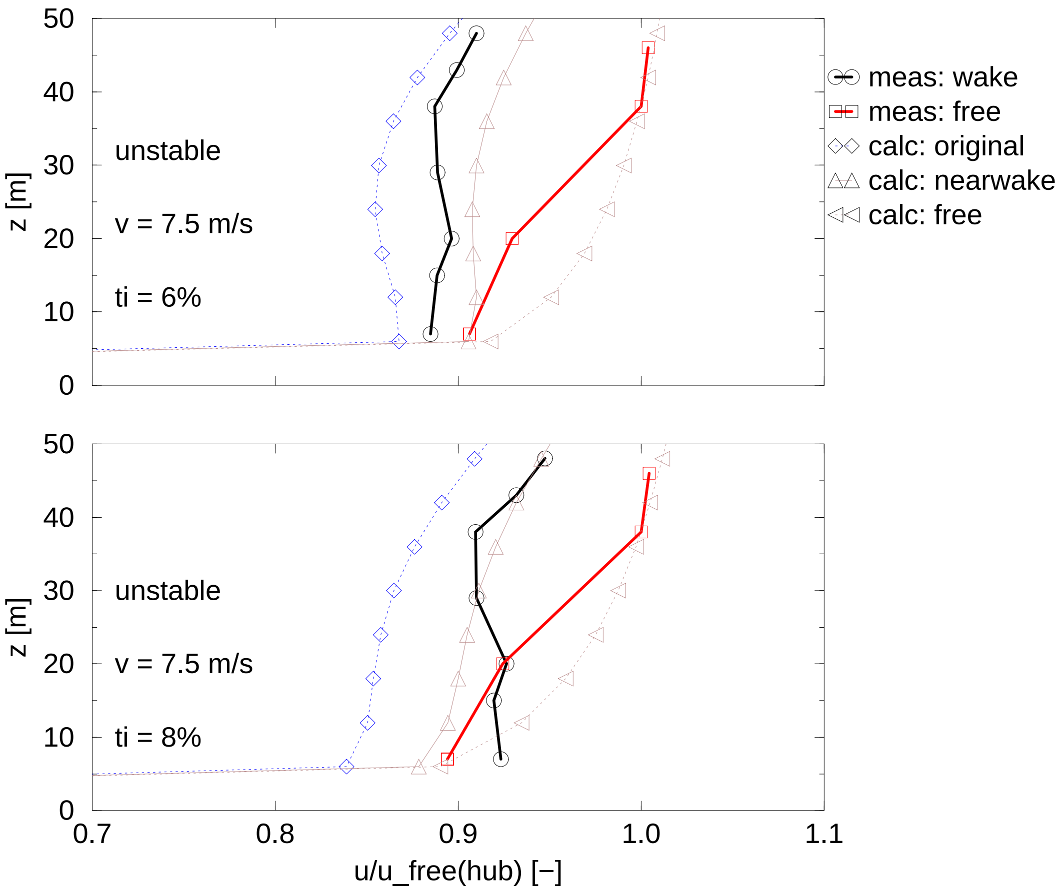

Figure 4.19 *Vindeby: Single wake: Measured and calculated non-dimensional velocity profiles at unstable conditions; Wind direction fluctuations are taken into account*

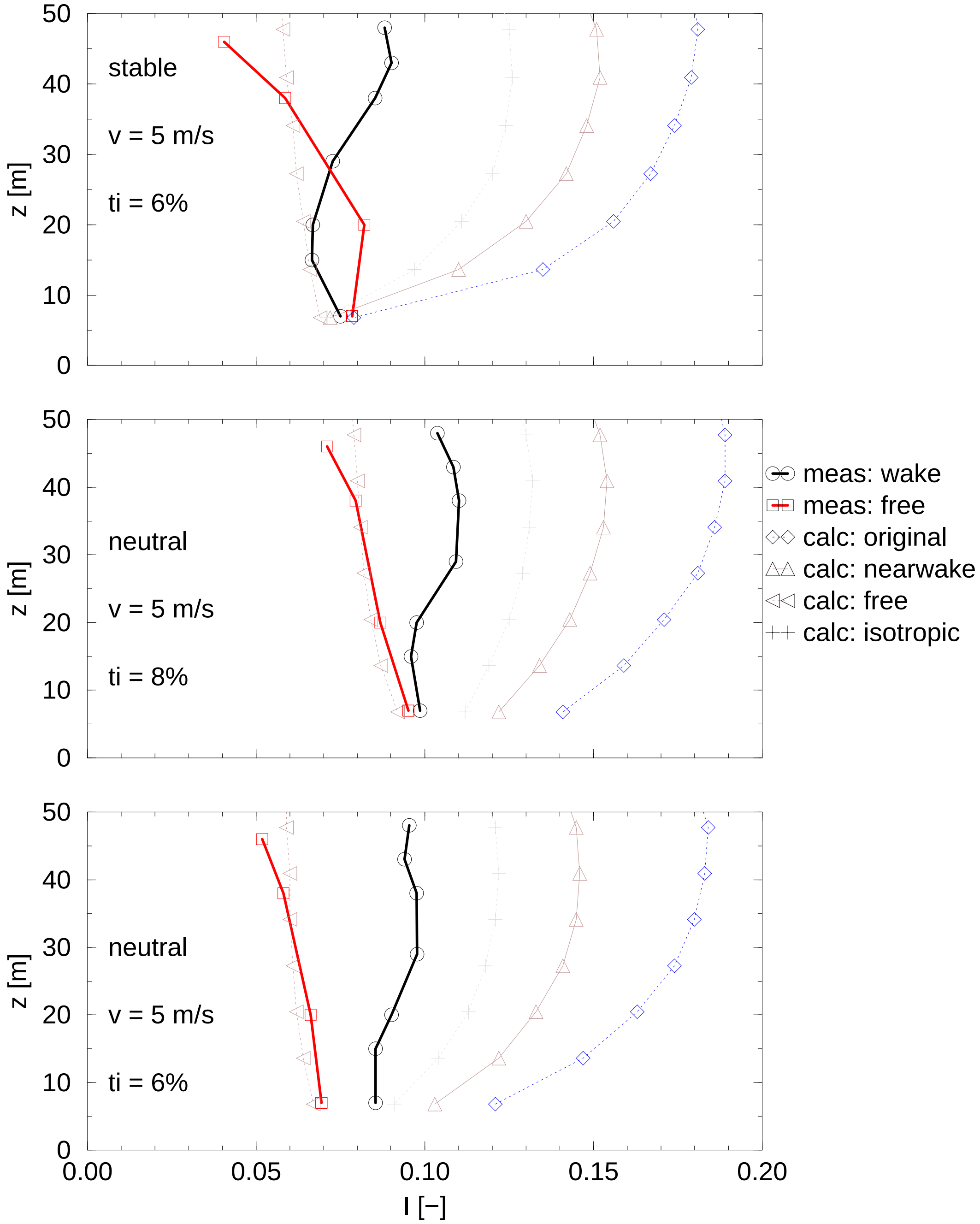

Figure 4.20 *Vindeby: Single wake: Measured and calculated turbulence intensities at neutral and stable conditions; Wind direction fluctuations are taken into account*

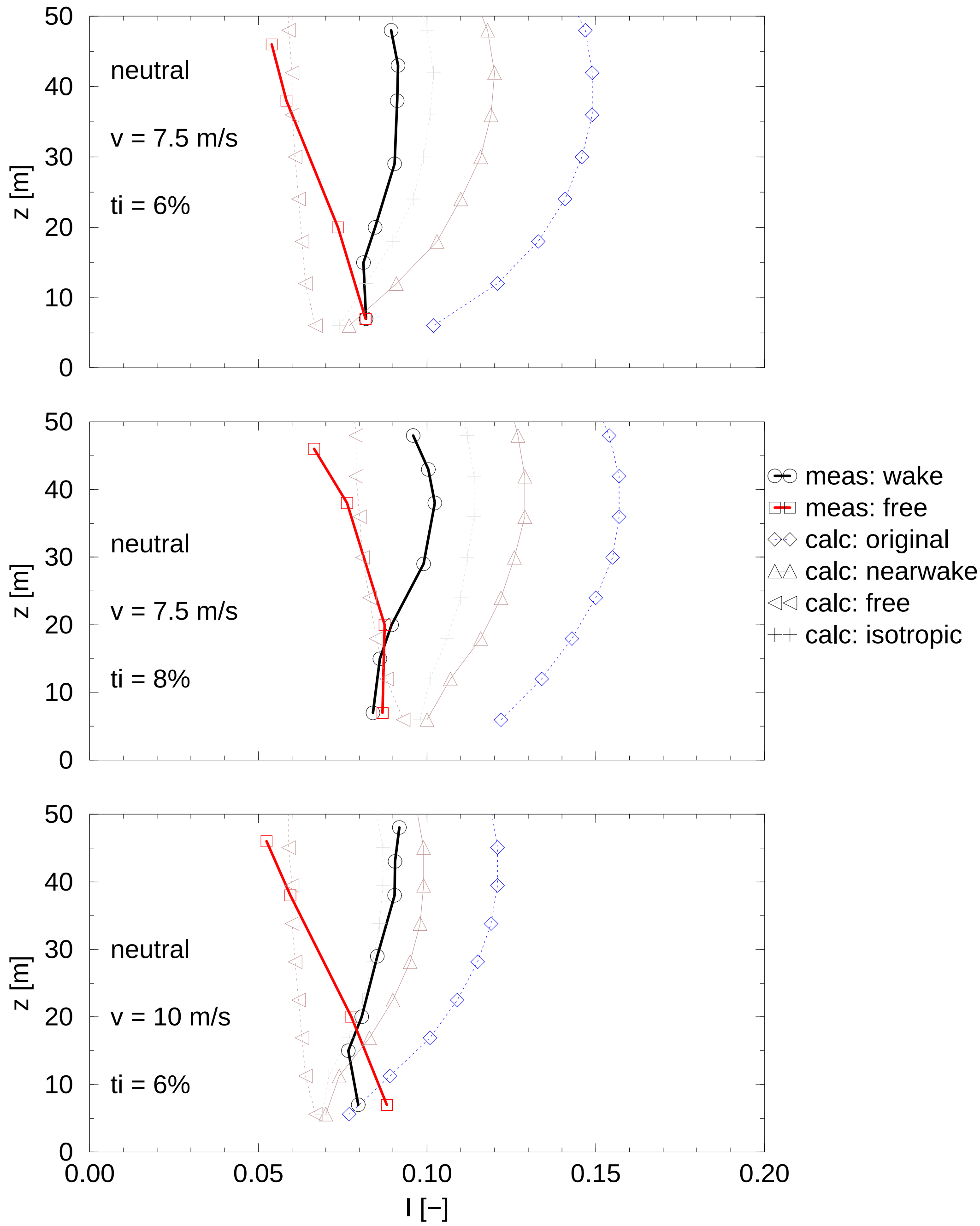

Figure 4.21 *Vindeby: Single wake: Measured and calculated turbulence intensities at neutral conditions; Wind direction fluctuations are taken into account*

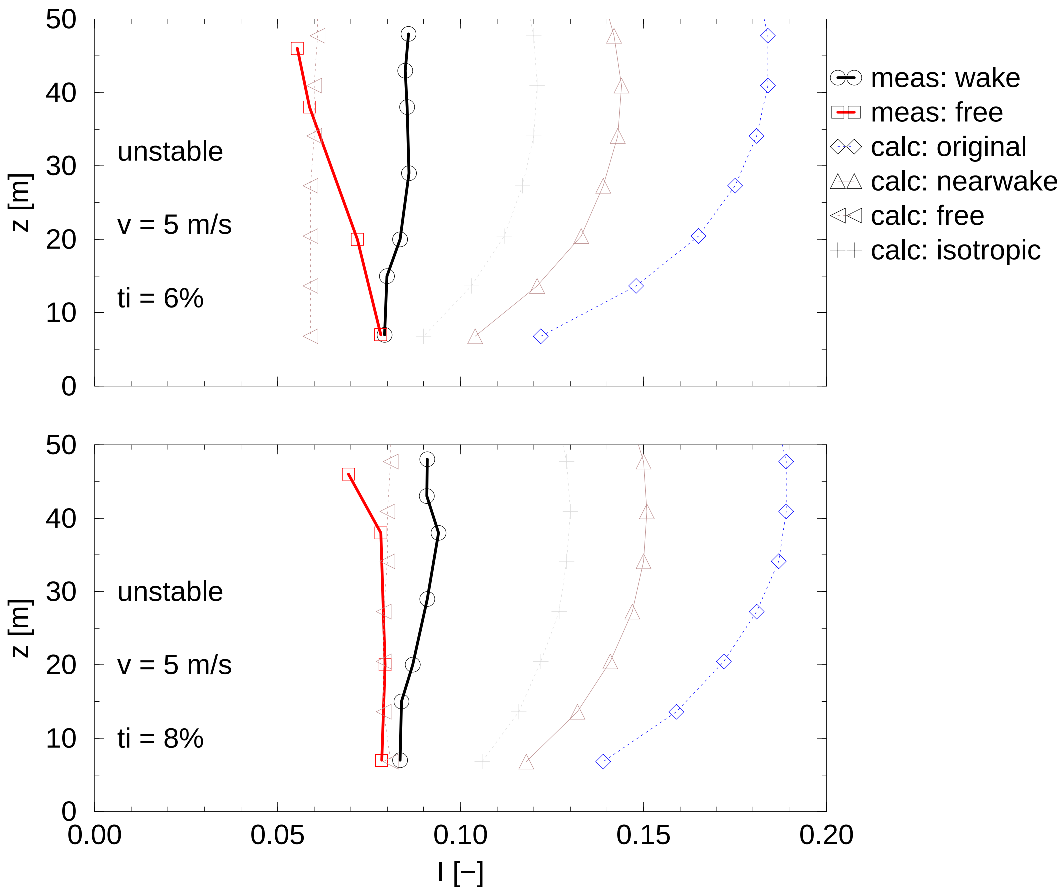

Figure 4.22 *Vindeby: Single wake: Measured and calculated turbulence intensities at unstable conditions; Wind direction fluctuations are taken into account*

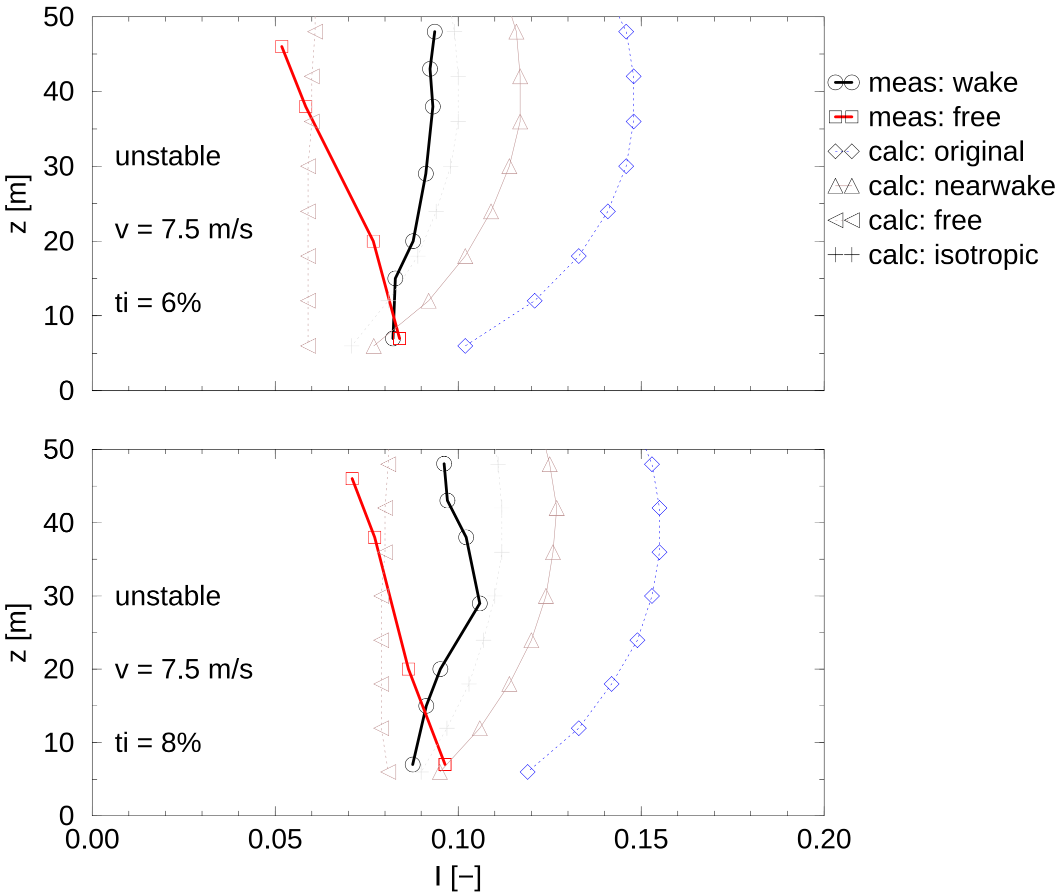

Figure 4.23 *Vindeby: Single wake: Measured and calculated turbulence intensities at unstable conditions; Wind direction fluctuations are taken into account*

4.3 Single wake: Validation with Alsvik measurements

4.3.1 Single wake: Description of Alsvik wind farm and measurements

The Alsvik wind farm is situated on the west coast of the Swedish island of Gotland in coastal flat terrain. The wind farm consists of 4 wind turbines. The turbines (180 kW Danwin) are stall controlled and fixed speed. The rotor diameter is 23.2 m and the hub height is 30 m. The thrust curve has been calculated from an aeroelastic model description, which was supplied by FFA within the EU-JOULE project Dynamic Loads in Wind Farms [12]. The 4 turbines are strategically placed, see figure 4.24.

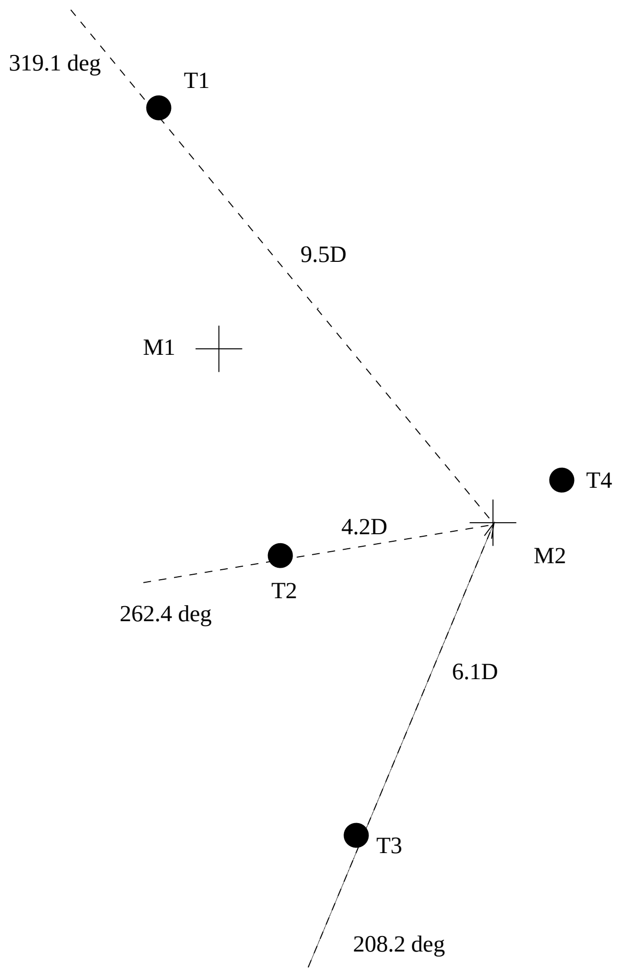

Figure 4.24 *Alsvik: Layout of wind farm, location of meteorological masts and wind directions with wake interferences*

Three of the turbines (turbines T1 to T3) stand on a line that runs along the shore at a direction of 345 degrees. The distance between turbine 1 and 2 is 8D, the distance between turbine 2 and 3 is 5D, the distance between turbine 1 and 4 is 9.4D and the distance between turbine 2 and 4 is 5D.

There are two meteorological masts, each with a height of 52 m, which measure wind speed and wind direction at 7 altitudes. The masts are indicated in figure 4.24. Mast 2 'feels', dependant on the wind direction, the wakes from turbine 1 (9.5D at a wind direction of 319.1 deg), turbine 2 (4.2D at a wind direction of

262.4 deg) or from turbine 3 (6.1D at a wind direction of 208.2 deg). At these wind directions, mast 1 is exposed to the free fream conditions.

A more detailed description of the site and the experimental layout is given in [13].

In the present study, a selection of 'summary' meteorological measurements is compared with calculations. The selected measurements are presented as function of wind direction, for a number of ambient wind speed bins. They show the velocities 'felt' by mast 2 at hub height, normalised with the corresponding velocities at mast 1, as well as the ratio of the standard deviation of the wind velocities 'felt' by mast 2 at hub height, normalised with the standard deviation of the velocities at mast 1. Note that the summary data are only binned on ambient wind speed. As such some differences may be apparent in a bin with respect to the remaining ambient conditions (turbulence intensity, wind shear etc).

The measurements have been supplied by FFA to ECN within the EU-JOULE project Dynamic Loads in WindFarms II, [12].

4.3.2 Single wake: Simulation of Alsvik measurements

The measurements as described in the previous section have been simulated with WAKEFARM. Again, calculational results from 3 WAKEFARM program versions have been supplied, see section 4.1.2

As explained in section 4.1.2. the input for the WAKEFARM program basically consists of the diameter, the hub height and the thrust coefficient. These quanties are straightforwardly known, see section 4.3.1.

Furthermore the incoming basic flow should be prescribed.

Alsvik wind farm: Basic flow
The basic flow has been modelled in the WAKEFARM program by tuning the values of friction velocity, roughness height, and Monin-Obukhov length scale to the appropriate centre bin values of hub height velocity. Furthermore FFA supplied a representative value of the turbulence intensity at hub height (5.5 %) and a wind shear to which the parameters were tuned.

4.3.3 Single wake: Validation with Alsvik measurements: Comparison between calculational and measured results

In the figures 4.25 and 4.28, the calculational results from the different model versions are compared with the measurement results. The velocity defects and the increase in turbulence (wind speed standard deviation) are given as function of wind direction for different wind speed bins. The calculational results which are indicated by 'original' denote the calculational results from the original WAKEFARM program. The results which are identified with 'nearwake' are the calculational results in which the Gaussian near wake profile is implementented, and the results which are identified with 'isotropic' are the results in which the added wake turbulence is assumed to be isotropic. The latter effect is obviously only relevant for the turbulence intensities (figures 4.27 and 4.6) and not for the wake profiles (figures 4.25 and 4.26).

The following observations can be made:

- Off-set in wind direction:
  As already observed in [13] some discrepancies are apparent between the wake directions given in figure 4.24 and the wind directions where the maximum wake effects are found in the figures 4.25 to 4.28. This is most likely caused by measurement uncertainties. Therefore the calculations have been adjusted, such that the prescribed wake lines coincide with the directions where the maximum wake effects occur. As a result, the 6.1D wake effects were supposed to occur at a wind direction of 214 degrees, the 4.2 D wake effects were supposed to occur at a wind direction of 273 degrees and the 9.5D wake effects were supposed to occur at a wind direction of 334 degrees.
- It was known from the Dynamic Loads in Wind Farms projects that the wake effects (both in terms of deficits as well as in terms of standard deviation ratios), were overpredicted by the original WAKEFARM program at the low wind speeds. At the higher wind speeds, the agreement was much better.
  A similar trend is still visible in the present results from the original WAKEFARM program. In the Dynamic Loads in Wind Farms projects, the large overprediction of wake effects at low wind speeds was partly attributed to the turbulent wake state and to the fact that the wind direction variations are more severe at the low wind speeds. Thereto it is reminded that the calculations assume a constant wind direction. In reality the wind direction varies and as such the wake effects will be 'smeared out'.
  The modified near wake profile generally improves the agreement with the measurements, although now the wake effects at high wind speeds are slightly underpredicted. A stronger underprediction may be expected if wind direction fluctuations are taken into account.

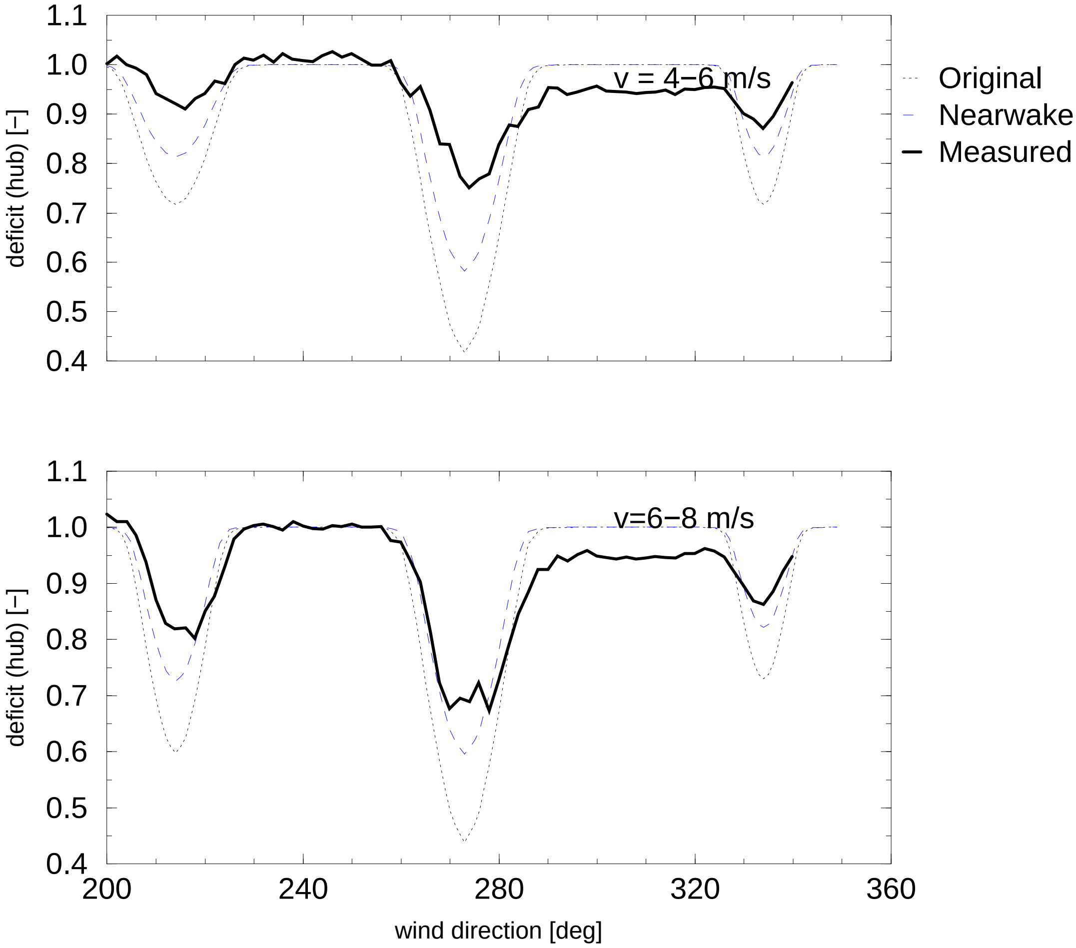

Figure 4.25 *Alsvik: Single wake: Measured and calculated wake velocity deficit at (free stream) wind speeds: 4-6 and 6-8 m/s*

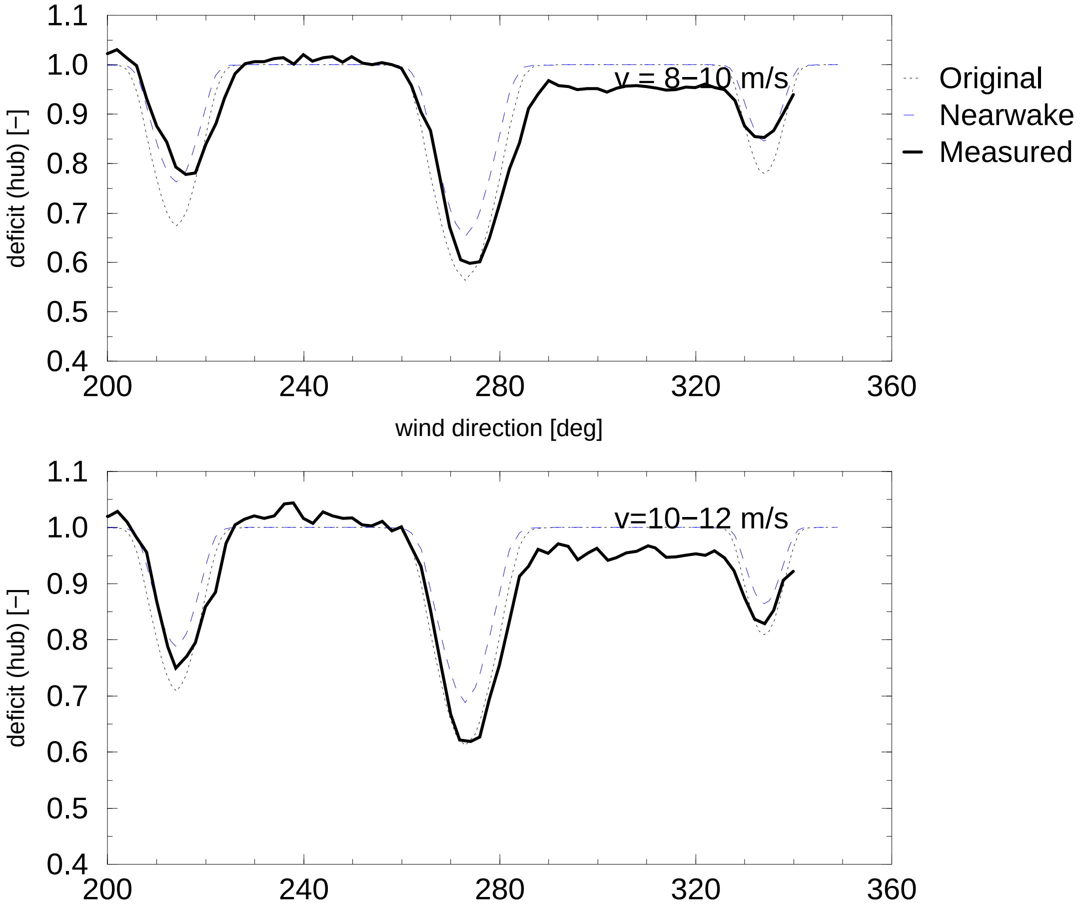

Figure 4.26 *Alsvik: Single wake: Measured and calculated wake velocity deficit at (free stream) wind speeds: 8-10 and 10-12 m/s*

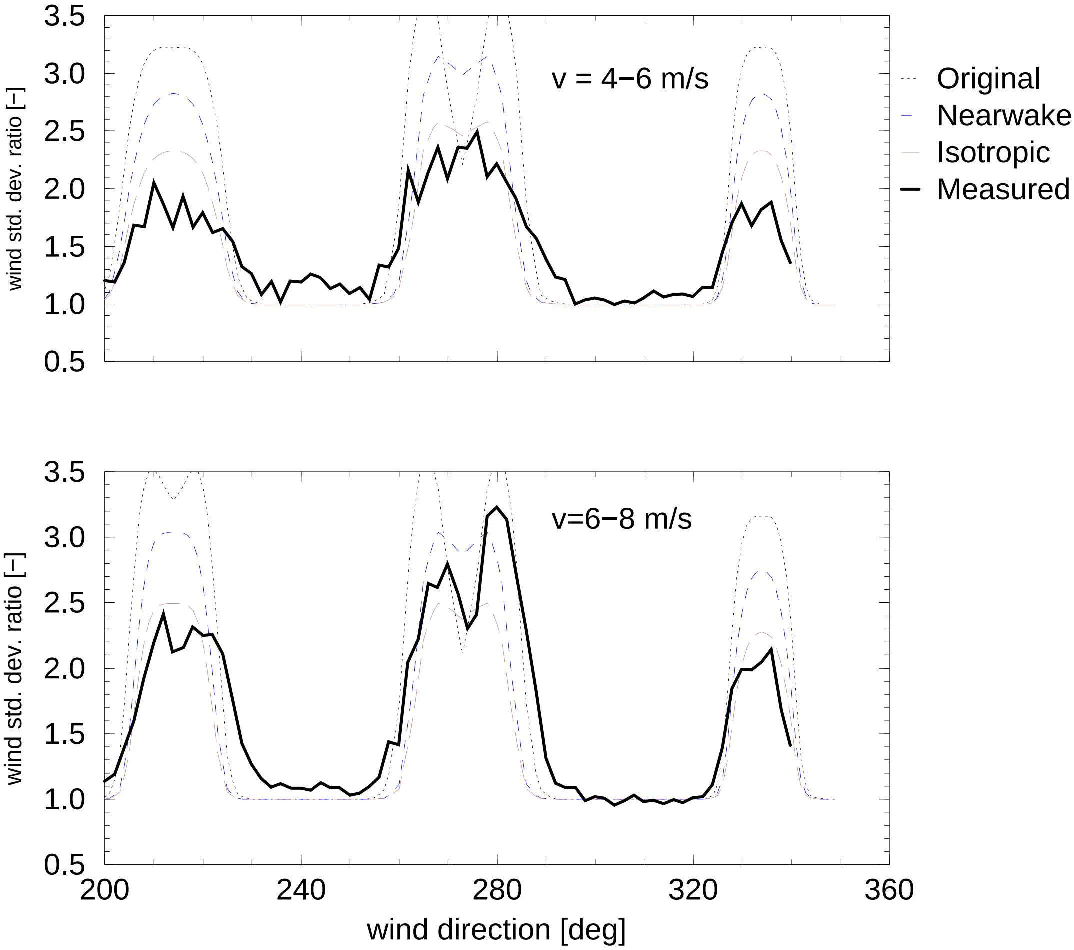

Figure 4.27 *Alsvik: Single wake: Measured and calculated wind speed standard deviation ratio at (free stream) wind speeds: 4-6 and 6-8 m/s*

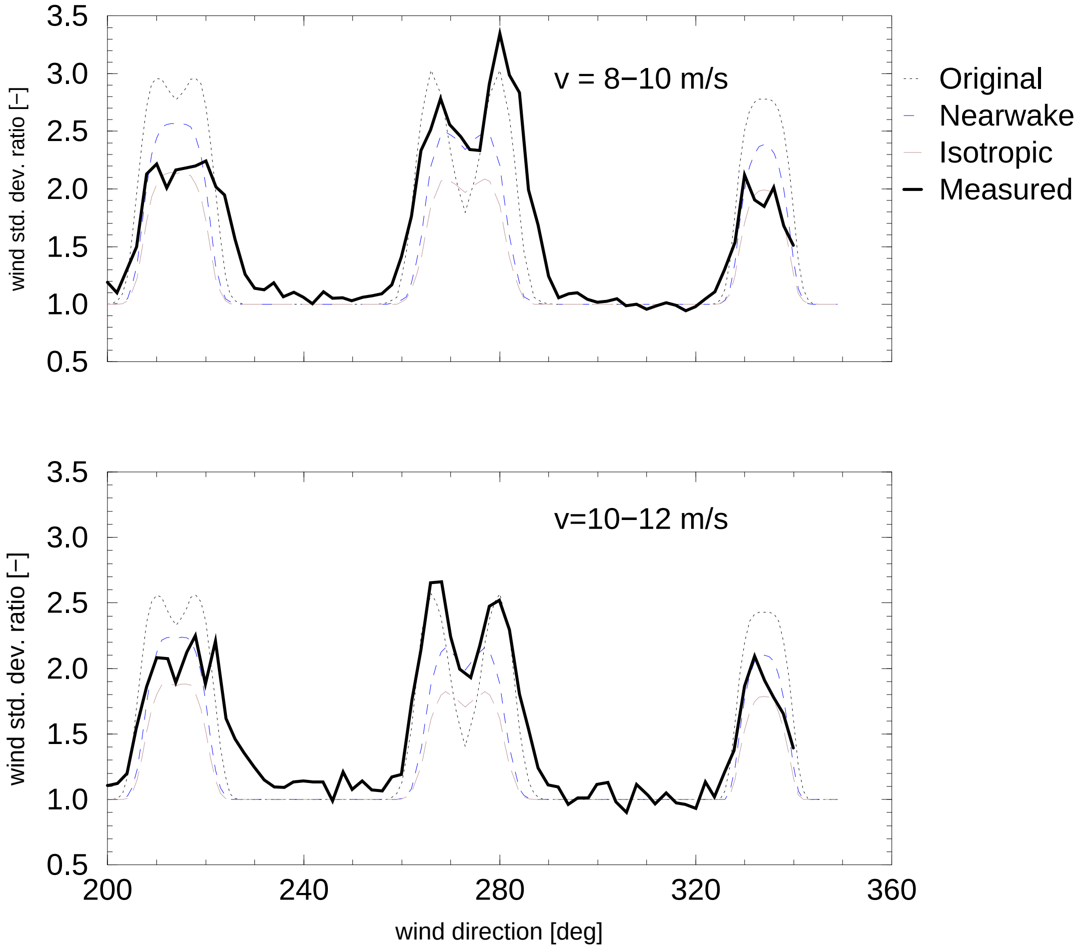

Figure 4.28 *Alsvik: Single wake: Measured and calculated wind speed standard deviation ratio at (free stream) wind speeds: 8-10 and 10-12 m/s*

5. MODELLING OF MULTIPLE WAKE SITUATIONS

In the previous chapters, single wake situations were considered only. In the ENDOW project, multiple wake conditions were also considered. Multiple wake conditions are modelled according to the following procedure (for the sake of simplicity only double wake is considered):

1. As a first step, a single wake calculation is performed. Hence the flow downstream of the first turbine is modelled. This yields the WAKEFARM output properties, see section 2.3, i.e. added velocity components ($u_{def,w,1}$, v_1 and w_1), the turbulent kinentic energy ($k_{add,1}$), the dissipation rate ($\epsilon_{add,1}$) and the temperature ($t_{add,1}$);
2. The single wake calculation behind the first turbine yields a rotor averaged wind speed at the location of the second turbine. This value is used to determine the axial force coefficient on the second turbine ($C_{D.ax,2}$);
3. The initial velocity deficit behind the second turbine ($u_{def,w,2}$) is calculated from the axial force on this turbine. The shape of the upstream wake flow is retained, see also figure 5.1:

$$u_{def,w,2}(y,z) = u_{def,w,1}(y,z) - (1.0 - 1.0\sqrt{1 - C_{D.ax,2}}) \cdot U_{w,1,ave} \quad (5.1)$$

   ($u_{def,w,1}$ is the velocity deficit just upstream of the second rotor and $U_{w,1,ave}$ is the rotor averaged velocity just upstream of the second rotor).

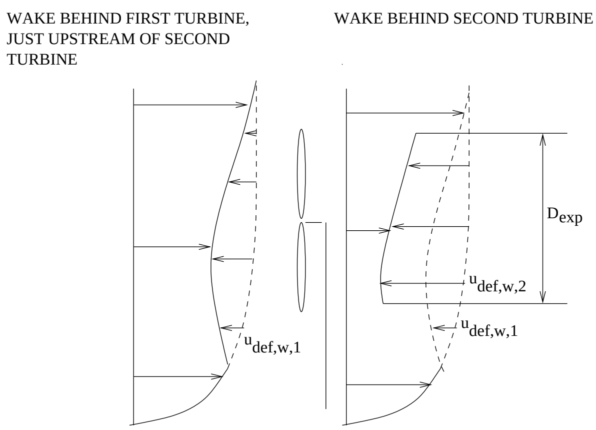

Figure 5.1 *Double wake deficit*

It should be noted that an initial velocity profile according to equation 5.1 is comparable to the original near wake initialisation, which was described for single wake situations in section 2.2 (compare equation 5.1 with equation 2.13).

Similar to the single wake situation, it has been made possible to model a Gaussian near wake profile for double wake situations as well. The resulting equation then becomes:

$$u_{def,2}(y,z) = u_{def,1}(y,z) - 1.3 \cdot (1.0 - 1.0\sqrt{1 - C_{D.ax}}) \cdot U_\infty \cdot (e^{-0.5(y/r\sigma_y)^2} e^{-0.5((z-h_t)/r\sigma_z)^2}) \quad (5.2)$$

The initialisation of the near wake profile at double wake condition was some point of concern. As stated before, the best agreement for single wake conditions was found when the initial wake profile is applied at a downstream distance from the rotor:
For double wake situations, experience learned that the application of an initial profile at x=2.25D did not yield a very good agreement between calculated and measured wake results. This may be due to the fact that the rotor operates in a shear stress profile which has already developed in the wake of the first turbine. This developed shear stress profile will lead to mixing of momentum from the ambient flow to the wake. Therefore, in the present calculations the double wake is initialized at the rotor plane opposite to the single wake situation. Hence the inviscid wake expansion is assumed to take place in the rotor plane. Although this is an unrealistic assumption from a physical point of view, it turned out to give the best results.

4. The v and w components of the wake velocities, the added turbulent kinetic energy, the dissipation and the temperature just upstream of the second turbine is transfered to the position just downstream of the second turbine:

$$
\begin{aligned}
v_2(h) &= v_1(h) \\
w_2(h) &= w_1(h) \\
k_{add,2}(h) &= k_{add,1}(h) \\
\epsilon_{add,2}(h) &= \epsilon_{add,1}(h) \\
t_{add,2}(h) &= t_{add,1}(h)
\end{aligned}
$$

It is obvious that the present procedure retains the free stream values for the velocity and turbulence profile at infinite, since all perturbation values will approach zero. This should be considered as an important requirement in modelling (multiple) wake situation.

6. MULTIPLE WAKE: VALIDATION

In this section, the validation for multiple wake situations is described. Thereto calculational results from different model versions of the WAKEFARM program are compared with double wake measurements from the Vindeby wind farm and from GH's wind tunnel experiments. Furthermore quintuple wake situations from the Vindeby wind farm are considered.

6.1 Double wake: Validation with wind tunnel measurements

6.1.1 Double wake: Description of wind tunnel measurements

In this section, the validation with double wake wind tunnel measurements is described. These wind tunnel measurements have been performed by GH.

The wind tunnel and the model turbines were the same as for the single wake sitation, see also [1] and section 4.1.1. The double wake measurements are made at several positions (i.e. 2.5D, 5D. 7.5D and 10D) downstream of a turbine, which is placed at 5D or 7.5D behind a first turbine. Note that a double wake measurement, which is made at a position 2.5D downstream of a turbine which is placed 5D behind the first turbine, is denoted through x = 5-2.5D. Mean velocity profiles in the wake are given for λ_1 = 4.0 and λ_1 = 2.9, the latter only for a 7.5D spacing between the turbines. Note that the values of λ_1 refer to the upstream turbine.

Added turbulence measurements are only given for λ_1 = 4.0 and a 7.5D spacing between the turbines.

6.1.2 Double wake: Simulation of wind tunnel measurements

The measurements of wake profiles and turbulence intensities at 2.5D, 5D and 7.5D downstream of the second turbine have been compared with different WAKEFARM model versions. Similar to the single wake situation, there are 3 WAKEFARM program versions:

- The original version with a 'hat shaped' near wake profile and wake anisotropy which is assumed to be similar to the ambient anisotropy (see section 2);
- A modified version with the 'Gaussian shaped' near wake profile (section 3.1 and equation 5.2), but the wake anisotropy is still assumed to the ambient anisotropy;
- A modified version with the 'Gaussian shaped' near wake profile, and assuming isotropic wake turbulence, see equation 3.18.

The input for the WAKEFARM program (diameter, hub height, thrust coefficient, the distances between the turbines, the location behind the turbine, and the basic conditions) are straightforwardly known from the tunnel conditions and the measurement set-up, see section 4.1.1 and section 4.1.2.

6.1.3 Double wake: Validation with wind tunnel measurements: Comparison between calculational and measured results

In the figures 6.1 to 6.4, the calculational results from the different model versions are compared with the measurements. The results which are identified with 'original' denote the calculational results from the original WAKEFARM program. The results which are identified with 'nearwake' are the calculational results in which the Gaussian near wake profile is implemented, and the results which are identified with 'isotropic' are the results in which the added wake turbulence is assumed to be isotropic. The latter results are obviously only relevant for the turbulence intensities (figure 6.4) and not for the wake profiles.

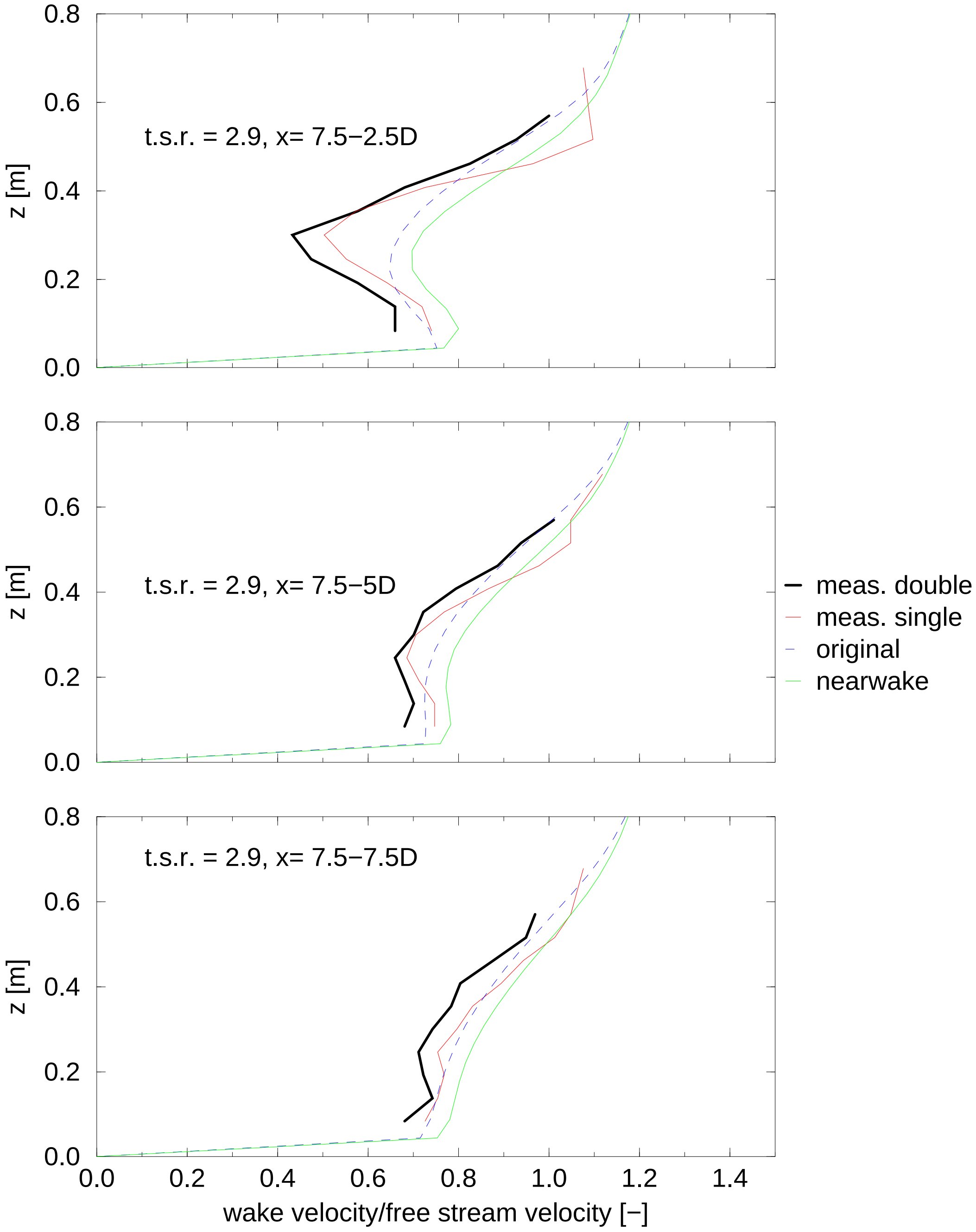

Figure 6.1 *Wind tunnel: Double wake, distance between the two turbines is 7.5D: Measured and calculated wake profiles at λ = 2.9 (y=0)*

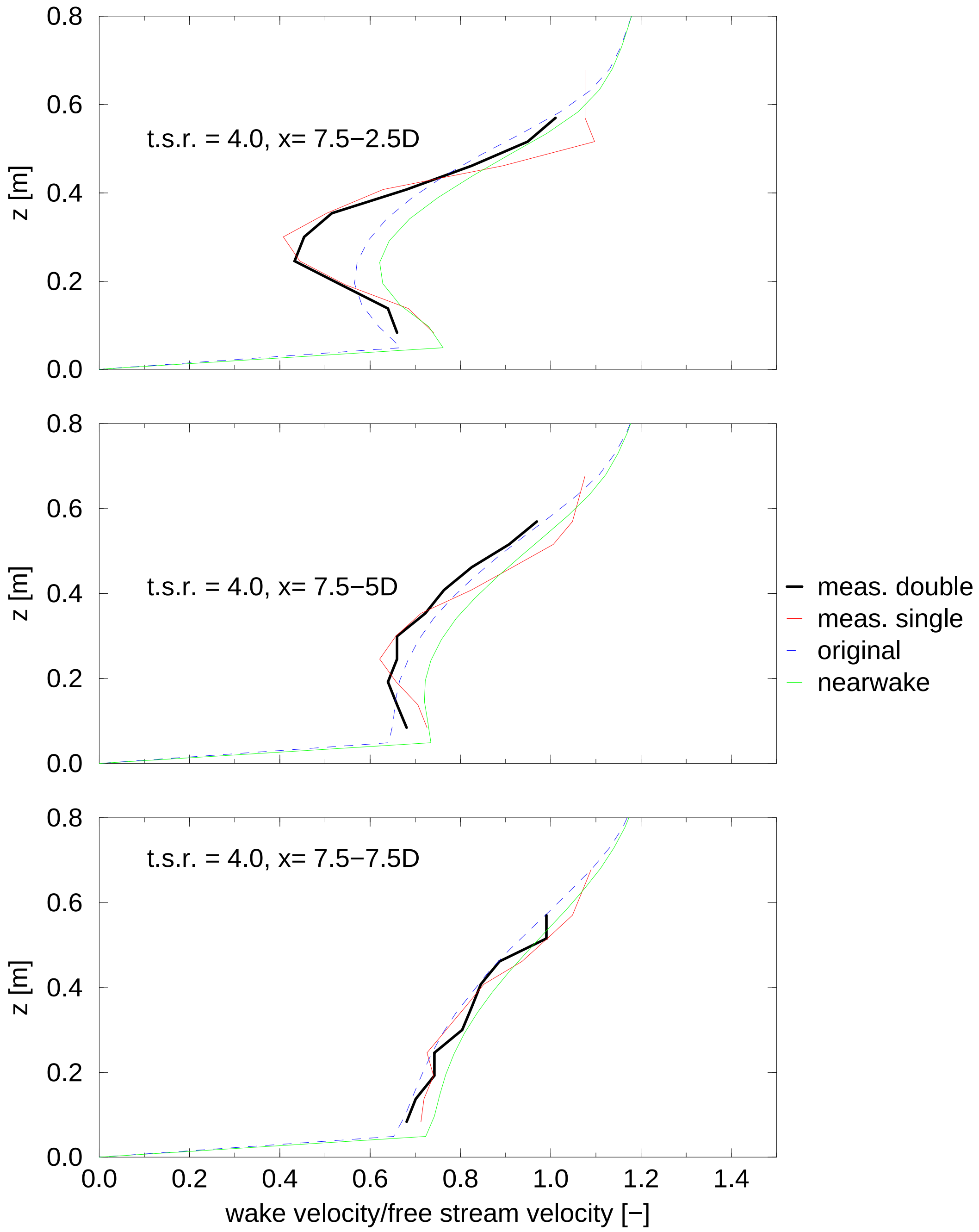

Figure 6.2 *Wind tunnel: Double wake, distance between the two turbines is 7.5D: Measured and calculated wake profiles at λ = 4.0 (y=0)*

Velocity profiles

It is interesting to note that the measurements indicate that the double wake velocity defect at $\lambda_1 = 4.0$ is often smaller than the single wake velocity defect. At first sight this is a surprising result: It would be expected that the reduction in wind speed in the first wake should, at least partly, be enhanced in the second wake. Furthermore the reduced wind speed at the second (constant speed) turbine increases the $C_{D.ax}$ and as such the induction factor is increased. However in [14] it was shown, that an increased loading due to a higher wind speed hardly effects

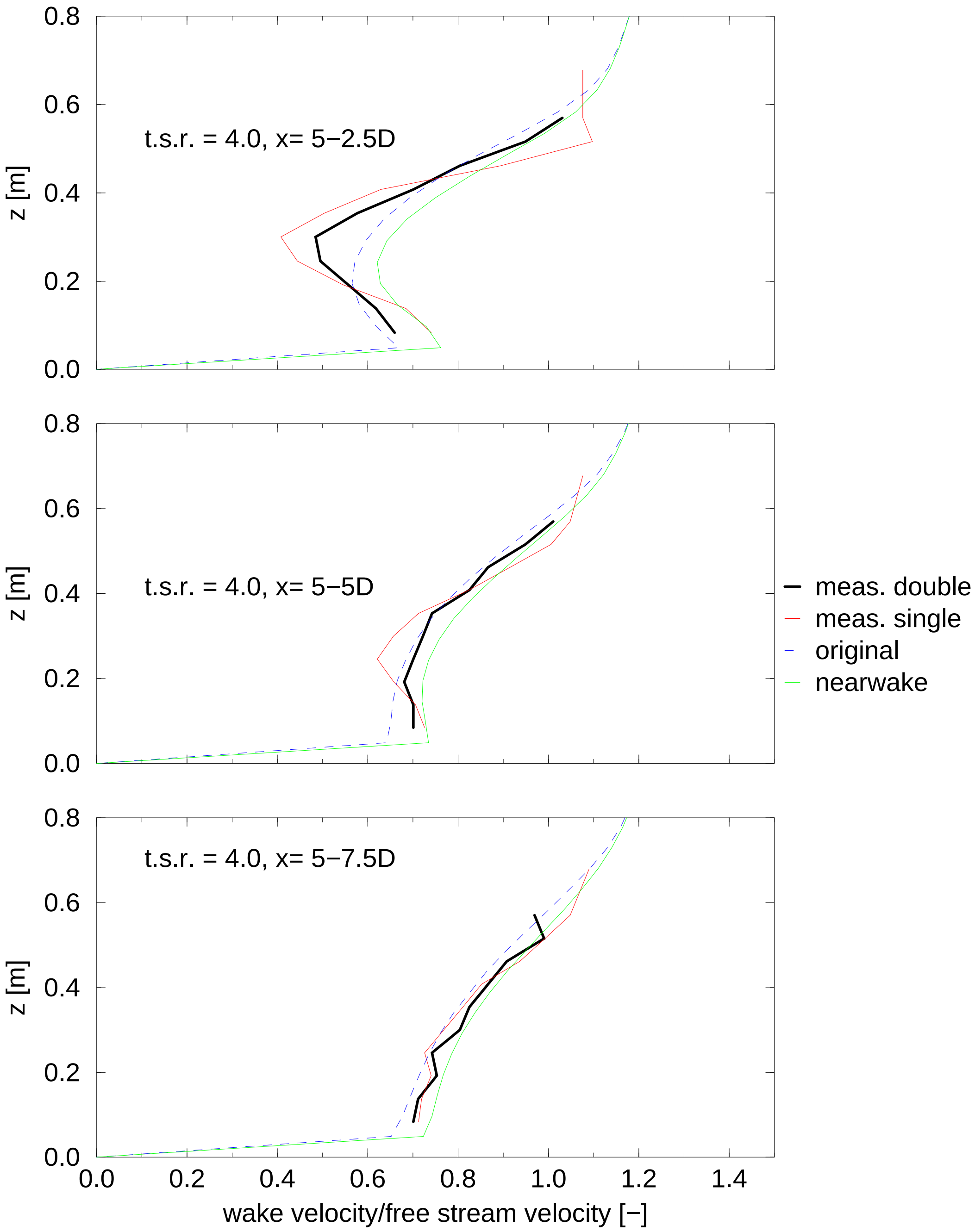

Figure 6.3 *Wind tunnel: Double wake, distance between the two turbines is 5.0D: Measured and calculated wake profiles at* λ *= 4.0 (y=0)*

the velocity deficit in the rotorplane: The axial induction factor is increased but this is compensated by the fact that this axial induction factor should be related to a lower incoming wind speed, $U_{w,1,ave}$, see equation 5.1.

A possible explanation for the reduced double wake deficit is given in [1]: The near wake length is reduced and as such the shear stress profile has developed much more rapidly, see also section 5.

When comparing calculational and measured results, it can be observed that, in

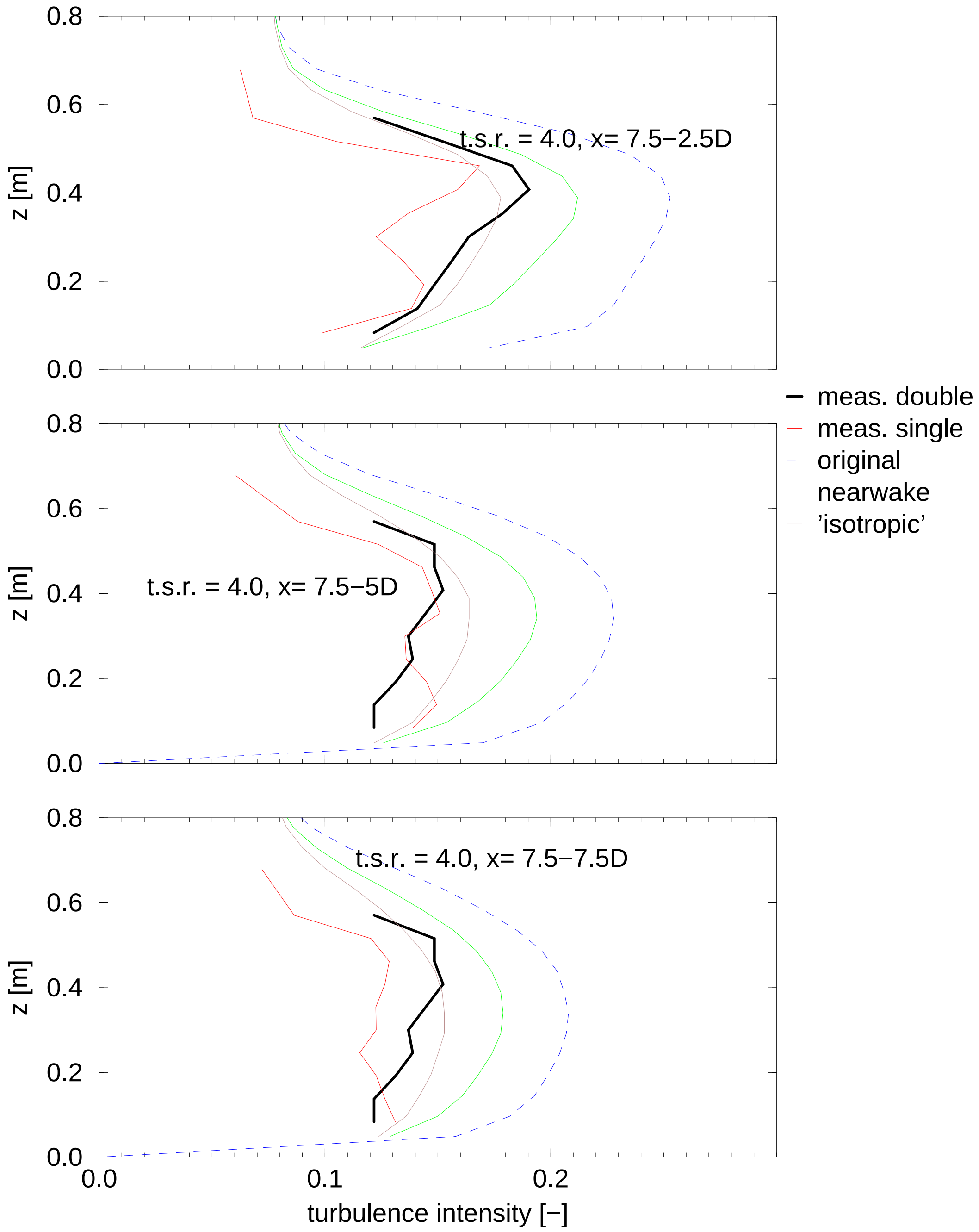

Figure 6.4 *Wind tunnel: Double wake, distance between the two turbines is 7.5D: Measured and calculated turbulence intensity profiles at* λ *= 4.0 (y=0)*

agreement with the measurement results, the calculations show a reduced double wake deficit at $\lambda_1 = 4.0$ as well.

However, in the $\lambda_1 = 2.9$ cases, the calculations again show a reduced double wake defect, where the measurements indicate a larger wake deficit. The explanation may be, that at $\lambda = 2.9$, the wake induced turbulence is smaller which leads to a longer near wake. This indicates that the position of the wake initialisation should be made dependent on the loading.

Despite these discrepancies, the velocity deficit at hub height is predicted within an accuracy of 20%, compared to 15% in single wake. For the near wake region (x=2.5D) the discrepancies are larger. In many cases the 'Gaussian' near wake model yields a poorer agreement with the measurement results, but the differences with the results from the 'original' near wake model are relatively small.

Another double wake effect is the broadening of the wake. This effect is also visible in the calculations.

Added turbulence
In figure 6.4, the turbulence intensity profiles are given. Note that results are only available for λ = 4.0. For this tip speed ratio, the shape of the added turbulence profiles is predicted reasonably well.

The turbulence in the double wake situation is clearly increased compared to the single wake situation. This increase is, to a much larger degree, also apparent in the calculations. Due to the fact that the turbulence in the single wake is underestimated, the agreement between calculated and measured turbulence becomes better in double wake.

With the exception of the 2.5D wake location, the maximum added turbulence is predicted within an accuracy of about 15% compared to 25% to 50% in single wake conditions.

The 'Gaussian' near wake model yields a better agreement with the measurement results than the original near wake model. The assumption of isotropic added turbulence improves the agreement further.

6.2 Multiple wake: Validation with Vindeby measurements

6.2.1 Double wake: Description of Vindeby wind farm and measurements

In this section, the validation with double wake measurements from the Vindeby wind farm are reported. The description of the Vindeby wind farm is given in section 4.2.1. The double wake measurements have been performed at a wind direction of approximately 77 degrees, where the Sea-Mast West is in the wake of the turbines 5E and 4W and the Sea-Mast South is located in the free stream. A number of Vindeby double wake measurements have been made available. They were sorted into bins of wind speed, turbulence intensity and stability. The centre bin values of wind speed, turbulence intensity and a measure for the stability for the selected cases is given in tabel 6.1. Note that these bin values are derived from Land Mast measurements.

6.2.2 Double wake: Simulation of Vindeby measurements

The free stream conditions for the simulations are based on the centre bin values of wind speed and turbulence intensity, given in table 6.1. For stable conditions the assumed Monin-Obukhov length scale is +200 m, for unstable conditions, the assumed Monin-Obukhov length scale is -200 m and for neutral conditions, the assumed Monin-Obukhov length scale is 10000 m. The geometrical input data for WAKEFARM (turbine diameter, hub height, turbine locations) is given in section 4.2.1

stability	wind speed [m/s]	turbulence intensity [%]
neutral	5	6
neutral	5	8
stable	5	6
stable	5	8
unstable	5	6
neutral	7.5	6
neutral	7.5	8
stable	7.5	6
stable	7.5	8
unstable	7.5	6
neutral	10	6
neutral	10	8

Table 6.1 *Vindeby double wake measurements: Wind speed, turbulence intensity and stability*

6.2.3 Double wake: Validation with Vindeby measurements: Comparison between calculational and measured results

In the figures 6.5 to 6.9, the wake profiles calculated with the different model versions are compared with the measurements. In the figures 6.10 to 6.14, the turbulence intensities are compared. The results which are identified with 'original' denote the calculational results from the original WAKEFARM program. The results which are identified with 'nearwake' are the calculational results in which the Gaussian near wake profile is implemented, and the results which are identified with 'isotropic' are the results in which the added wake turbulence is assumed to be isotropic. The latter effect is obviously only relevant for the turbulence intensities.

The following comments can be made:

- The effect of stability can be distinguished by considering the steepness of the free stream wind profiles: In unstable atmosphere the velocity profile is 'steeper';
- Some clear differences can be observed between the measured and calculated free stream wind profiles. This holds in particular for the lower part of the profiles at neutral and stable conditions at v = 5 m/s.
  Differences, to a smaller degree, can also be found in the free stream turbulence intensity profiles. The calculated free stream profiles are determined from the centre bin values of hub height wind speed, turbulence intensity and Monin-Obukhov length scale. The differences are partly caused by the fact that the free stream profiles presented in the figures are based on the Sea Mast South measurements, where the prescribed WAKEFARM input data was based on the Land Mast measurements.
  It should be realised that these differences in free stream profiles will also effect the wake profiles;
- In many cases the wake anemometer at h = 43 m is malfunctioning;
- In most cases the wake deficits are overpredicted by the WAKEFARM program. Moreover the calculated and measured shape of the wake profiles often differs;
- Generally speaking the agreement between measured and calculated wake deficits becomes better at increasing wind speed. The agreement at neutral

and unstable conditions also seems to be somewhat better than the agreement for stable conditions.

- The modification in near wake profile improves the agreement with the measurement results;
- The turbulence intensities in the wake are clearly overpredicted. This holds in particular for the low wind speeds; The modification in near wake profile and the assumption of isotropic wake turbulence improves the agreement between calculations and measurements considerably;
- It must be emphasised that the above mentioned differences can at least partly be attributed to the neglect of wind speed variations. The calculations assume a 100% wake operation, where in reality this is not the case. For single wake conditions, the effect of wind direction fluctuations has been determined in section 4.2.4.

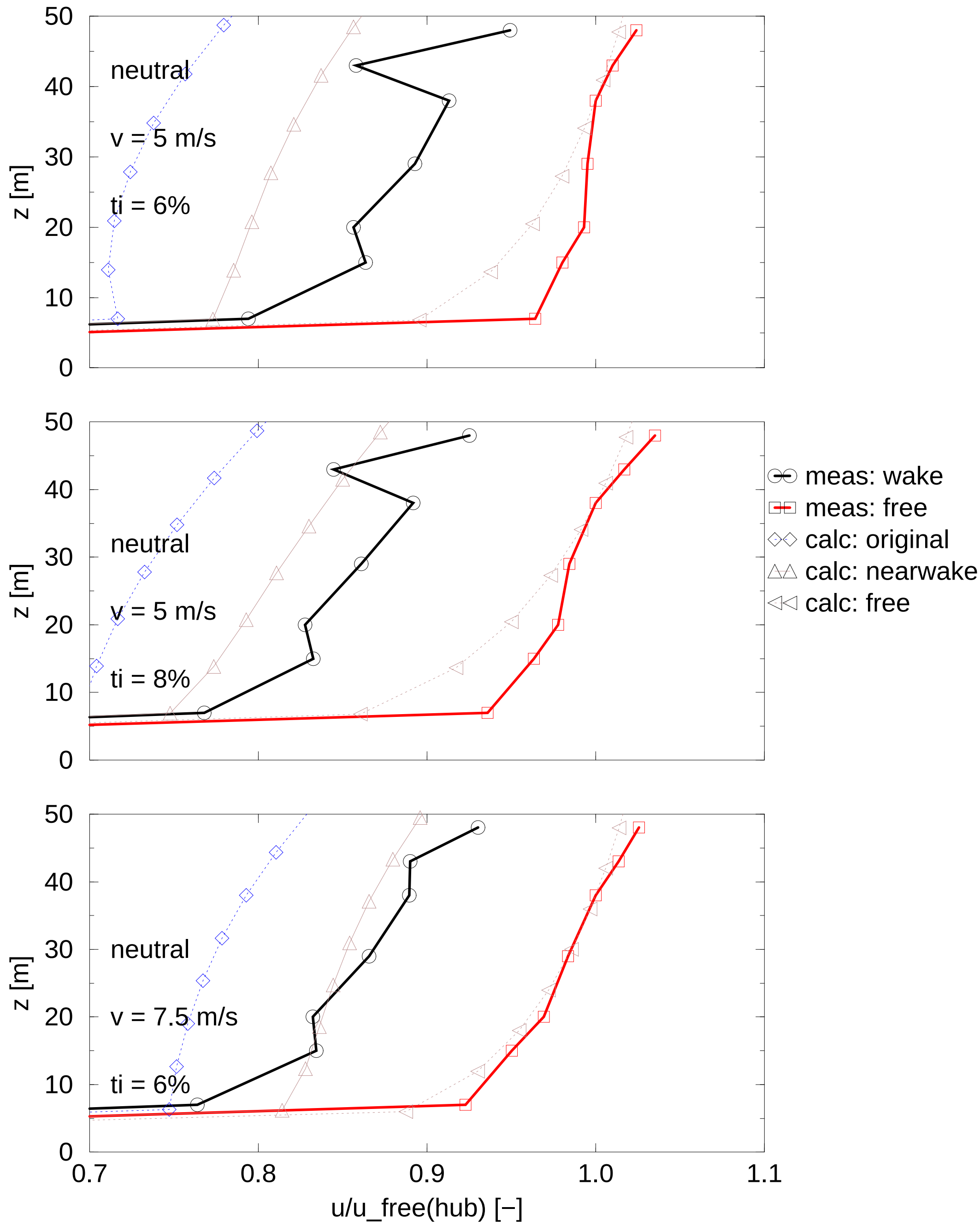

Figure 6.5 *Vindeby: Double wake: Measured and calculated non-dimensional velocity profiles at neutral conditions*

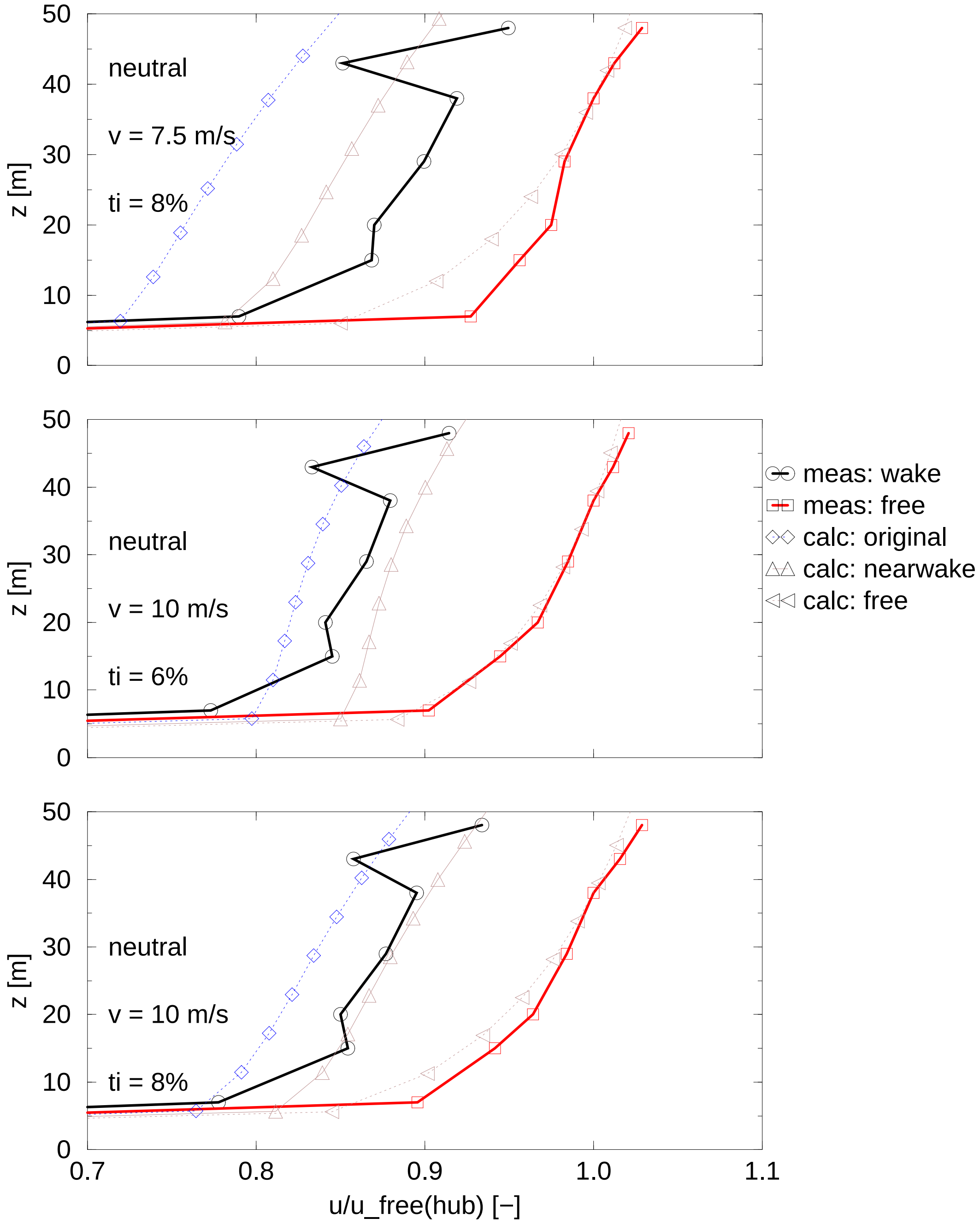

Figure 6.6 *Vindeby: Double wake: Measured and calculated non-dimensional velocity profiles at neutral conditions*

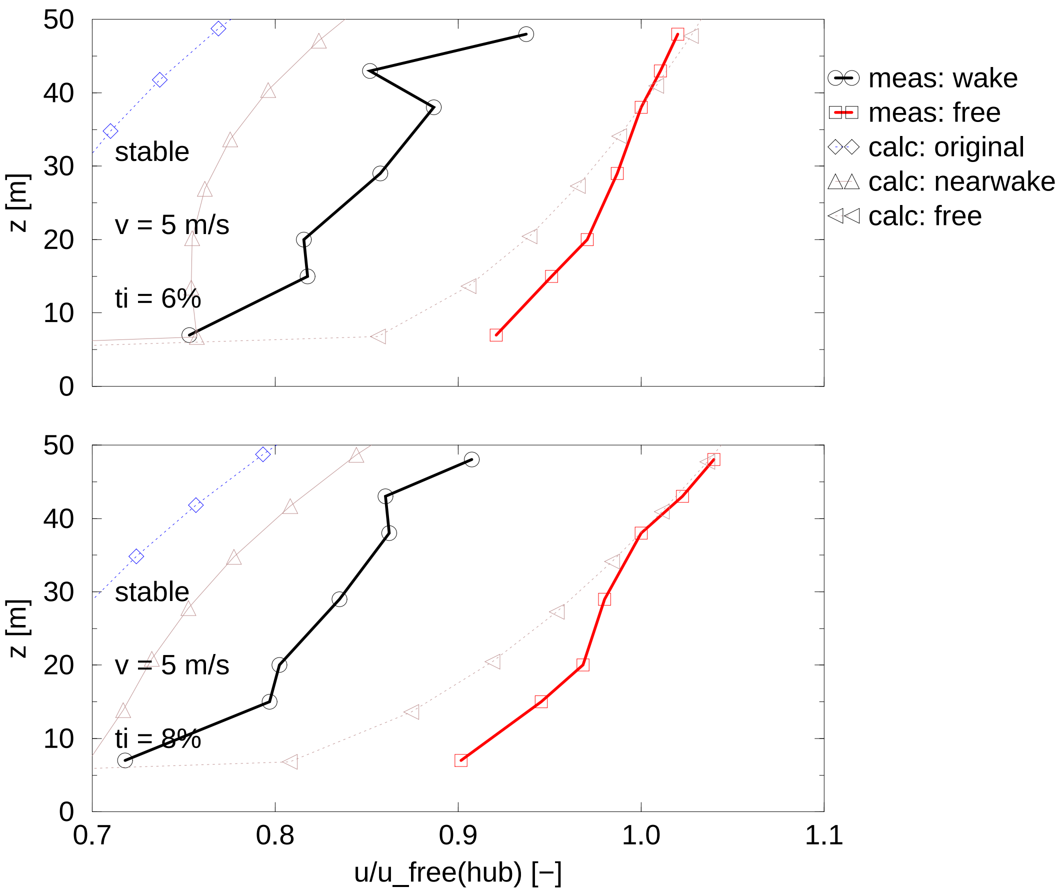

Figure 6.7 *Vindeby: Double wake: Measured and calculated non-dimensional velocity profiles at stable conditions*

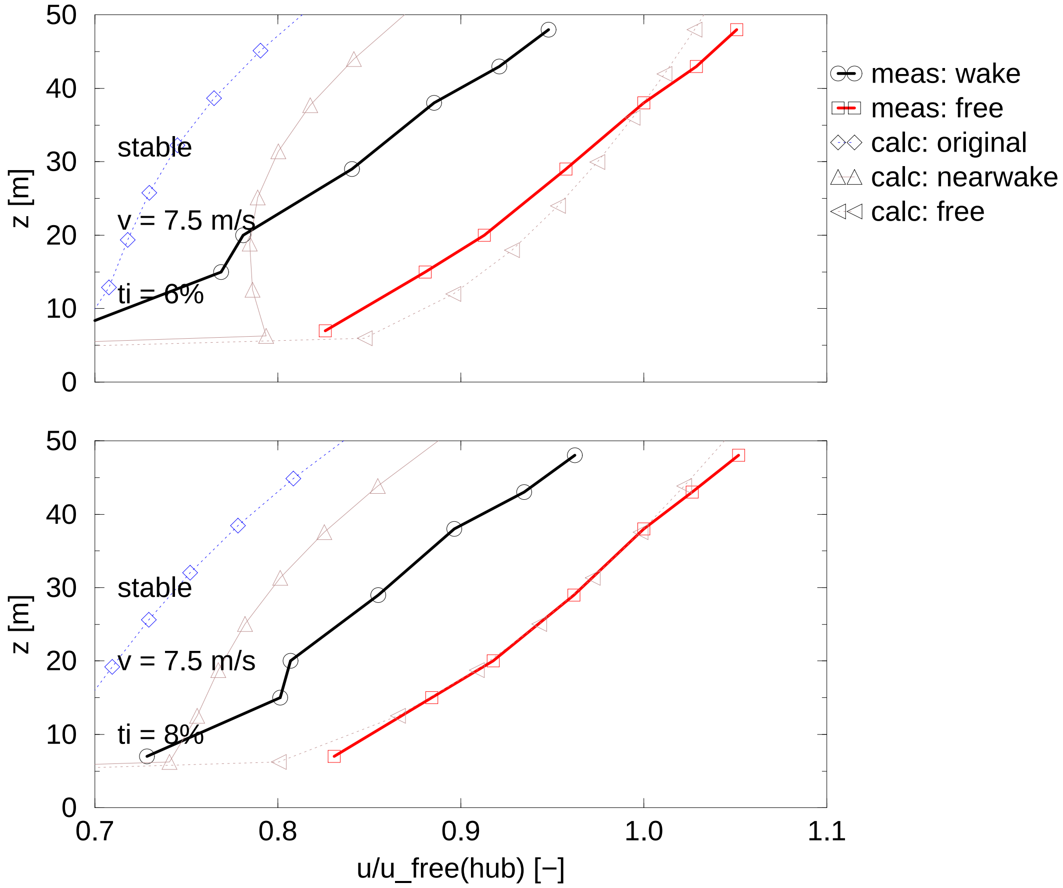

Figure 6.8 *Vindeby: Double wake: Measured and calculated non-dimensional velocity profiles at stable conditions*

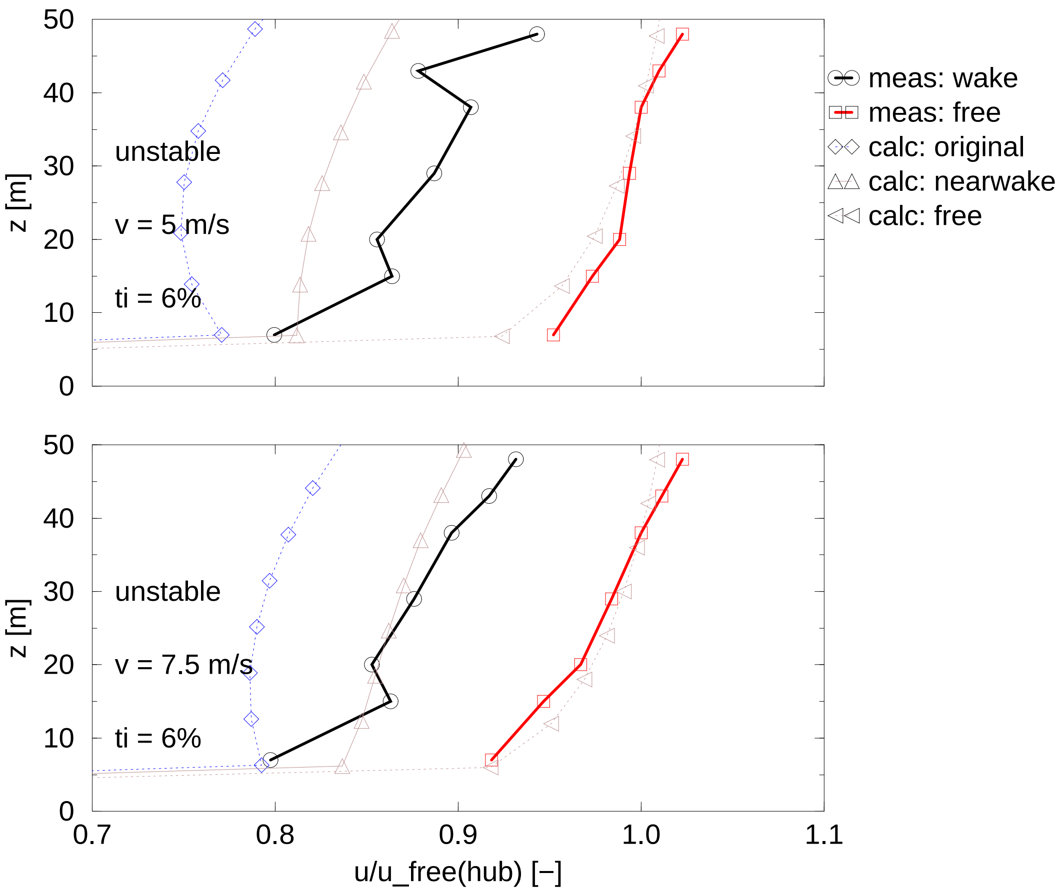

Figure 6.9 *Vindeby: Double wake: Measured and calculated non-dimensional velocity profiles at unstable conditions*

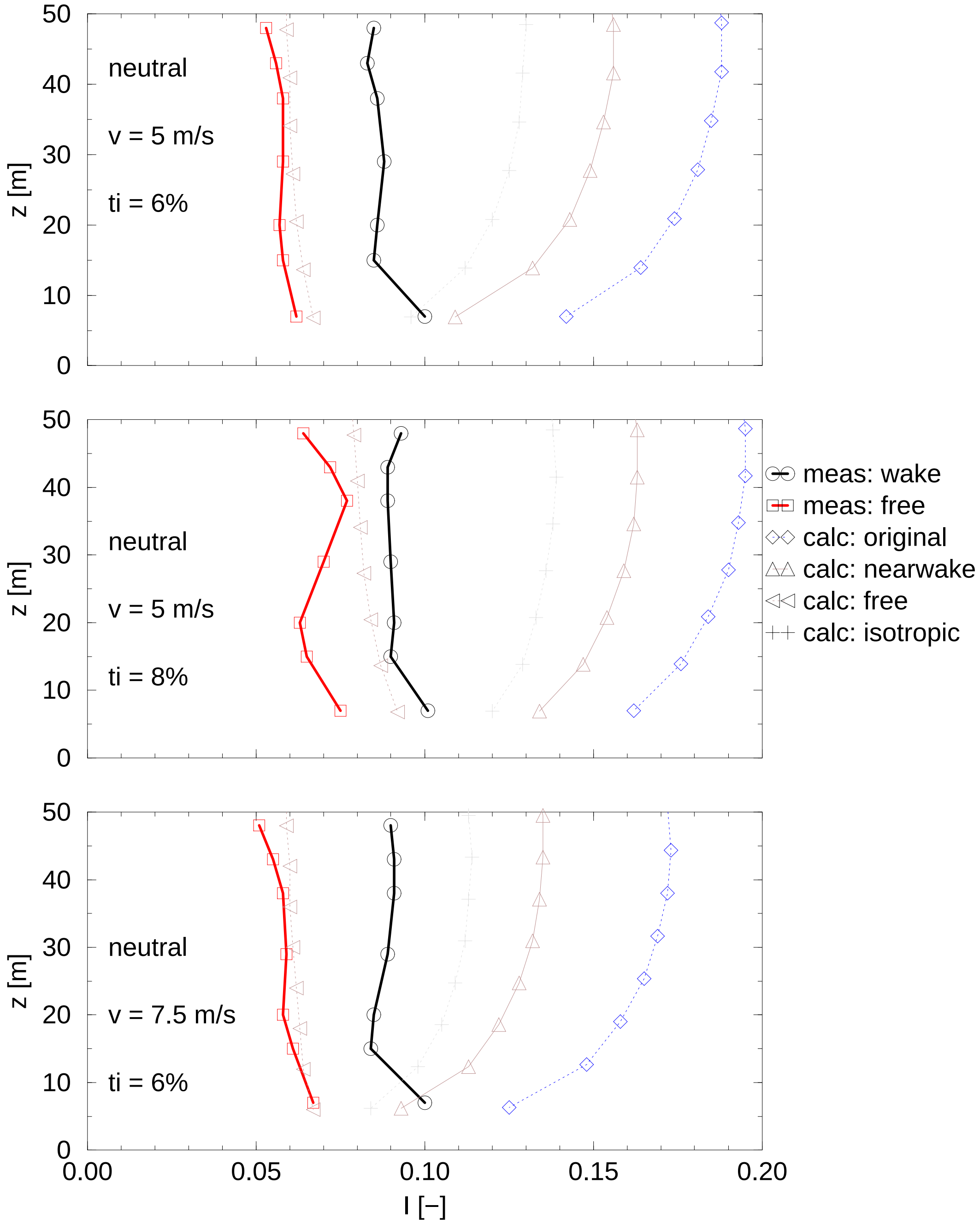

Figure 6.10 *Vindeby: Double wake: Measured and calculated turbulence intensity profiles at neutral conditions*

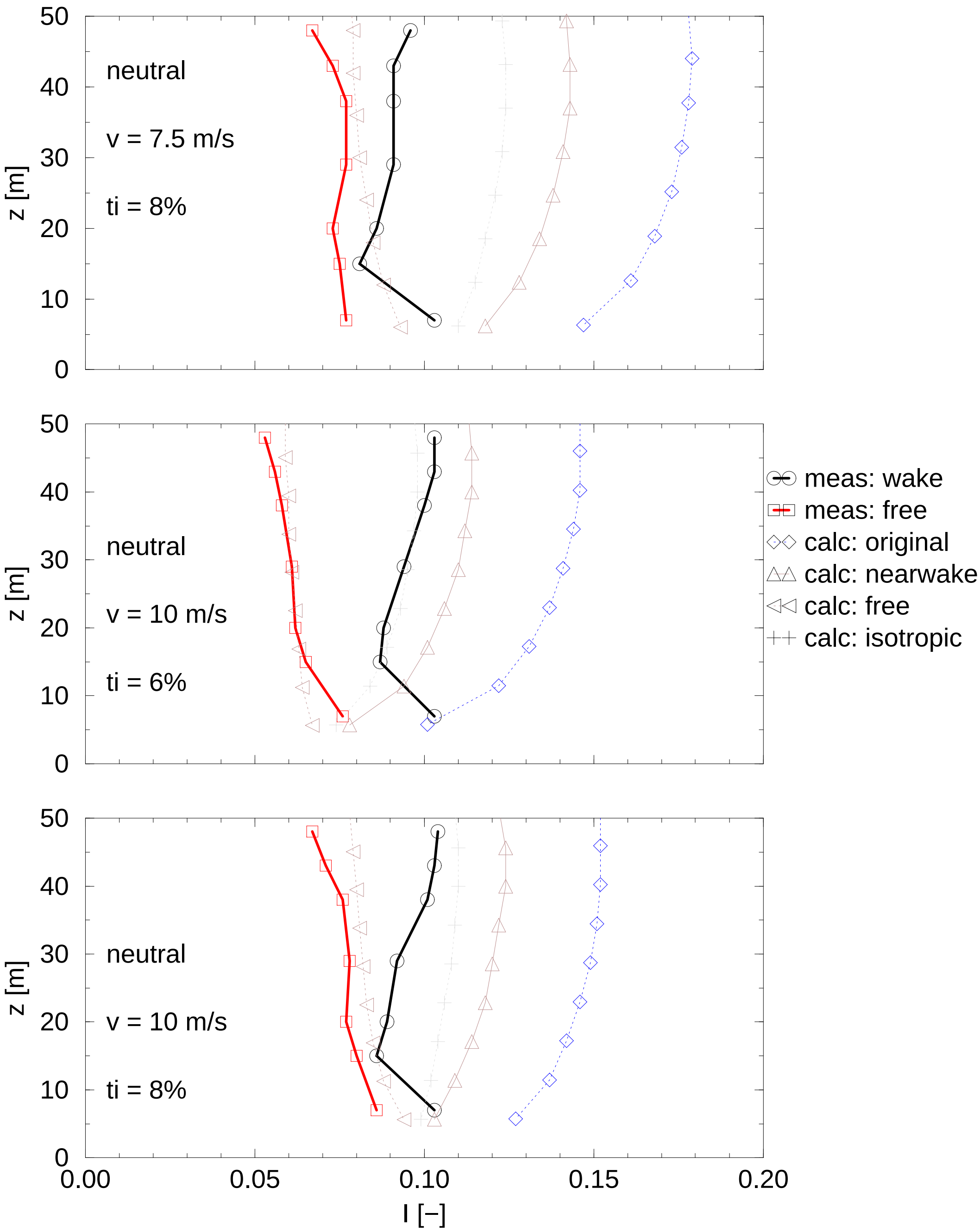

Figure 6.11 *Vindeby: Double wake: Measured and calculated turbulence intensity profiles at neutral conditions*

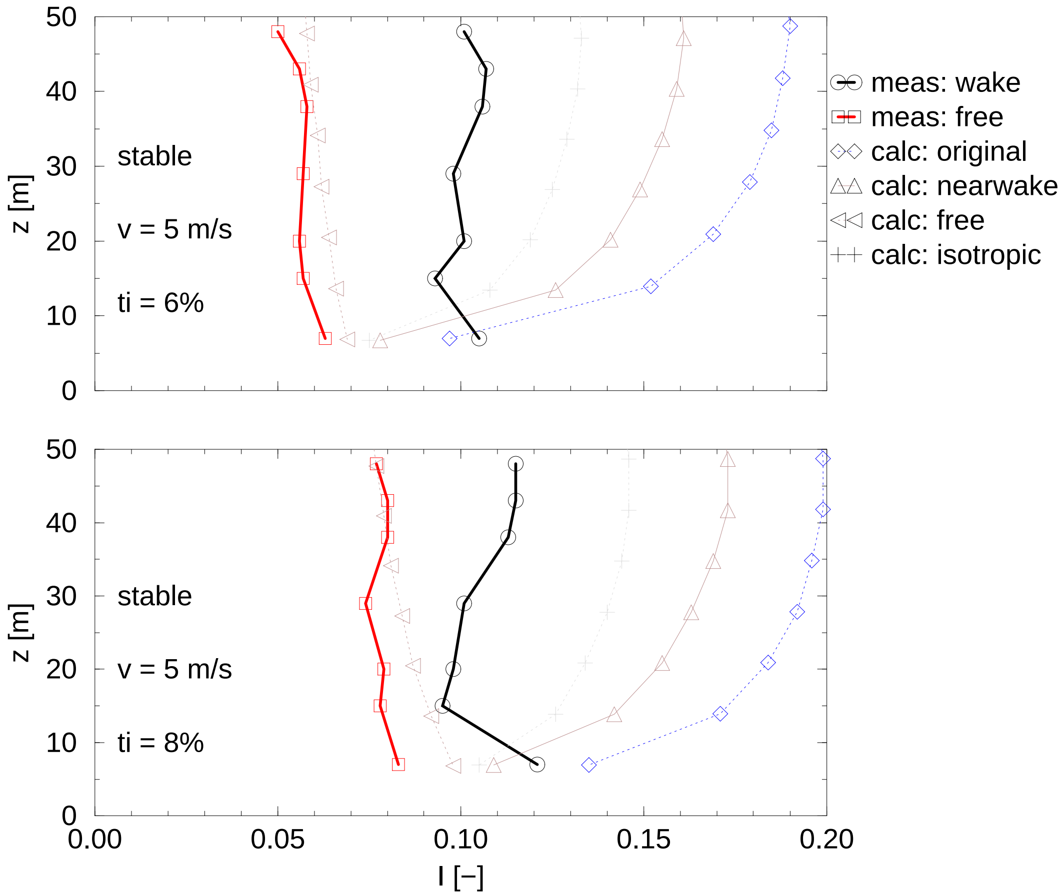

Figure 6.12 *Vindeby: Double wake: Measured and calculated turbulence intensity profiles at stable conditions*

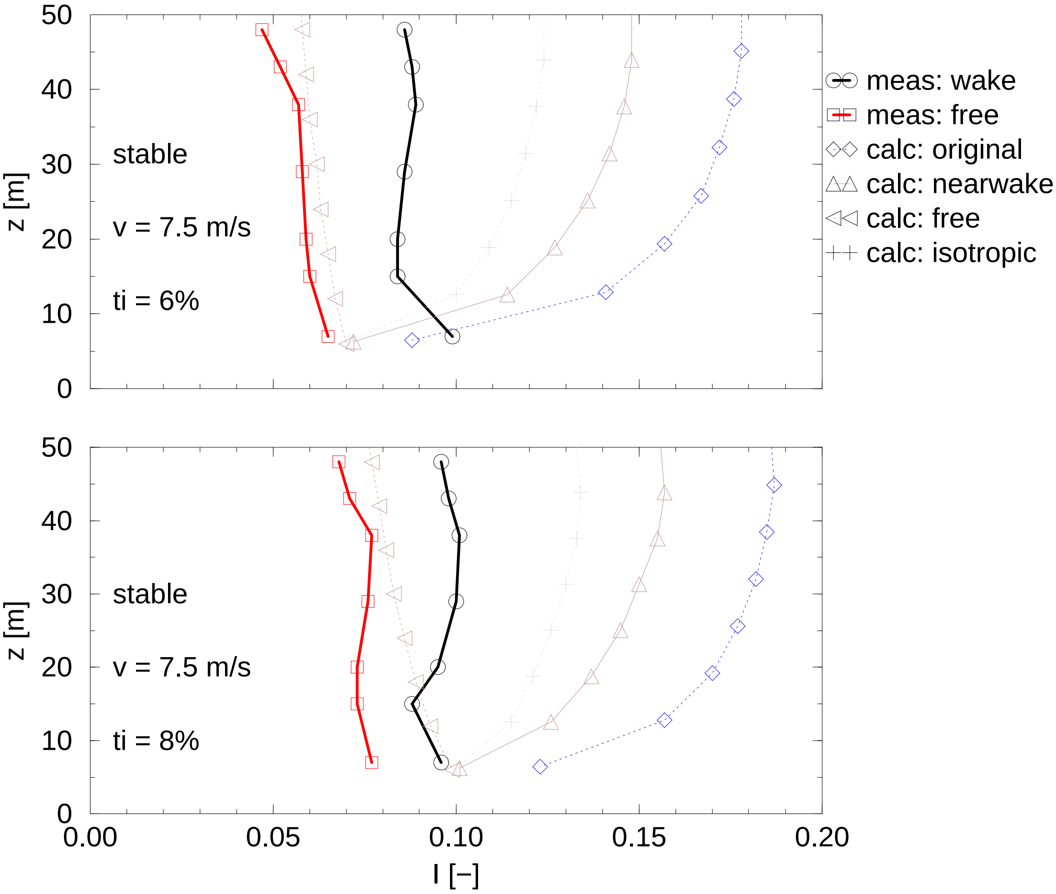

Figure 6.13 *Vindeby: Double wake: Measured and calculated turbulence intensity profiles at stable conditions*

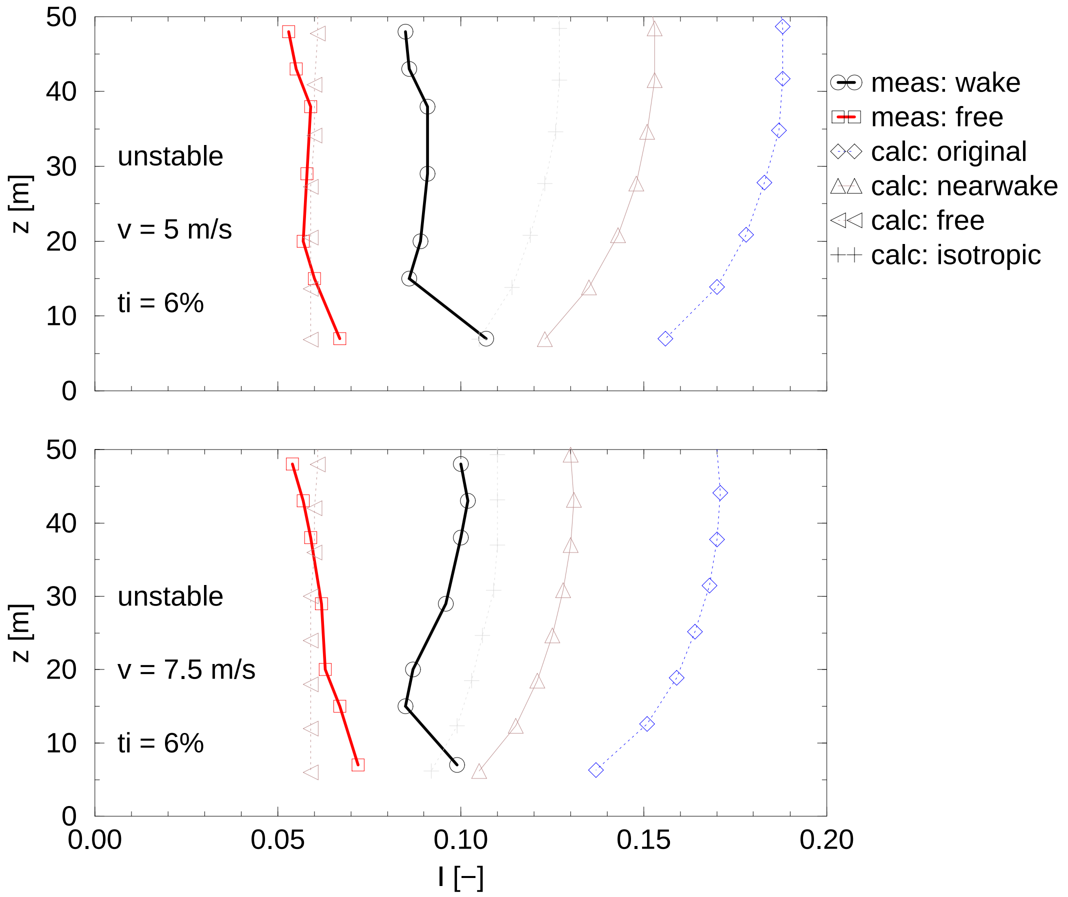

Figure 6.14 *Vindeby: Double wake: Measured and calculated turbulence intensity profiles at unstable conditions*

6.3 Quintuple wake: Validation with Vindeby measurements

6.3.1 Quintuple wake: Description of Vindeby wind farm and measurements

In this section, the quintuple wake cases from the Vindeby wind farm are reported. The description of the wind farm is given in section 4.2.1. The quintuple wake measurements have been performed at a wind direction of approximately 320 degrees, where the Sea-Mast South is in the wake of the turbines 1W to 5W. The Sea-Mast West is in the free stream. A number of Vindeby quintuple wake measurements have been made available. They were sorted into bins of wind speed, turbulence intensity and stability. The centre bin values of wind speed, turbulence intensity and a measure for the stability for the selected cases are given in table 6.2. Note that these conditions are derived from the Land Mast measurements.

stability	wind speed [m/s]	turbulence intensity [%]
neutral	5	6
neutral	5	8
stable	5	6
stable	5	8
unstable	5	6
unstable	5	8
neutral	7.5	6
neutral	7.5	8
stable	7.5	6
unstable	7.5	6
unstable	7.5	8
neutral	10	6
neutral	10	8
unstable	10	6

Table 6.2 *Vindeby quintuple wake measurements: Wind speed, turbulence intensity and stability*

6.3.2 Quintuple wake: Simulation of Vindeby measurements

For stable conditions the assumed Monin-Obukhov length scale is +200 m, for unstable conditions, the assumed Monin-Obukhov length scale is -200 m and for neutral conditions, the assumed Monin-Obukhov length scale is 10000 m. The geometrical input data for WAKEFARM (turbine diameter, hub height, turbine locations) is given in section 4.2.1.

In some of the quintuple cases the WAKEFARM program did not convergence. Table 6.3 lists whether of not convergence is reached for the different model versions. Convergence is indicated through +. The cases where the WAKEFARM program did not converge are indicated through -. Note that the 'near wake' and the 'isotropic' version are similar with regard to the numerical aspects and hence these two versions perform similarly in terms of convergence.

The item of convergence is further discussed in section 8.

Vw	turb	stability	orig	near wake	isotropic
5	6	neutral	+	-	-
5	8	neutral	+	+	+
5	6	stable	+	-	-
5	8	stable	+	-	-
5	6	unstable	+	+	+
5	8	unstable	+	+	+
7.5	6	neutral	+	+	+
7.5	8	neutral	+	+	+
7.5	6	stable	+	+	+
7.5	6	unstable	+	+	+
7.5	8	unstable	+	+	+
10.	6	neutral	+	+	+
10.	8	neutral	+	+	+
10.	6	unstable	+	+	+

Table 6.3 *Vindeby quintuple wake measurements: Convergence (+) or no convergence (-)'*

6.3.3 Quintuple wake: Validation with Vindeby measurements: Comparison between calculational and measured results

In the figures 6.15 to 6.19, the calculated wake profiles from the different model versions are compared with the measurements. In the figures 6.20 to 6.24, the calculated turbulence profiles are compared with the measurements.

The results which are identified with 'original' denote the calculational results from the original WAKEFARM program. The results which are identified with 'nearwake' are the calculational results in which the Gaussian near wake profile is implemented, and the results which are identified with 'isotropic' are the results in which the added wake turbulence is assumed to be isotropic. The latter effect is obviously only relevant for the turbulence intensities and not for the wake profiles.

The comments, which can be made are almost similar to those made at the double wake cases:

- The effect of stability can be found by considering the steepness of the free stream wind profiles: In unstable atmosphere the velocity profile is 'steeper'.
- Some clear differences can be observed between the measured and calculated free stream wind profiles. This holds in particular for the lower part of the profiles at stable and unstable conditions. Differences are also found in the turbulence intensity profiles, not only in shape but also in absolute values.
  The calculated free stream profiles are determined from the centre bin values of hub height wind speed, turbulence intensity and Monin-Obukhov length scale. The differences can then at least partly be attributed to the fact that these centre bin values are based on land mast measurements, where the free stream profiles, which are presented in the figures, are measured with the Sea Mast West (SMW). It should anyhow be realised that these differences in free stream profiles will also effect the wake profiles.
- In many cases the free stream anemometer at h = 43 m is malfunctioning;
- In most cases the wake deficits are overpredicted by the WAKEFARM program. This holds in particular for the lower part of the rotor plane;
- Generally speaking the agreement between measured and calculated wake deficits becomes better at increasing wind speed. At neutral/unstable con-

ditions the agreement seems to be better than for stable conditions;

- The modification in near wake profile improves the agreement with the measurements;
- The turbulence intensities in the wake are clearly overpredicted, although it should be realised that the actual free stream ('starting') turbulence intensity in the calculations is higher than the actual value. This holds in particular for the cases at low ambient wind speeds A higher 'starting' turbulence intensity is expected to yield a higher wake turbulence intensity as well;
- The modification in near wake profile and the assumption of isotropic wake turbulence improves the agreement between calculations and measurements;
- It must be emphasised that at least part of the above mentioned differences can be attributed to the neglect of wind speed variations. The calculations assume a 100% wake operation, where in reality this is not the case. For single wake conditions, the effect of wind direction fluctuations has been determined in section 4.2.4.

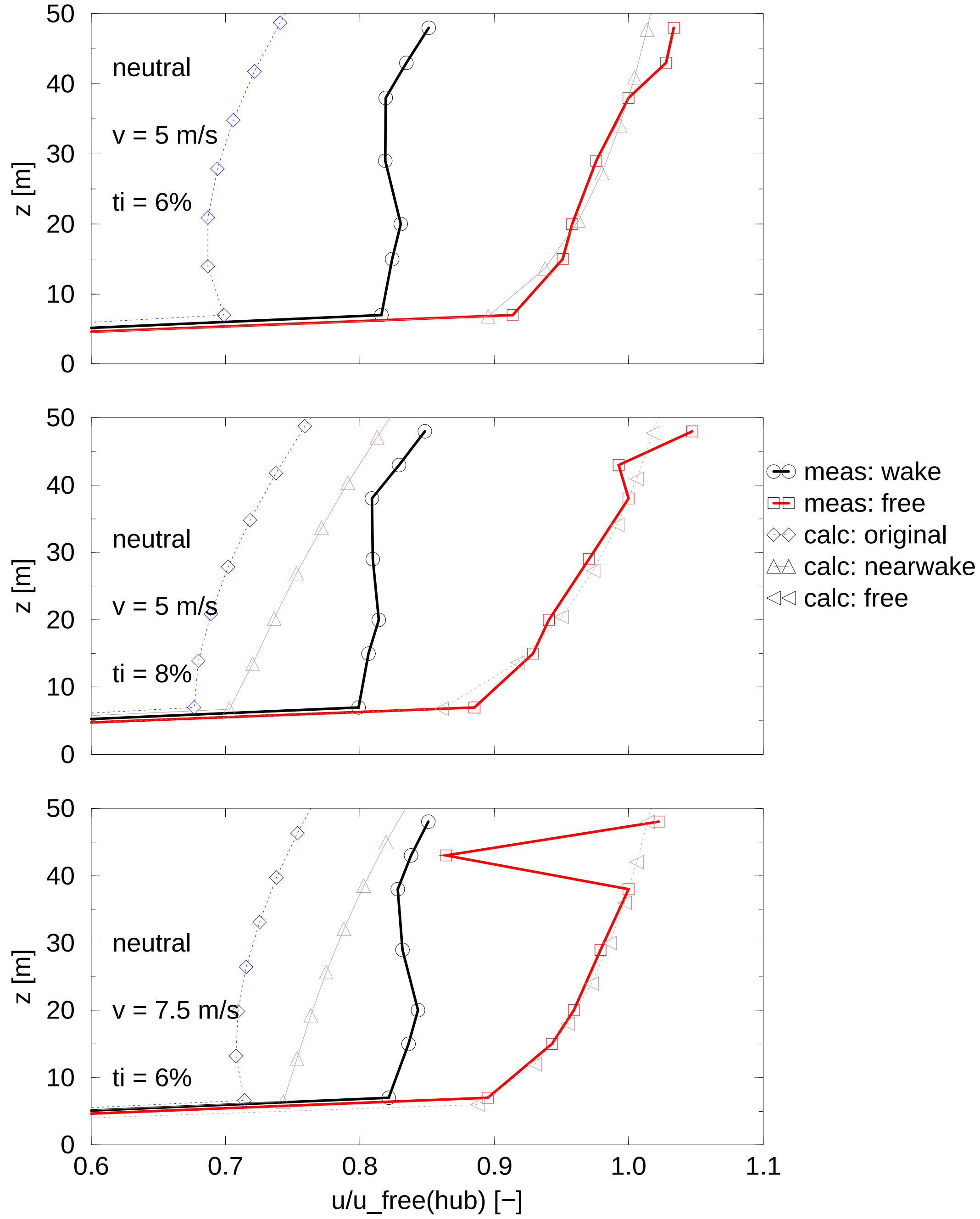

Figure 6.15 *Vindeby: quintuple wake: Measured and calculated non-dimensional velocity profiles at neutral conditions*

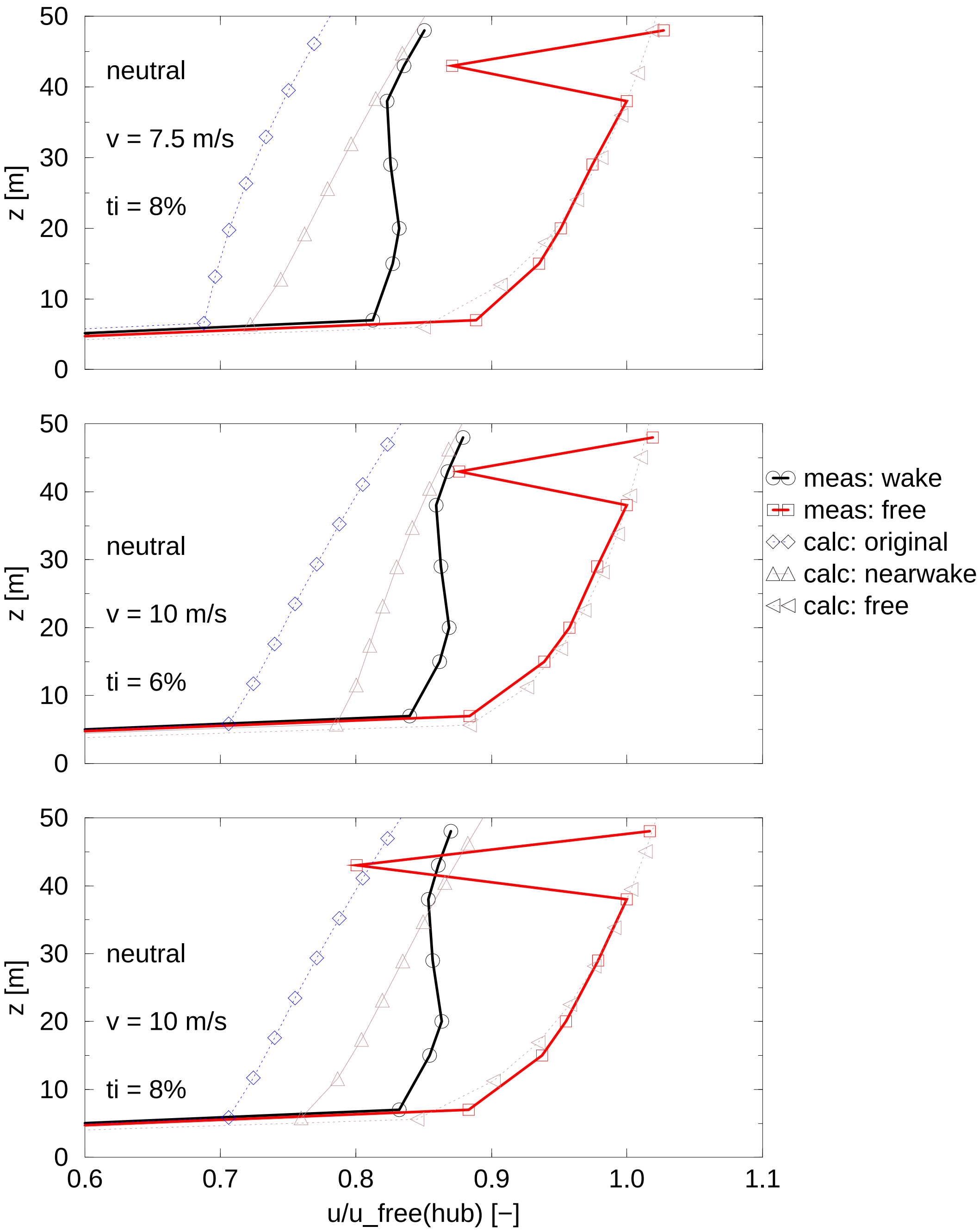

Figure 6.16 *Vindeby: quintuple wake: Measured and calculated non-dimensional velocity profiles at neutral conditions*

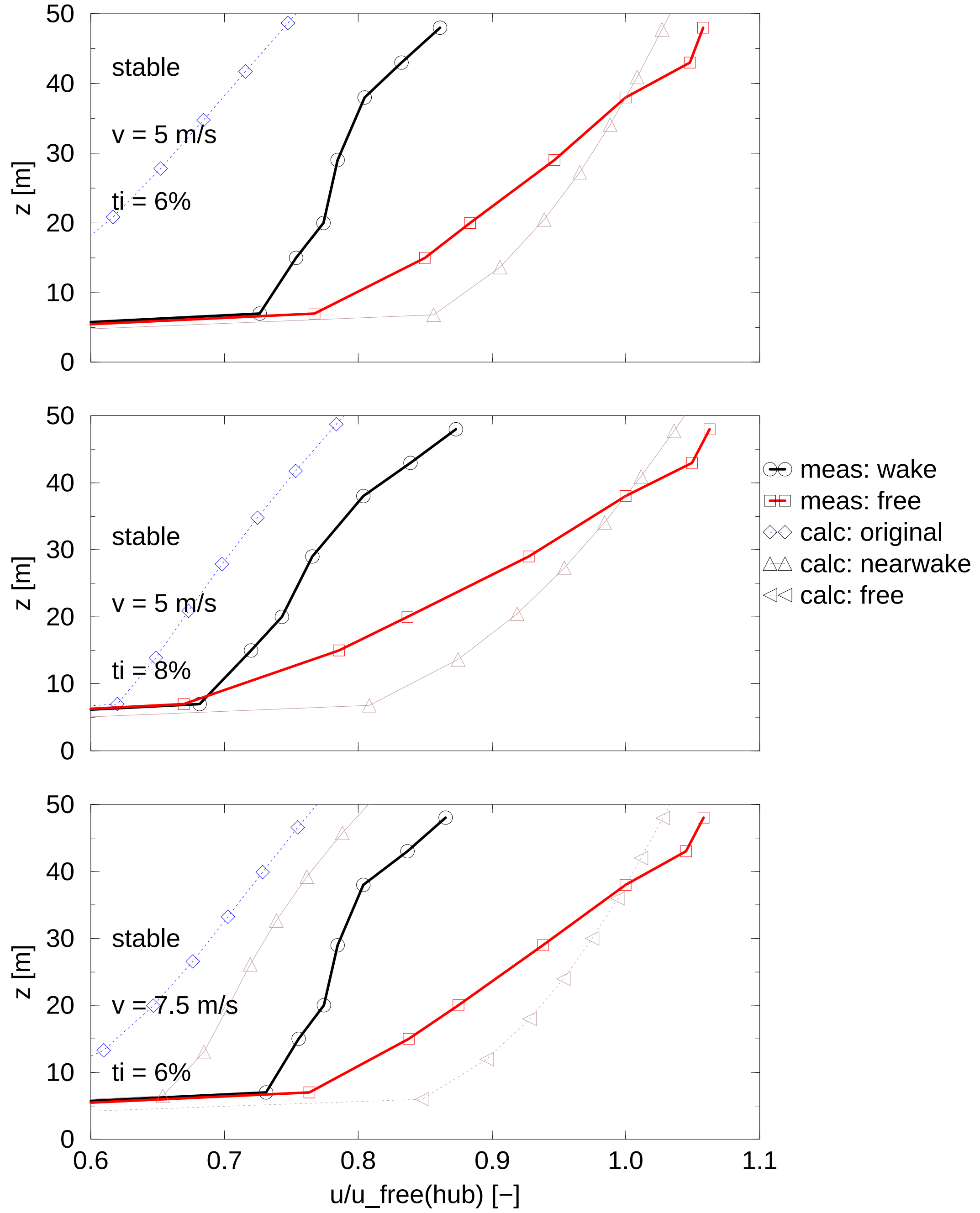

Figure 6.17 *Vindeby: quintuple wake: Measured and calculated non-dimensional velocity profiles at stable conditions*

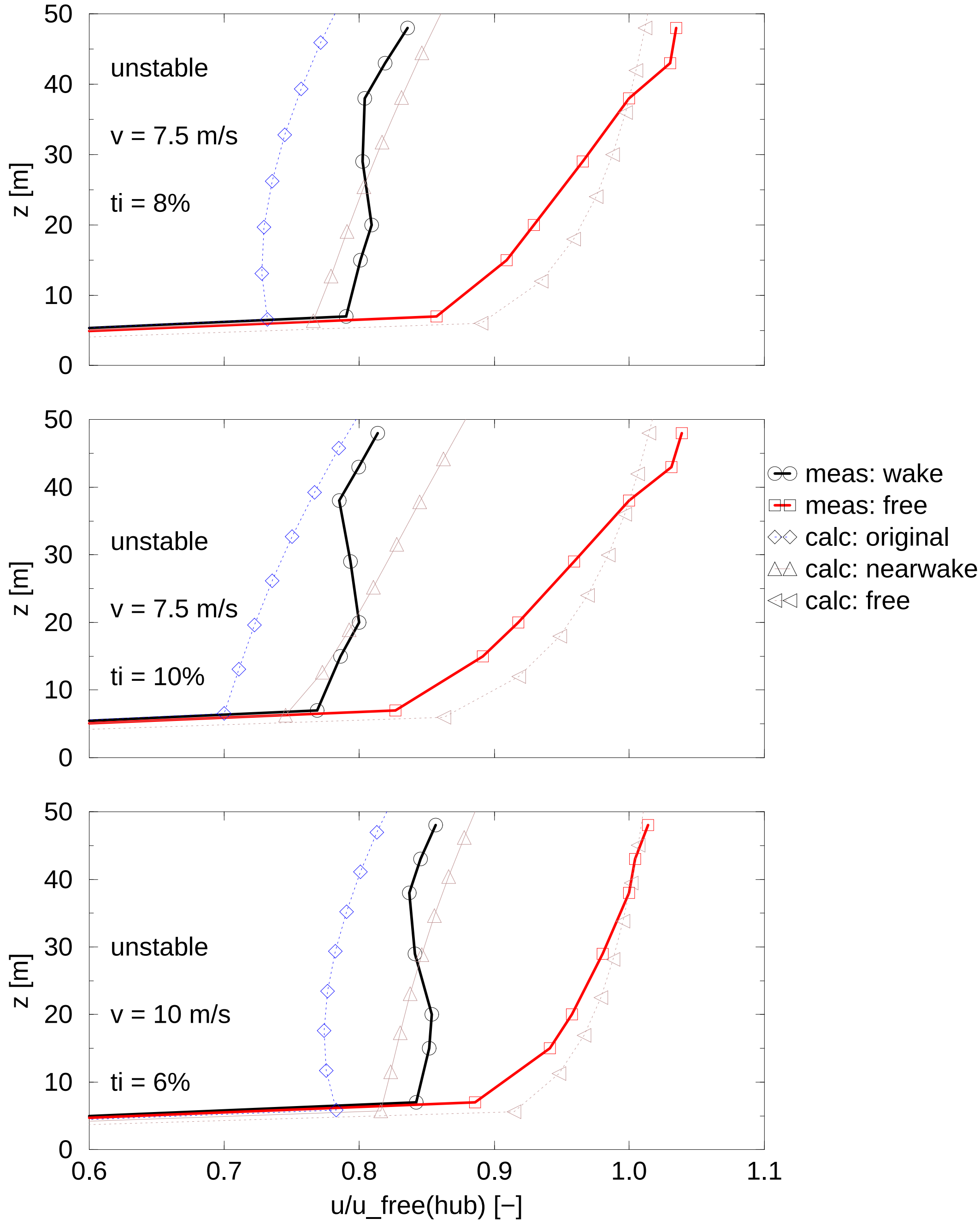

Figure 6.18 *Vindeby: quintuple wake: Measured and calculated non-dimensional velocity profiles at unstable conditions*

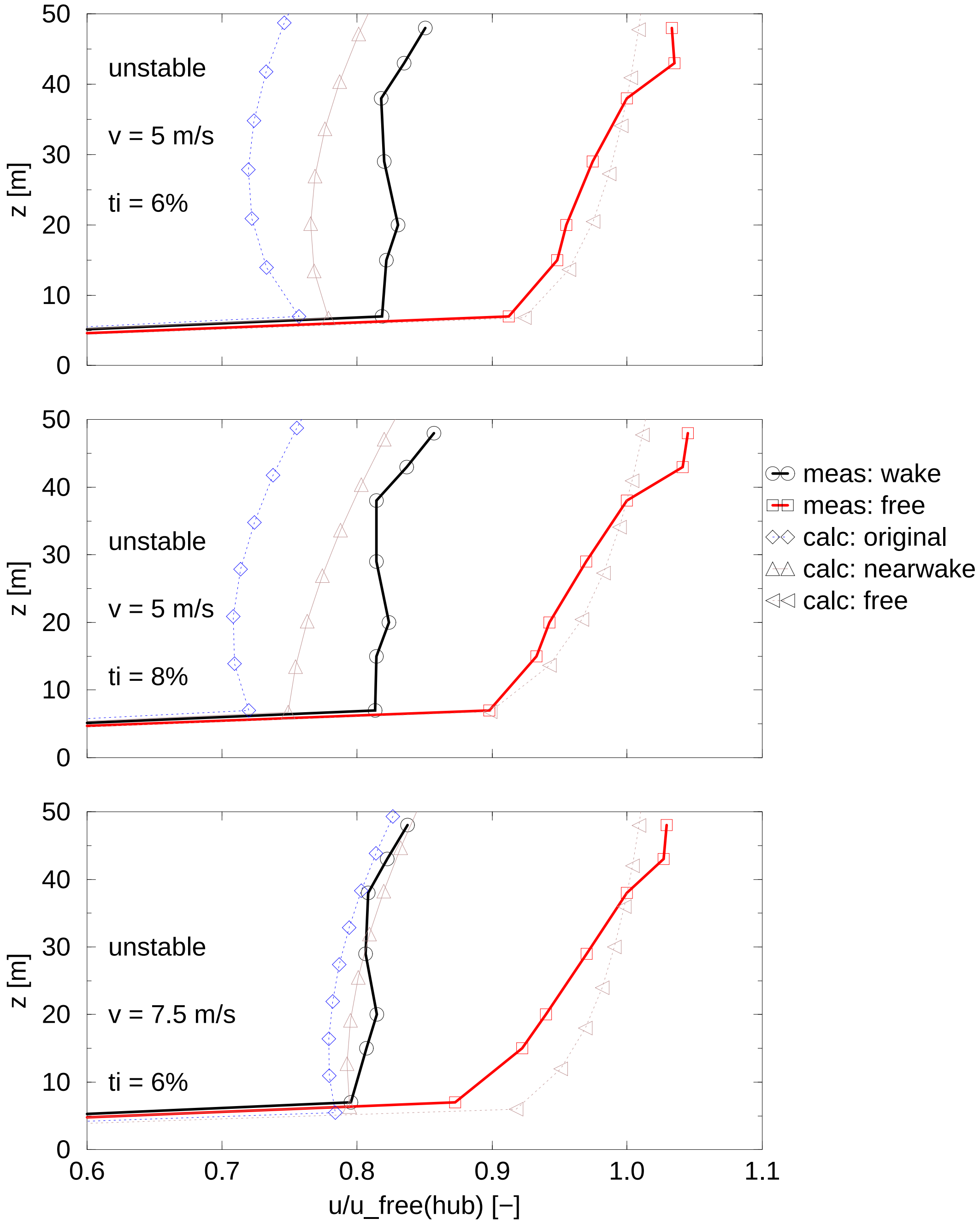

Figure 6.19 *Vindeby: quintuple wake: Measured and calculated non-dimensional velocity profiles at unstable conditions*

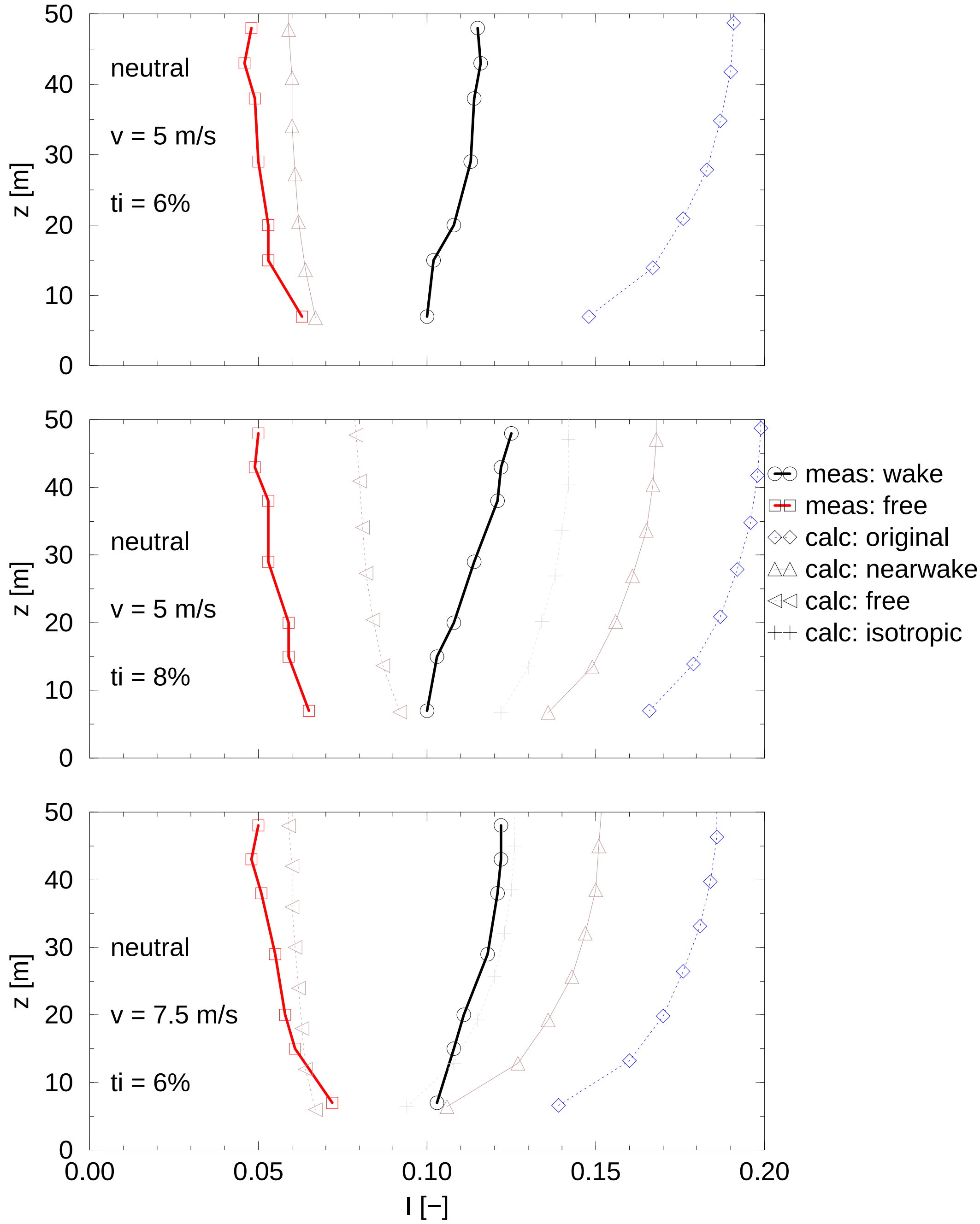

Figure 6.20 *Vindeby: quintuple wake: Measured and calculated turbulence profiles at neutral conditions*

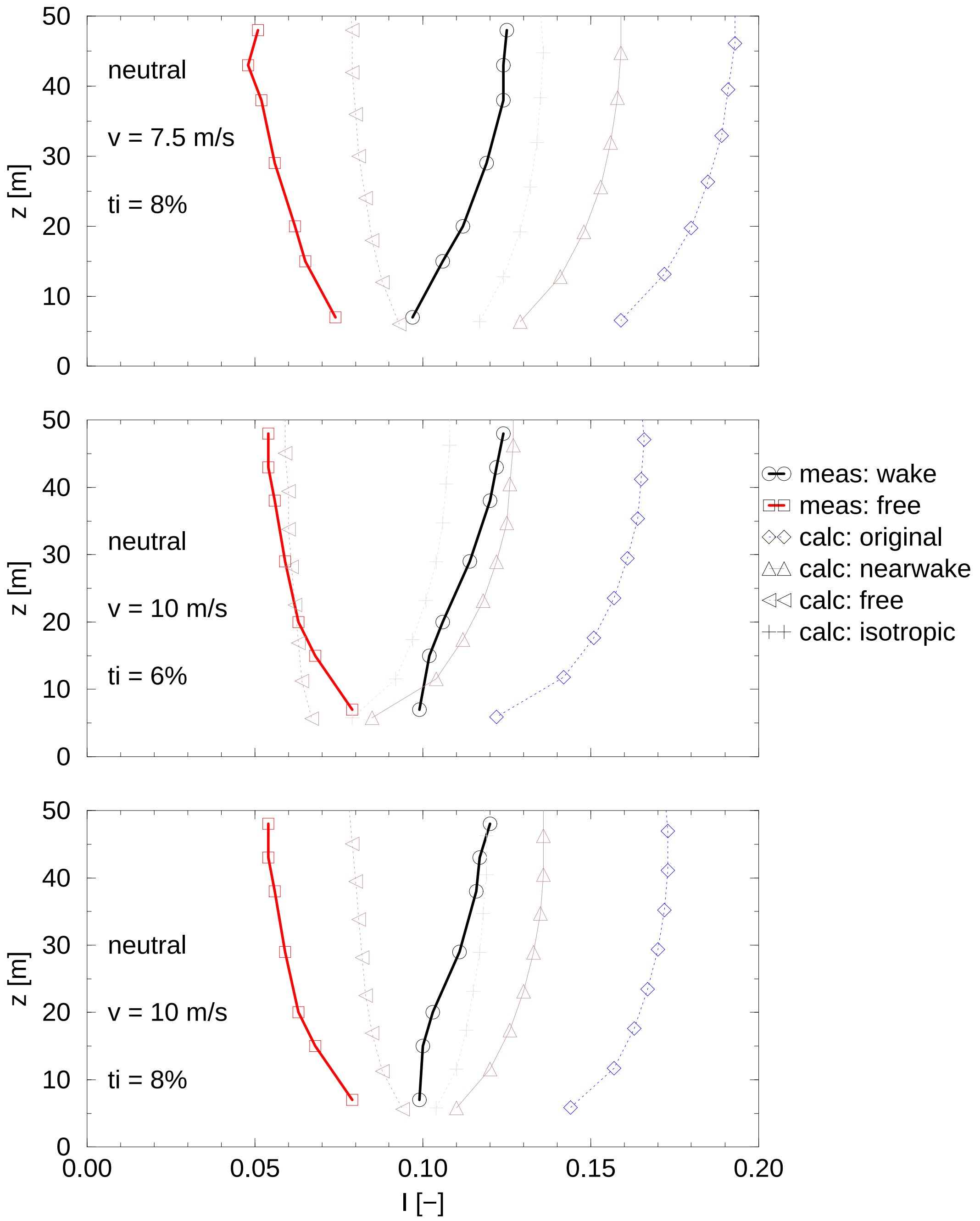

Figure 6.21 *Vindeby: quintuple wake: Measured and calculated turbulence intensity profiles at neutral conditions*

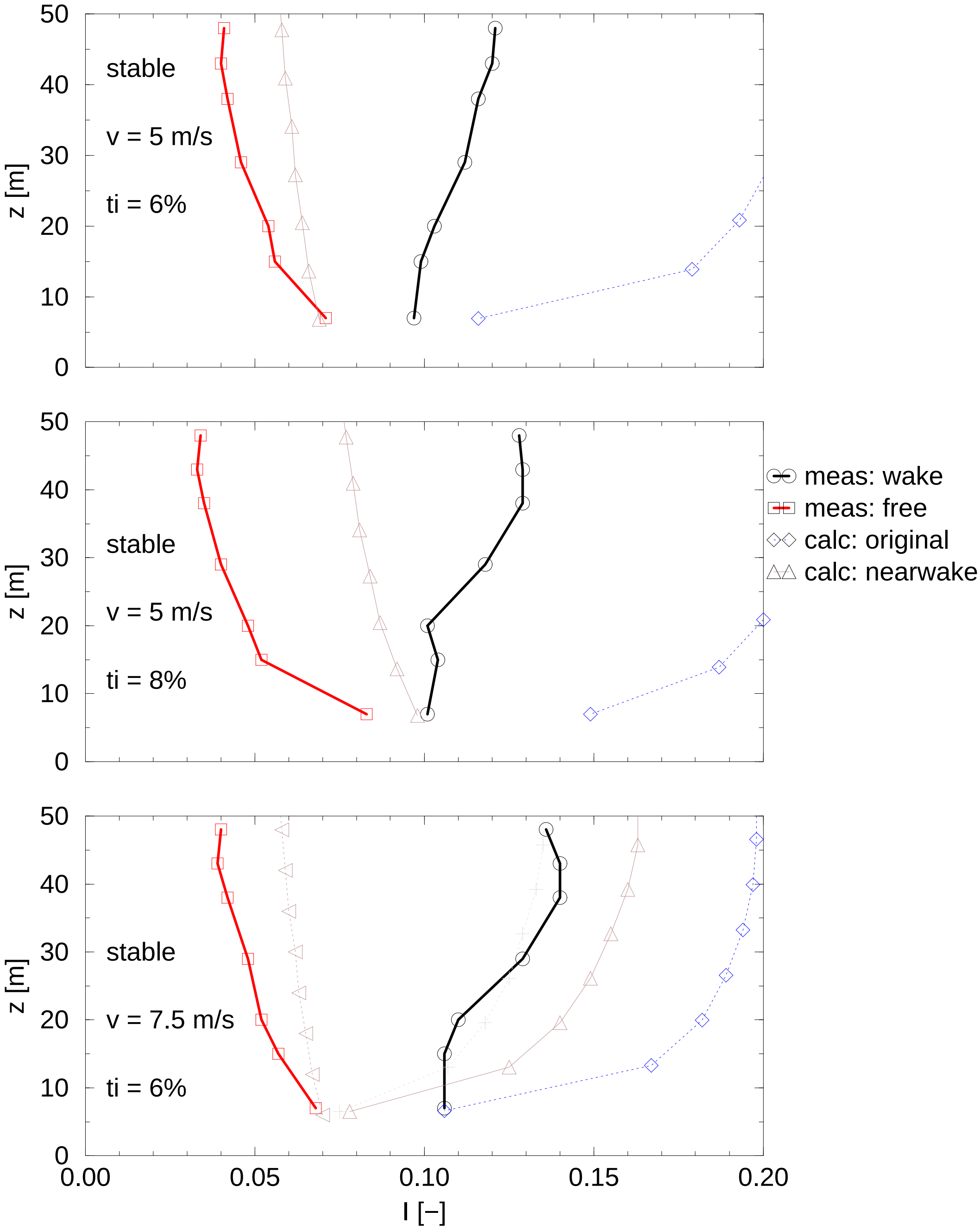

Figure 6.22 *Vindeby: quintuple wake: Measured and calculated turbulence intensity profiles at stable conditions*

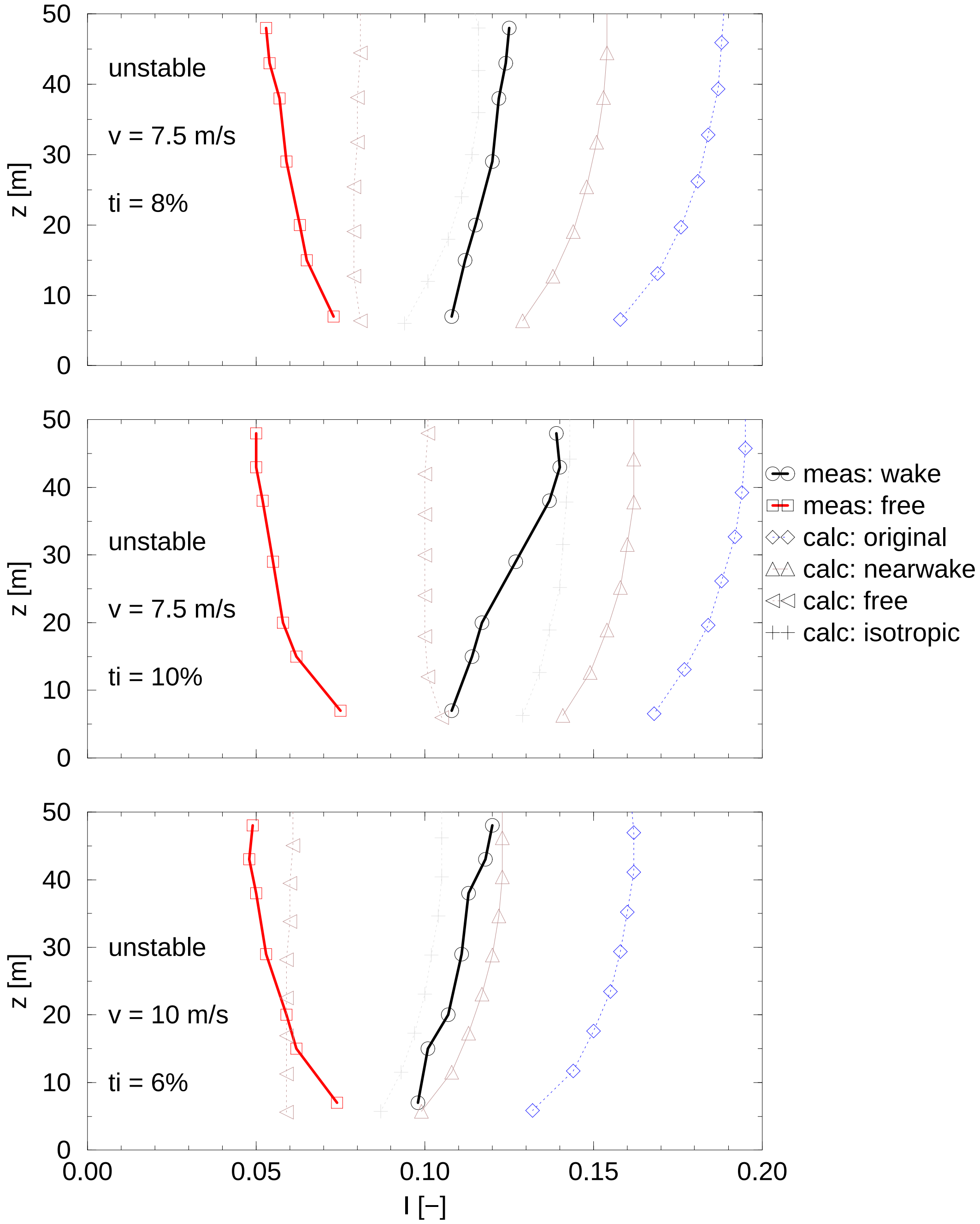

Figure 6.23 *Vindeby: quintuple wake: Measured and calculated turbulence intensity profiles at unstable conditions*

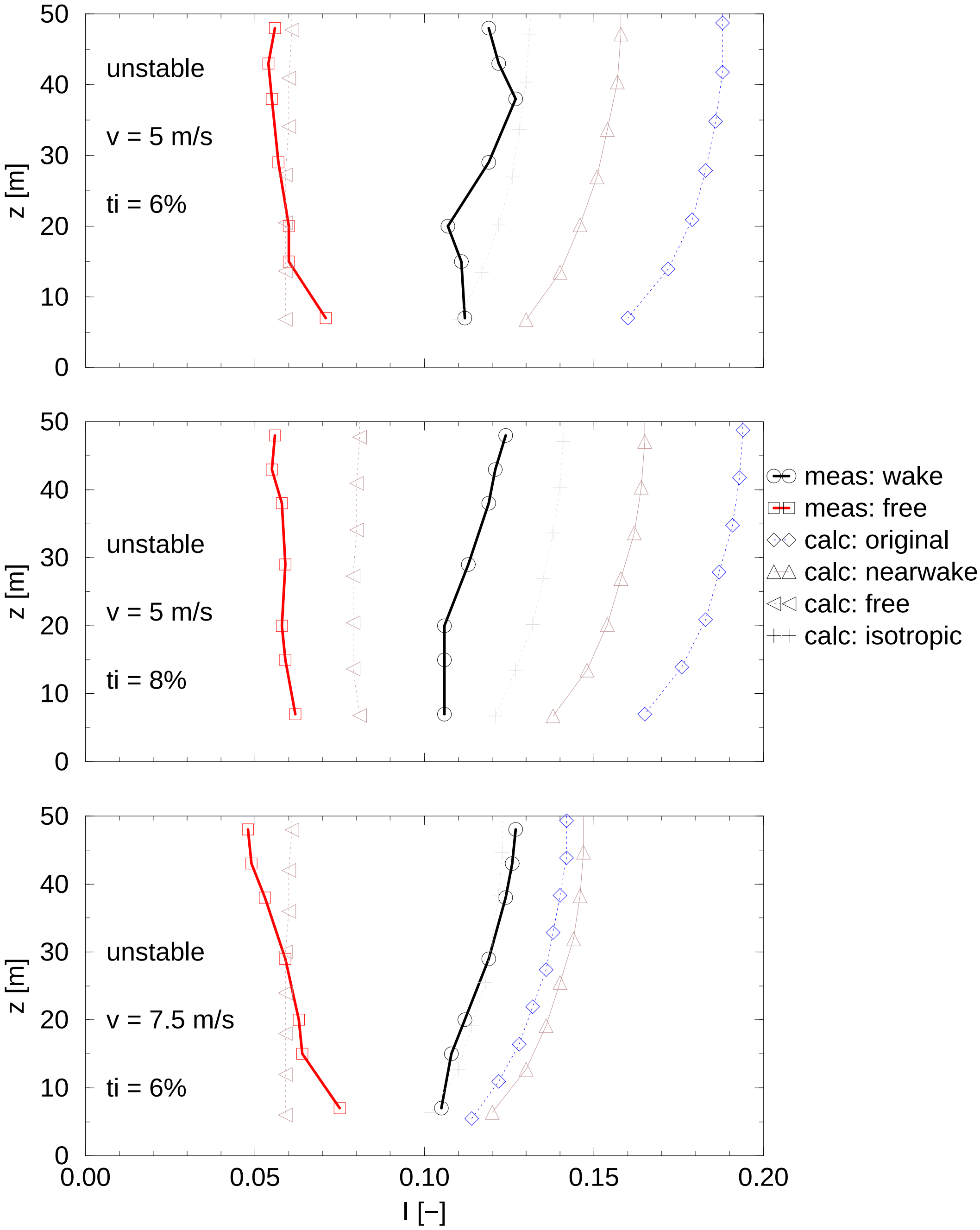

Figure 6.24 *Vindeby: quintuple wake: Measured and calculated turbulence intensity profiles at unstable conditions*

7. VALIDATION WITH SODAR MEASUREMENTS

7.1 SODAR: Introduction

In this section, the SODAR measurements, which have been taken in the Vindeby wind farm are reported and compared with calculations from the WAKEFARM program. The SODAR system was mounted on a boat and measurements of the vertical wind speed profile were recorded at several locations behind a turbine.

The SODAR measurements form a valuable addition to the conventional measurements in the Vindeby wind farm, which have been performed with anemometers on fixed meteorological masts. Although the SODAR measurements are expected to be less accurate than these conventional measurements, the transportability and the relative low costs are important advantages of the SODAR measurement system.

Note that a more detailed description of the experiments can be found in [15].

7.2 SODAR: Description of experiments

Within the ENDOW project, 6 SODAR measurement campaigns from the Vindeby wind farm have been supplied to the project group for validation of the wake models.

In these campaigns, vertical wind speed profiles have been measured at several locations behind a turbine (all in single wake) using a SODAR measurement system, which was mounted on a boat. The transportability of the SODAR measurement system made it possible to perform measurements at several locations in the wake of an operating turbine. The majority of the measurements was taken in the near wake, but a few measurements were performed in the far wake.

The measurements which are presented in this chapter are averaged over the measurement period, which was typically in the order of 30 minutes.

Due to the fact that there was only 1 SODAR system available, the free stream profile could not be measured simultaneously with the wake profile. In order to get an indication for the free stream wind conditions of a campaign, the turbine was switched off immediately after the wake measurements were done. Then SODAR measurements were taken behind the non-operational turbine. These measurements were assumed to be representative for the free stream conditions during the wake measurement campaign. Hence it is implicitly assumed that the changes in free stream conditions during a period of 0.5 to 1.0 hour are limited.

In addition to the SODAR measurements, the ’common’ meteorological masts were operational as well. The specifications and locations of these masts are described in section 4.2.1. Comparison of the SODAR data and the meteorological mast data gives some indication for the accuracy/reliability of the SODAR measurements. The data from the meteorogical masts also formed additional input for the calculational cases.

The conditions of the 6 measurement campaigns are summarized in table 7.1.

Case number	U_{hub} [m/s]	U_{48} [m/s]	Distance [D]	I_{hub} [-]	L [m]
4	6.5	5.74	2.8	0.0425	130
7	6.37	6.37	3.4	0.0778	668
9	8.51	6.90	1.7	0.0777	380
10	7.95	7.54	2.9	0.0934	1103
11	7.03	5.91	7.4	0.0760	231
12	9.19	8.19	3.4	0.0873	990

Table 7.1 *Conditions at SODAR validation cases*

In this table, the free stream hub height wind speed (U_{hub}) was found from the 'free stream' SODAR measurements, i.e. the SODAR data which were recorded behind the non-operational wind turbine, just after the actual wake campaign. The value for U_{48} is measured at the Land Mast at a height of 48 m. The value for the turbulence intensity at hub height (I_{hub}) and the Monin-Obukhov Length scale have also been been found from the Land Mast measurements.

7.3 SODAR: Simulation of experiments

The input data for the SODAR simulations were provided by the Robert Gordon University. Within the ENDOW project group it was decided to follow three different approaches in the calculations:

- Procedure 1:
  - Basic Assumptions:
    - Neutral atmosphere, hence L = ∞ (in practice a large value for L is prescribed);
    - z_0 from Charnock, i.e.

$$z_0 = 0.018 \cdot u^{*2}/g \tag{7.1}$$

    - u^* is calculated, such that U_{48} is obtained (see table 7.1: U_{48} is the measured velocity of the Land Mast at a height of 48 m):

$$U_{48} = 2.5 \cdot u^* \cdot \ln(48/z_0) \tag{7.2}$$

      Hence (using z_0 from equation 7.1):

$$U_{48} = 25.43u^* - 5 \cdot u^* \cdot \ln(u^*) \tag{7.3}$$

      From equation 7.3, a value for u^* can be derived such that the Land Mast measured value of U_{48} is obtained.

  - The resulting values of u^*, z_0 and L form sufficient input for the WAKEFARM free stream model;
  - It is noted that the present procedure, hardly uses any measured data: Although a measured velocity at h = 48 m is used, this measurement value is determined from the Land Mast, which is some distance away from the turbine position. The resulting velocity profiles, and the turbulence intensity do not rely on any measured data at all. As an example: The turbulence intensity (for neutral atmosphere) at h = 48 m follows from equation 2.11:

$$I = 2.4\frac{u^*}{U_{48}} = \frac{2.4}{25.43 - 5 \cdot \ln(u^*)} \tag{7.4}$$

This turbulence intensity does not necessarily correspond to the measured value (in practice it is close to 7 %).
The main motivation for applying the present procedure is given by the fact that it resembles a 'blind' procedure. Such blind procedure needs to be followed in the design tool (one of ENDOW's deliverables), when no measurement data are available.

- Procedure 2:
  - Basic assumptions:
    - Wind speed at hub height from U_{hub} (see table 7.1, U_{hub} is measured with the SODAR);
    - Turbulence intensity at hub height from I_{hub} (see table 7.1, I_{hub} is measured with the Land Mast);
    - u^* follows from the turbulence intensity (assuming a neutral atmosphere), see equation 2.11:

$$I(h) = 2.4 \frac{u^*}{U_{hub}} \tag{7.5}$$

    - z_0 and the Monin-Obukhov Length scale are found by a fit to the Sodar wind speed profile (equation 2.1).
  - The resulting values for u^*, z_0 and L, form the input for the WAKEFARM free stream model;
  - The present procedure can be considered as a best fit to the measured data: The wind speed profile follows the SODAR measured velocity profile closely and the turbulence intensity agrees with the measured value at the Land Mast.
- Procedure 3:
  - Basic assumptions:
    - Wind speed at hub height from U_{hub} (see table 7.1, U_{hub} is measured with the SODAR);
    - Turbulence intensity at hub height from I_{hub} (see table 7.1, I_{hub} is measured with the Land Mast);
    - Monin-Obukhuv length scale from table 7.1 (i.e. measured with the Land Mast);
    - The values for u^*, z_0 are determined by the above mentioned wind speed and turbulence intensities, using the Monin Obukhov Length from the Land Mast and equations 2.1, 2.7 and 2.8;
  - The resulting values for u^*, z_0 and L, form the input for the WAKEFARM free stream model;
  - To some extent the present procedure makes the maximum use of measured data. Nevertheless the resulting velocity profile may differ from the measured profile. This is due to the fact that the Monin Obukhov Length scale from the Land Mast is used, which is some distance away from the wind farm.

It is recalled that the free stream hub height wind speed which is used for procedure 2 and 3 has been measured 30 minutes after the actual wake measurements were performed.

Apart from the free stream input parameters, WAKEFARM also requires the thrust and some geometrical input data. Thereto RISØ specified the thrust($C_{D.ax}$) values for the particular cases. ECN did not use these values but derived the $C_{D.ax}$ values in the same way as for the previous validation cases on the Vindeby wind farm, see the sections 4.2 and 6.2. Thereto the $C_{D.ax}$ is found by interpolation of U_{hub} in the $C_{D.ax}$ curve, The resulting values for $C_{D.ax}$ are only slightly different from the values specified by RISØ.

The geometrical input data (turbine data, farm lay-out) are described in section 4.2.1.

Calculations have been performed with two versions of the WAKEFARM model:
- The original version with a 'hat shaped' near wake profile and wake anisotropy which is assumed to be similar to the ambient anisotropy (see section 2);
- A modified version with the 'Gaussian shaped' near wake profile (section 3.1), but the wake anisotropy is still assumed to be similar to the ambient anisotropy.

7.4 SODAR: Comparison between calculational and measured results

7.4.1 SODAR: Comparison between calculational and measured results, Procedure 1

In the figures 7.1 and 7.2 the comparison between calculated and measured vertical wind speed profiles is presented, where the various cases are calculated according to procedure 1, i.e. the procedure which uses the wind speed from the Land Mast and which assumes neutral atmosphere.

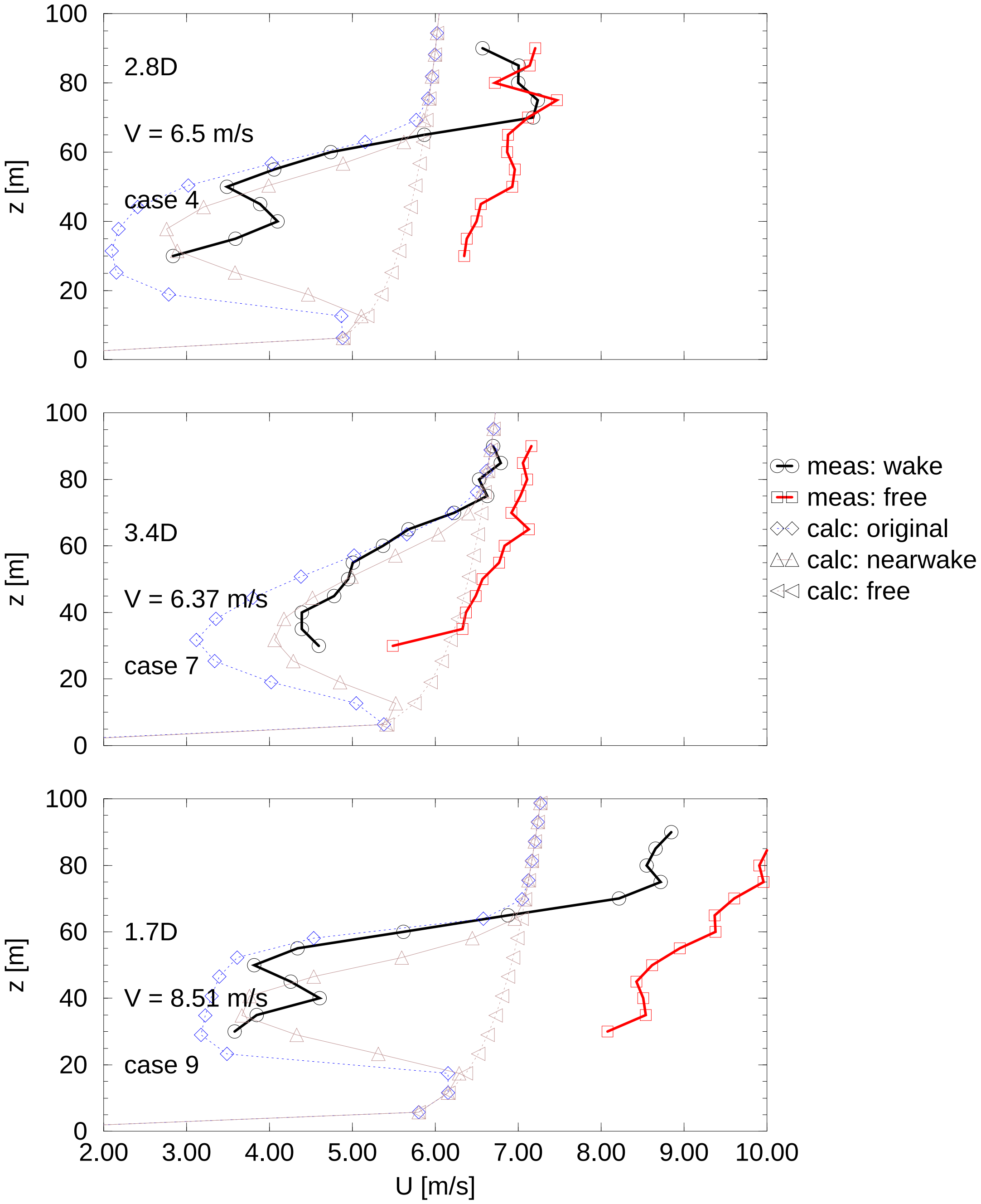

Figure 7.1 *Vindeby, Procedure 1: Single wake SODAR data: Comparison between calculations and SODAR experiments, (1)*

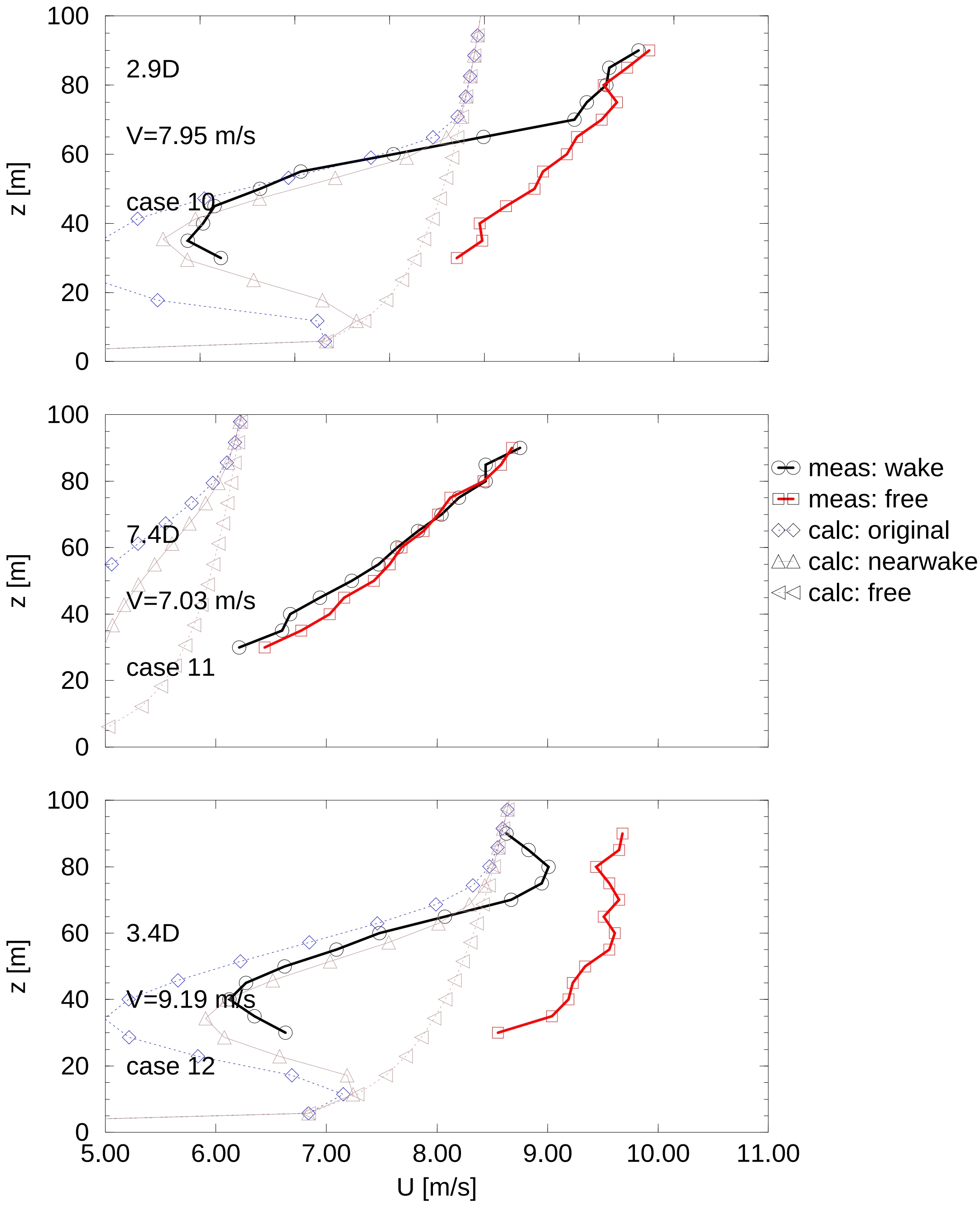

Figure 7.2 *Vindeby, Procedure 1: Single wake SODAR data: Comparison between calculations and SODAR experiments (2)*

7.4.2 SODAR: Comparison between calculational and measured results, Procedure 2

In the figures 7.3 and 7.4 the comparison between calculated and measured vertical wind speed profiles is presented, where the various cases are calculated according to procedure 2, i.e. the procedure which uses the tuned free stream profile and the turbulence intensity from the Land Mast.

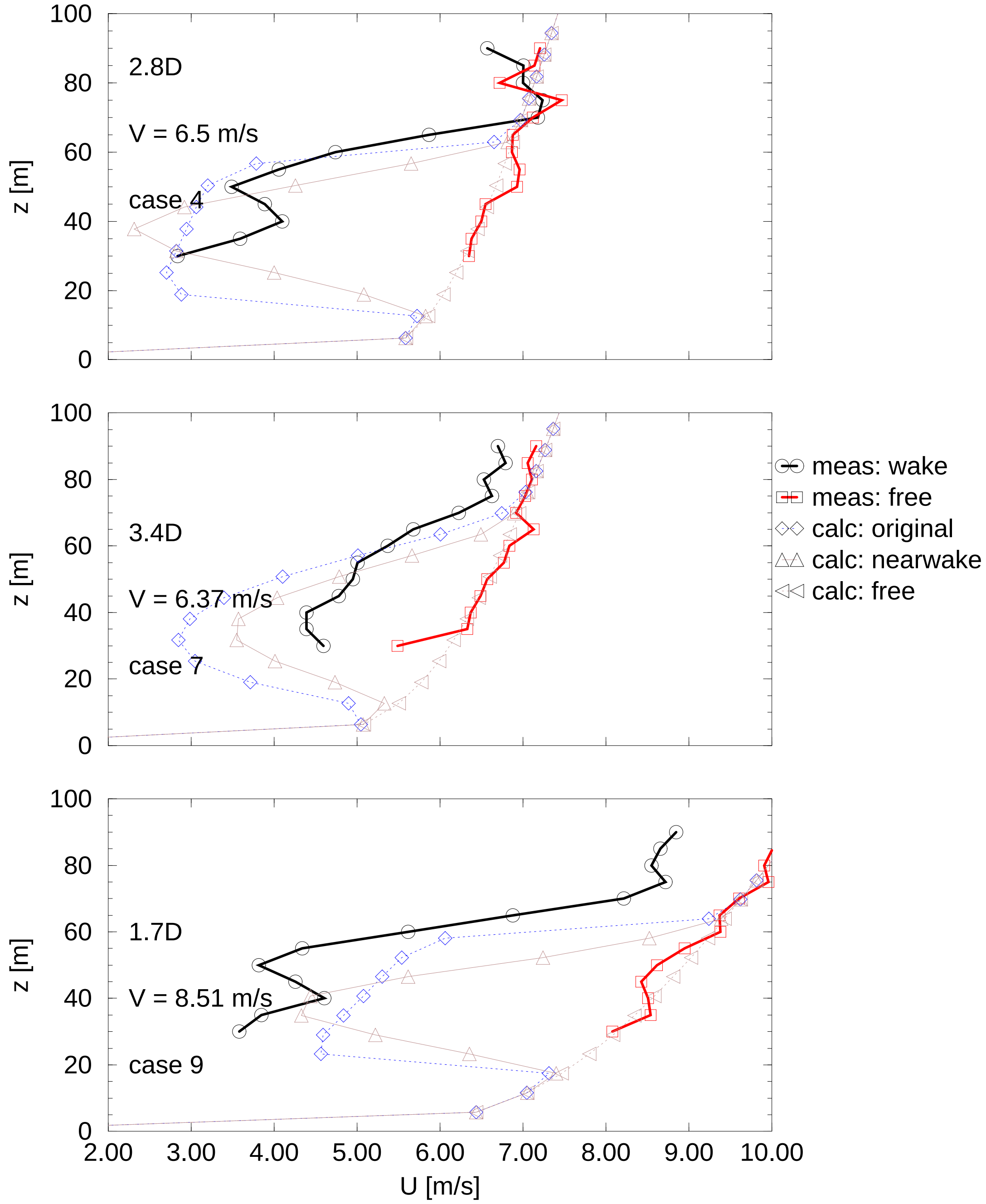

Figure 7.3 *Vindeby, Procedure 2: Single wake SODAR data: Comparison between calculations and SODAR experiments (1)*

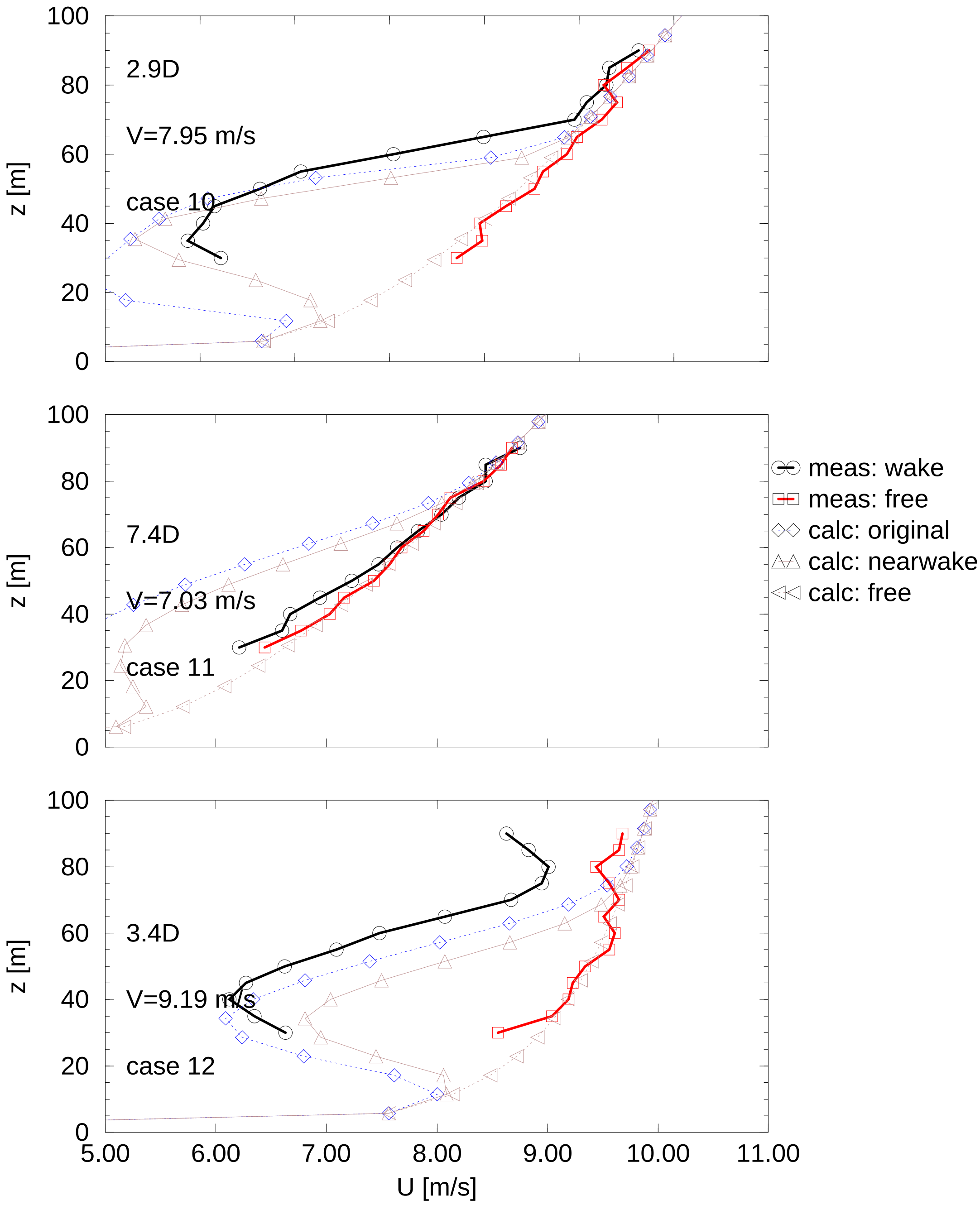

Figure 7.4 *Vindeby, Procedure 2: Single wake SODAR data: Comparison between calculations and SODAR experiments (2)*

7.4.3 SODAR: Comparison between calculational and measured results, Procedure 3

In the figures 7.5 and 7.6 the comparison between calculated and measured vertical wind speed profiles is presented, where the various cases are calculated according to procedure 3, i.e. the procedure which uses the Monin-Obukhov length scale from the Land Mast.

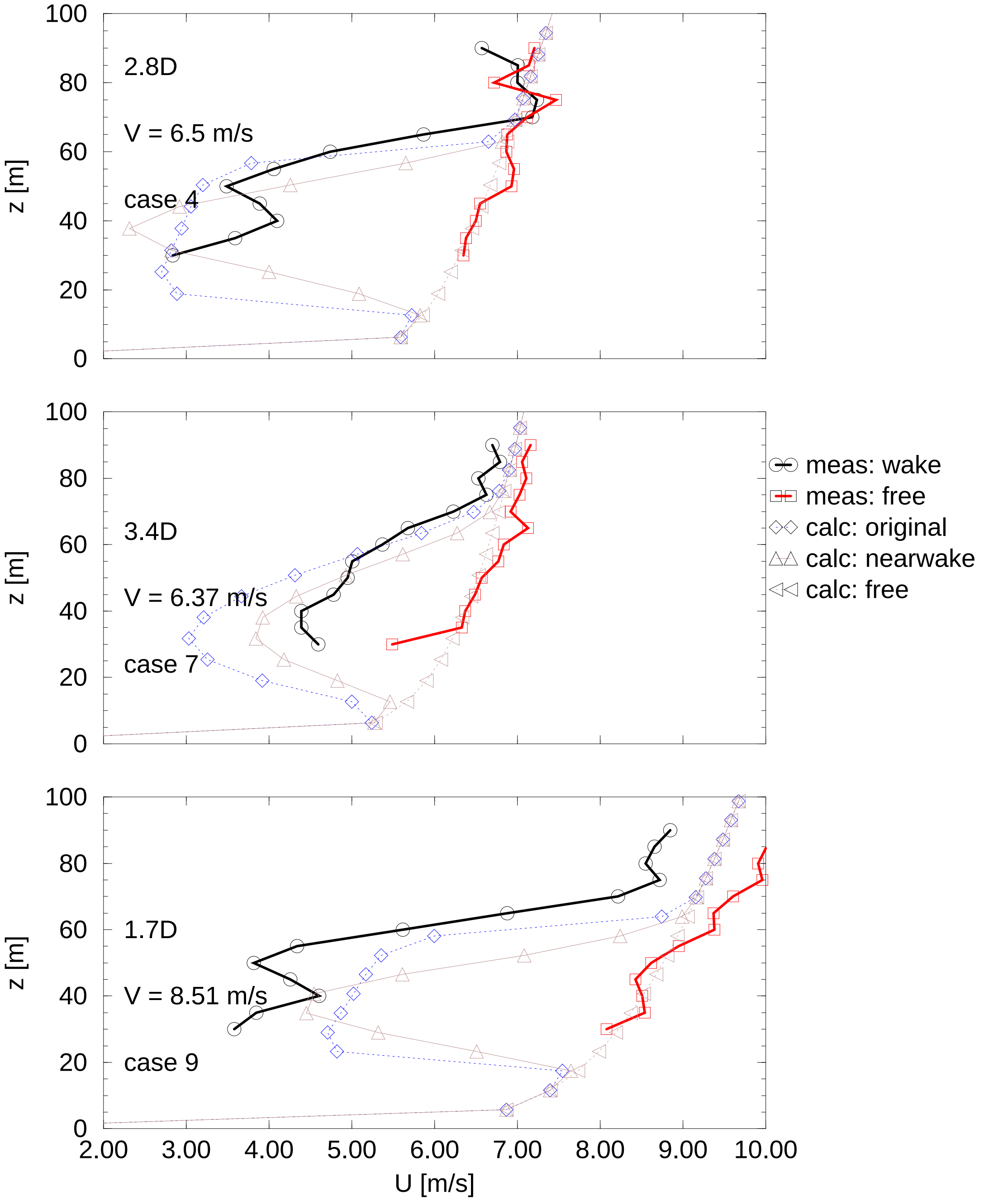

Figure 7.5 *Vindeby, Procedure 3: Single wake SODAR data: Comparison between calculations and SODAR experiments (1)*

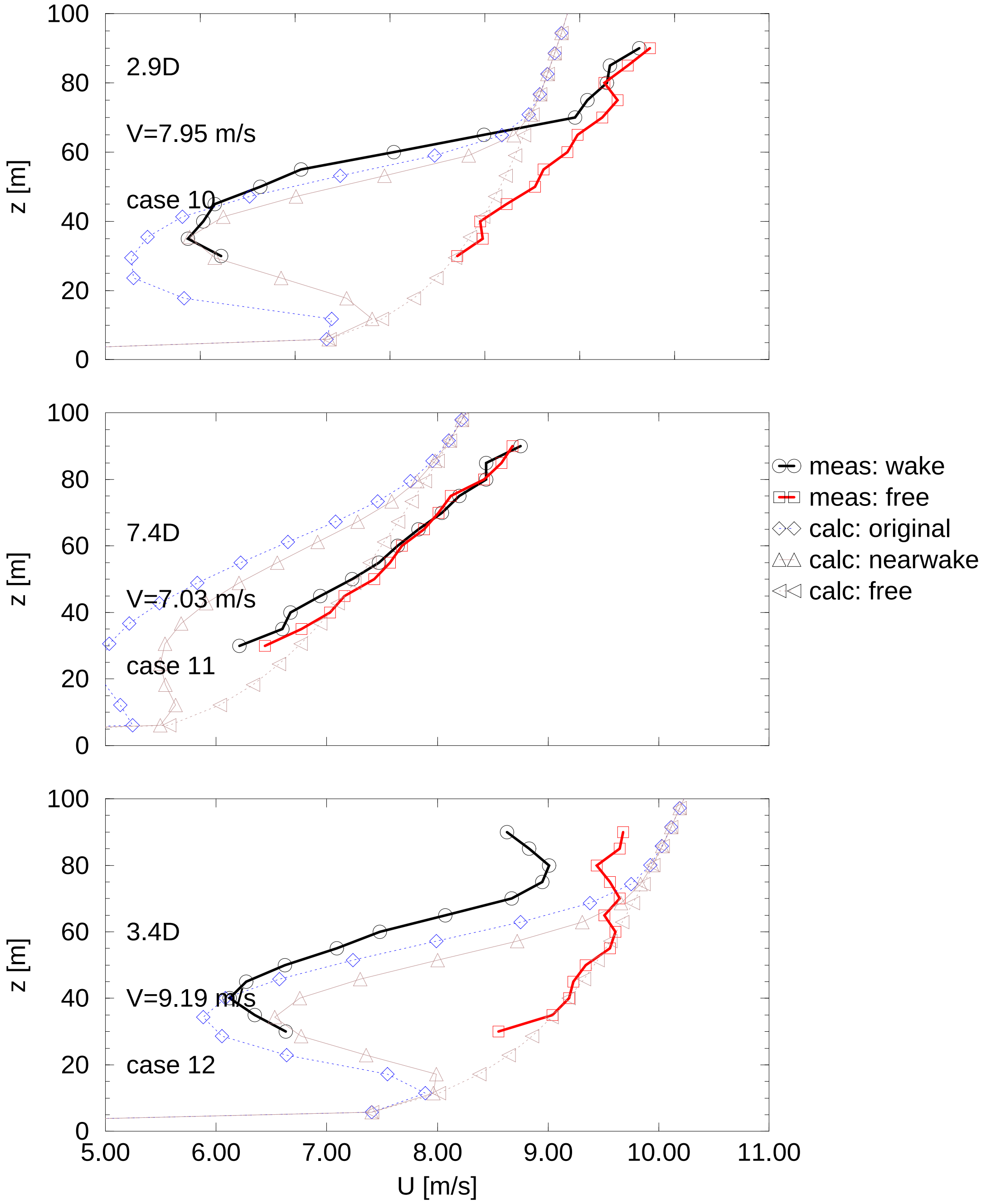

Figure 7.6 *Vindeby, Procedure 3: Single wake SODAR data: Comparison between calculations and SODAR experiments (2)*

7.4.4 SODAR: Comparison between calculational and measured results, Discussion of results

The most important comments which can be given to the results are:

- The comparison from procedure 1 is heavily obscured by the large differences between calculated and measured free stream profile. This is already apparent from table 7.1: This table shows considerable differences between the Land Mast measurement of U_{48} (which forms the basis for the WAKEFARM input of procedure 1) and the actual SODAR measurement of U_{hub}. It is noted that these differences are far too large to be attributed to the different heights at which U_{48} and U_{hub} are measured. Hence the comparison of procedure 1 has limited value for the purpose of validating WAKEFARM. Nevertheless the comparison learns that the land conditions differ considerably from the farm conditions.
- To a smaller extent the same observations can be made for the results of procedure 3. The use of the Monin-Obukhov length scale from the Land Mast yields a free stream wind profile which differs considerably from the actual profile. This is in particular true for the cases 7, 9, 10 and 11. This observation is consistent with the comparison of the Monin-Obukhov lenghts in the table below. The table compares the Monin-Obukhov length scales from the Land Mast (L_{LM}, prescribed for procedure 3) with the Monin-Obukhov length scales from RGU's best fit (L_{fit}, prescribed for procedure 2). For the cases 7,9,10 and 11 large differences are found, where the differences are limited for case 4 and for case 12 (note that both $L = \infty$ and $L = 990$ m can be considered to be a neutral atmosphere).

Case number	L_{fit} proc. 2 [m]	L_{LM} proc. 3 [m]
4	129	130
7	261	668
9	158	380
10	162	1103
11	115	231
12	∞	990

- Then the following remarks mainly refer to the results of procedure 2:
  - In the cases 7, 9 and 12, the velocities at the edge of the wake (say at h = 80m) do not approach the free stream values. This can be caused by the fact that the free stream measurements and the wake measurements are not performed at the same time but in subsequent campaigns. Of course the measurement uncertainty may play a role as well;
  - The most interesting observations are definitely found in the near wake cases 4 and 9: The SODAR measurements show a local maximum of the wake velocity near $z \approx z_{hub}$ and a local minimum near $z \approx 50$ m. Hence, the present measurements indicate a 'double dip' wake profile. The following comments can be made on this behaviour:
    * The double dip behaviour conflicts with the Gaussian shape which was assumed in the the near wake model from section 3.1. In such Gaussian profile the lowest wake velocity is found at hub height.
    * Other near wake models (i.e. the conventional 'potential core' model) also show a minimum wake velocity at hub height. As such, these models conflict with the present measurement results as well;

  - ∗ The ’double dip’ reflects the load variation over the blade: The loading and the resulting wake defect is expected to be maximum near 2/3 blade span and approaches zero at the hub and the tip. Hence the wake velocity is expected to be minimum at 2/3 blade span and increases towards the tip and the hub;
  - ∗ ECN’s near wake model is based on a rotor disc averaged value of the thrust and does not account for the variation in load over the blade. As such a Gaussian profile is modelled and the double dip behaviour is not taken into account;
  - ∗ Some more detailed models have been able to predict (at least part) of the present near wake structure:
    - · RISØ performed some calculations, using a coupled general CFD code and an aeroelastic wind turbine model. These calculations show a characteristic "double dip", see figure 7.7. This is consistent with the measurement results of the cases 4 and case 9.
    - · Some recent CFD results of the near wake structure have been published in [16]. These near wake profiles are partly comparable to the present results: A local minimum in wake velocity is found near 2/3R blade span, in agreement with the present results. However, opposite to the present results, another minimum is found near the hub. This minimum was explained by the large drag forces on the inner part of the blade.
  - ∗ It is remarkable to see that at case 10 (at 2.9 D) the double dip is hardly visible, even though the distance to the turbine is almost similar to the distance in case 4; The difference may be caused by the much higher ambient turbulence level at case 10. This indicates that the empirical near wake model should depend on the ambient conditions: At a higher ambient turbulence level, the near wake profile at 2.25 D is more ’developed’;
  - ∗ The wake profile approaches a ’normal’ Gaussian shape at larger wake distances. This is consistent with the calculations from figure 7.7.
- – Opposite to the previous validation cases, which are described in section 6.1 and section 6.2, the present results indicate only a slight improvement from the Gaussian near wake profile. This is somewhat difficult to believe, since many of the previous validation cases have been performed on the same wind farm at more or less similar conditions. However, a firm conclusion on this subject is difficult to made, because of the larger measurement uncertainty. It is recalled that these uncertainites are among others due to the fact that the wake and free stream measurements are not performed simultaneously. Furthermore, the validation cases of section 6.2 are done for far wake situations, where most of the present results refer to near wake situations;
- – Finally it is remarkable to see, that in case 11 (7.4D) the measured wake deficit is very small, where the calculated wake deficit is much larger. This may be caused by wind direction fluctuations and the meandering of the wake.

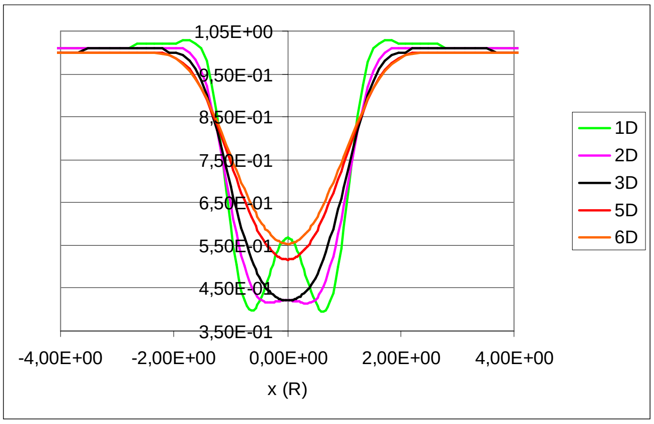

Figure 7.7 *Example of RISØ calculation showing 'double Gaussian' behaviour in the near wake)*

8. NUMERICAL ASPECTS OF WAKEFARM PROGRAM

For some of the calculations, which were reported in the previous chapters, no solution was produced by the WAKEFARM program, because of numerical problems. In most of these cases the basic problem turned out to be a convergence problem, which led to an infinite number of iterations. In other (more rare) cases, a so called 'core dump' could occur as well. A core dump usually implies a division by zero, an array index outside its bounds etc.

In order to get more insight into the cause of these numerical problems an inventory has been made of the cases which led to numerical problems. This was followed by a detailed analysis of the numerical approach in WAKEFARM (see [6]. In the present report the inventory is described and the analysis from [6] is summarized.

In the tables below a '+' denotes a case, at which a solution was found, a '-' denotes a case at which a convergence problem occurs, and a CD denotes a case at which a core dump occurred.

8.1 Inventory of cases at which numerical problems occur

8.1.1 Single wake cases

In none of the single wake calculations, which have been performed within the framework of ENDOW, see section 4, numerical problems occurred. Similar experiences are found from other projects, where convergence problems never occurred at single wake conditions.

However, in the sequel it will be shown that some multiple wake cases may suffer from convergence problems, in particular if the calculation is performed at free stream conditions which combine a low wind speed with a low turbulence intensity and a stable atmosphere.

For this reason it was attempted to define artificial single wake cases where the conditions were even more 'extreme', i.e. an even lower wind speed and turbulence intensity were prescribed in conjunction with a stable atmosphere. The calculations have been performed for the Vindeby wind farm.

Then two cases have been found in which numerical problems occurred. In both cases the Monin Obukhov Length scale was 200 m. In the first case the wind speed = 4 m/s and the turbulence intensity is 2.5%. In the second case the wind speed = 3 m/s and the turbulence intensity = 4.5 %. The core dump of the first case can be attributed to the very low roughness height which corresponds to a turbulence intensity of 2.5%. The core dump is then a result of a division through z, where z is extremely small, since the numerical grid starts at $z = z_0$

The second core dump is treated in section 8.2.

8.1.2 Multiple wake cases

Double wake cases
Within the framework of the ENDOW project, double wake cases have been calculated for a wind tunnel situation, see section 6.1 and for the Vindeby wind farm, see section 6.2. In all of these cases, convergence was reached.

Vindeby wind farm: Quintuple wake cases

In section 6.3 the quintuple wake cases are described which have been performed on the Vindeby wind farm. Table 6.3 shows that some quintuple cases suffer from convergence problems. For this reason the table is copied below: It shows that the original near wake model gave a solution for all quintuple wake cases. However the adjusted near wake model gives convergence problems at a wind speed of 5 m/s and a turbulence intensity of 6%. Furthermore a stable atmosphere, a wind speed of 5 m/s and a turbulence intensity of 8% yields convergence problems as well.

Hence, the results indicate that the modified near wake model is more sensitive to convergence problems. A combination of low wind speed, a low turbulence intensity and a stable atmosphere also seems to be a likely source for convergence problems.

Vw	turb	stability	orig	near wake	isotropic
5	6	neutral	+	-	-
5	8	neutral	+	+	+
5	6	stable	+	-	-
5	8	stable	+	-	-
5	6	unstable	+	+	+
5	8	unstable	+	+	+
7.5	6	neutral	+	+	+
7.5	8	neutral	+	+	+
7.5	6	stable	+	+	+
7.5	6	unstable	+	+	+
7.5	8	unstable	+	+	+
10.	6	neutral	+	+	+
10.	8	neutral	+	+	+
10.	6	unstable	+	+	+

Table 8.1 *Vindeby quintuple wake measurements: Convergence (+) or no convergence (-)*

Artificial wind farm: Quadruple wake cases

The table below shows the 'convergence status' of many WAKEFARM calculations, which have been performed within an internal ECN project.

U_{hub} (m/s)	4	6	8	10	12	14	16	18	20	25
I (%)										
5	—-	—-	—-	-579	-579	-579	3579	3579	3579	3579
7	—-	–79	3579	3579	-579	3579	3579	3579	3579	3579
9	—-	3579	3579	-579	3579	3579	3579	3579	3579	3579
11	—-	3579	3579	3579	3579	3579	3579	3579	3579	3579
13	-579	3579	3579	3579	3579	3579	3579	3579	3579	3579
15	-579	3579	3579	3579	3579	3579	3579	3579	3579	3579
17	—9	3579	3579	3579	3579	3579	3579	3579	3579	3579
19	CD	3579	3579	3579	3579	3579	3579	3579	3579	3579
21	CD	3579	3579	3579	3579	3579	3579	3579	3579	3579
30	CD	3579	3579	3579	3579	3579	3579	3579	3579	3579

Within this project, a large number of quadruple wake cases have been calculated for a range of wind speeds (4 to 25 m/s) and turbulence intensities (5 to 30%). Neutral atmosphere was assumed. The wake distances were 9D, 7D, 5D and 3D. A - indicates that somewhere in the quadruple wake calculational procedure, a convergence problem occurred. A ’CD’ denotes a core dump. The 3, 5, 7 and 9 indicate that the 3D, 5D, 7D and 9D quadruple wake calculation was completed succesfully.

The table shows that convergence problems mainly occur at a low wind speed in combination with a low turbulence intensity. A core dump mainly occurs at a low wind speed in combination with a high turbulence intensity. A small distance between the turbines, also yields more convergence problems.

8.1.3 Summary of inventory

The inventory described above shows that numerical problems are more likely at particular conditions: Convergence problems mainly occur:

- At multiple wake situations;
- At a low ambient wind speed;
- At a low ambient turbulence intensity;
- At stable atmosphere.

In addition, the WAKEFARM version with the modified near wake model seems to be more sensitive to convergence problems.

In the next section it is described that eventually it was possible to solve all convergence problems.

Finally it must be noted that even if the convergence problems could not be solved, they are of little importance from a practical point of view: Low wind speeds contribute little to energy production and fatigue damage. A practical solution would be to use the solution of the preceeding (converging) wake calculation, which is expected to differ little from the next (non-converging) wake result.

8.2 Analysis/solution of convergence problems

In section 2.3 the equations on which the WAKEFARM/UPMWAKE far wake model is based, are mentioned, but no full description of the equations and the numerical aspects is presented. This information can be found in [6], where a detailed analysis of the model has been performed, in order to understand and solve the convergence problems.

Without doubt, the most important result from [6] is the fact that eventually it was possible to reach a convergent solution for all of the cases which turned out to be a problem in the previous sections. This was among others the result of some modifications/corrections in the WAKEFARM model. These modifications often had a small influence on the outcome of WAKEFARM. In some cases the effect was more substantial. The corrected calculational results anyhow show less wake effects. Since in almost all validation cases, the wake effects are overpredicted, the corrections will improve the agreement with the measurements.

The numerical approach in the WAKEFARM model is based on the pressure correction technique in which the continuity equation is solved in conjunction with the three momentum equations. This is a method which basically determines the pressure field (and the corresponding wind speed components) in an iterative way. As a first guess the pressure field is estimated. This pressure field determines the velocity components through the momentum equations. Then the resulting velocity components are substituted into the continuity equation. Since the velocity components are obtained from guessed values the continuity equation is not necessarily satisfied. Therefore a new pressure correction is determined which is expected to bring the velocity field in closer agreement with the continuity equation and the whole process is repeated until convergence is found.

Now it has been possible, see again [6], to define a criterium, which determines whether or not convergence is reached. This criterium shows that increasing the local wake speed or decreasing dx, i.e. the grid size in x-direction, act positively on the convergence process. On the other hand, a low ambient wind speed in conjunction with a high loading (high initial wake defect) makes convergence less likely. The criterium also helps understanding why the modified near wake model is more sensitive to convergence problems: The lower minimum wake speed from the adjusted wake model act negatively on the convergence process.

The criterium offers insight how to solve possible convergence problems. The most straightforward solution appears to be a decrease of the grid size in x-direction.

Another way of reaching convergence is offered by neglecting the 4 least relevant equations from the WAKEFARM model. This implies that only 3 equations are solved:

- The momentum equation in x-direction
- The equation for the turbulent kinetic energy k
- The equation for the dissipation ϵ.

In this reduced WAKEFARM model, the pressure correction technique is not needed and numerical problems are less likely, where the most relevant wake properties, i.e. the wake wind speed in x-direction and the turbulence intensities in all three directions are still calculated. However in many cases, the reduced WAKEFARM model gives somewhat different results results for these wake properties than the elaborate '7 equation' WAKEFARM model.

Finally, it should be emphasized that numerical problems cannot be avoided in case of flow reversal in the wake. This can be understood by realising the parabolisation of the WAKEFARM model, which implies that only upstream influences are taken into account. This is obviously not an appropriate way of modelling backflow. Backflow in the wake is however very unlikely: In practice the turbulent wake model, see [17] prevents the model from these negative wake velocities. In the rare case of backflow, it is suggested to increase the local negative wake speed slightly to a positive value.

9. CONCLUSIONS AND RECOMMENDATIONS

- Within the ENDOW project, ECN spent most of its efforts on modelling and validating the near wake:
  - The modelling results from other partners and the SODAR measurements, see section 7.4.4, clearly showed that the previously assumed 'hat shaped profile' is incorrect;
  - Instead of the 'hat-shaped profile, a Gaussian profile has been implemented, see section 3.1;
  - The Gaussian profile usually improves the agreement with the measurements; Generally speaking, the modified near wake model still overpredicts the turbulence intensities, but to a smaller extent than previously, see chapter 4 and chapter 6;
  - The need of an empirical near wake model can be avoided by including all elliptic terms, similar to RGU's approach. However, from RGU's experience it is known that such procedure increases the computational effort enormously (3 seconds versus 12 hours, see [18]). Furthermore the 'elliptic results' have not shown a convincing improvement in accuracy yet (although there are also other slight differences between RGU's and ECN's model than the elliptic/parabolic approach). Hence, until now, there are no reasons to implement an elliptic procedure;
  - Although ECN's model has been pushed to its limits, some improvements are still expected to be possible:
    - The SODAR measurements, as well as calculations from other institutes indicate that a 'double dip' near wake profile, is more realistic, see section 7.4.4. This double dip reflects the load distribution along the upstream rotor. As such, the correct wake modelling, requires a more detailed aerodynamic modelling of the upstream turbine than previously assumed.

      Hence it is recommended to develop an empirical near wake model which takes into account the effect of the blade load distribution on the near wake profile;
    - The SODAR measurements indicate that the near wake profile depends on the ambient conditions (turbulence level, stability), see section 7.4.4. Taking into account such dependancy is expected to improve the agreement.
    - A further improvement can be expected from the intialisation of more quantities than the wake speeds alone. In particular the turbulent kinetic energy and the dissipation should be initialised as well. An estimate for these additional initialisations could be obtained from RGU's calculational results.

- Part of the differences between calculations and measurement were caused by the fact that the comparison was not fully 'fair'. This holds in particular for the inevatable averaging over the wind direction in the full scale environment, which should be reproduced as close as possible in the calculations. In almost all cases the inclusion of the wind direction fluctuations improved the agreement between calculations and measurements, see section 4.2.4;
- Generally speaking, the agreement between calculations and measurements is better for neutral/unstable conditions, high wind speeds and in the upper part

of the rotor plane, see the sections 4.2.3, 4.3.3, 6.2.3 and 6.3.3;

- Within the project some uncertainties had to be faced:
  - Inherent modelling uncertainties: It should be realised that a k-ϵ turbulence (like any other turbulence model) does not predict the '100% truth'. The relatively crude modelling of the near wake adds to these uncertainties;
  - The validation results should always be considered with care. This is partly caused by measurement uncertainties itself but also by uncertainties in the input: Inevatible uncertainties in thrust curve, ambient conditions (i.e. the instationarities and non-uniformities in wind speed and direction) makes a fair comparison difficult. These uncertainties are generally expected to increase differences between calculations and measurements. However some examples are known [19] where a good agreement between calculational and measured results turns out to be misleading; The good agreement was caused by model errors, which were compensated by input errors.
- Convergence problems in WAKEFARM mainly occurred at multiple wake conditions at low wind speeds. Eventually it was possible to solve these convergence problems, see chapter 8. Thereto a criterium has been defined which determines whether or not convergence is reached.

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EXECUTIVE SUMMARY

Within Europe many off-shore wind energy projects are planned and it is expected that several thousand megawatts will be installed in the first decade of the millennium.

In order to estimate the power production of such off-shore windfarms it is necessary to estimate the wake effects, i.e. the velocity decrease experienced by a wind turbine which is placed downstream of another wind turbine. Indications exist that these wake effects in off-shore conditions differ considerably from the wake effects on land. In particular it is expected that the wakes in off-shore conditions will be propagated over a larger distances. The likely result is that in order to optimise power output, offshore wind farms will require larger distances between rows than is common in design of onshore wind farms.

In order to gain insight into these effects, the EU 5th framework project 'EfficieNt Development of Offshore WindFarms', (ENDOW) was carried out. This project started on March 1 2000 and ended on March 1 2003. In this project ECN cooperated with the following partners:

- RISØ National Laboratory, Dept. of Wind Energy and Atmospheric Physics (Dk), Coordinator;
- Uppsala University, Dept. of Earth Sciences, Meteorology (S);
- Garrad Hassan and Partners (GH), (UK);
- Robert Gordon University (RGU), School of Mechanical and Offshore Engineering (UK);
- University of Oldenburg (OU), Dept. Of Energy and Semiconductor Research EHF, Faculty of Physics (FRG);
- Seas Distribution (Dk);
- Techwise A/S, (Dk);
- NEG Micon Project (Dk);
- ECOFYS, energy and environment (NL)

The main aim of the project is to validate, evaluate, enhance and interface wake and boundary-layer models for offshore utilisations.

In the present report the activities are described which are performed by ECN within the ENDOW project. All described activities are related to the validation and improvement of the wake model WAKEFARM. Note that another task of ECN was to perform SODAR measurements. These activities are described in [15].

WAKEFARM

The WAKEFARM program models the wake flow downstream of a turbine, using the basic flow field parameters and some wind turbine properties as input. Output properties from the model are among others the wake wind speeds and the turbulence intensities in the wake.

The WAKEFARM model is based on the UPMWAKE model, which has been developed by the Universidad Polytecnica de Madrid. Although some modifications have been made to the UPMWAKE program, these modifications are relatively minor and the basic modelling can be considered similar for both programs.

The UPMWAKE/WAKEFARM model is a 3D Parabolized Navier Stokes Code

using a k-ϵ turbulence model which accounts for the turbulent processes in the far wake. The parabolisation yields an efficient calculational procedure, which requires the near wake to be initialised with an (empirical) velocity profile. A true physical modelling of the near wake would require a full elliptic, time consuming approach.
A comparison with calculational results from Robert Gordon University offered much insight into the effects of the initialisation. The difference between ECN's and RGU's model lies mainly in the fact that RGU uses a full elliptic, and as such a more physical approach. On the other hand, the calculational time for a single wake calculation with RGU's model is in the order of 12 hours, where ECN's model needs only 3 to 5 seconds.

The comparison with RGU's results led to the insight that ECN's model could be improved by changing the original initial near wake model, which was 'hat-shaped', into a Gaussian profile.

In addition the isotropy of the turbulence has been considered as an option to improve the agreement between calculated and measured turbulence intensities in the wake. The effect of these modifications is validated with many measurements.

Validation results

A comparison is made between calculational results from the WAKEFARM program and:
- Wind tunnel measurements, made by Garrad and Hassan;
- Full scale measurements from the Vindeby off-shore wind farm. A very interesting validation was offered by SODAR measurements which were taken in this farm. Thereto the SODAR measurement system was mounted on a boat. This made it very easy to perform measurements on different locations in the wake;
- Full scale measurements from the Alsvik wind farm (supplied by FFA);
- RGU's calculations (not described in this report, but they can be found in different ENDOW reports).

The validation showed that the model with the Gaussian profile yields, generally speaking, results which compare much better to the measurements. Nevertheless the turbulence intensities are still overpredicted, but to a smaller extent than previously. A further improvement in the prediction of turbulence intensity was possible by assuming the turbulence generated by the wake to be of a more isotropic kind. The SODAR measurements indicated that a 'double dip' near wake profile, is more realistic than the Gaussian profile. The double dip is a result of the load distribution along the blade. As such, the correct modelling of the wake requires that the aerodynamic behaviour of the upstream turbine is modelled in more detail than previously assumed.

Conclusions

- The modelling results from other partners and the SODAR measurements clearly showed that the previously assumed 'hat shaped profile' is incorrect;
- Instead of the 'hat-shaped profile, a Gaussian profile has been implemented;
- The Gaussian profile usually improves the agreement with the measurements; Generally speaking, the modified near wake model still overpredicts the turbulence intensities, but to a smaller extent than previously;

- The need of an empirical near wake model can be avoided by including all elliptic terms similar to RGU's approach. However, from RGU's experience it is known that such procedure increases the computational effort enormously. Furthermore the 'elliptic results' have not shown a convincing improvement in accuracy yet (although there are also other slight differences between RGU's and ECN's model than the elliptic/parabolic approach). Hence, until now, there are no reasons to implement an elliptic procedure;
- Although ECN's model has been pushed to its limits, some improvements are still expected to be possible:
  - The SODAR measurements, as well as calculations from other institutes indicate that a 'double dip' near wake profile, is more realistic. This double dip reflects the load distribution along the upstream rotor. As such, the correct modelling of the wake requires that the aerodynamic behaviour of the upstream turbine is modelled in more detail than previously assumed. Hence it is recommended to develop an empirical near wake model which takes into account the effect of the blade load distribution on the near wake profile;
  - The SODAR measurements indicate that the near wake profile depends on the ambient conditions (turbulence level, stability). Taking into account such dependancy is expected to improve the agreement.
  - A further improvement can be expected from the intialisation of more quantities than the wake speeds alone. In particular the turbulent kinetic energy and the dissipation should be initialised as well. An estimate for these additional initialisations could be obtained from RGU's calculational results.
- Part of the differences between calculations and measurement were caused by the fact that the comparison was not fully 'fair'. This holds in particular for the inevatable averaging over the wind direction in the full scale environment, which should be reproduced as close as possible in the calculations. In almost all cases the inclusion of the wind direction fluctuations improved the agreement between calculations and measurements;
- Within the project some uncertainties had to be faced:
  - Inherent modelling uncertainties: It should be realised that a k-ϵ turbulence (like any other turbulence model) does not predict the '100% truth'. The relatively crude modelling of the near wake adds to these uncertainties;
  - The validation results should always be considered with care. This is partly caused by measurement uncertainties itself but also by uncertainties in the input: Inevatible uncertainties in thrust curve, ambient conditions (i.e. the instationarities and non-uniformities in wind speed and direction) makes a fair comparison difficult.
- Convergence problems in WAKEFARM mainly occurred at multiple wake conditions at low wind speeds. Eventually it was possible to solve these convergence problems. Thereto a criterium has been defined which determines whether or not convergence is reached.